The High-Frequency Shape of the Source Spectrum for Earthquakes in Eastern and Western Canada

by Gail M. Atkinson

Abstract The high-frequency shape of the earthquake spectrum strongly influences the amplitude of the peak ground acceleration and of the response spectrum at frequencies of 10 Hz and greater. A key parameter for the description of high-frequency ground motions is "kappa," which is the decay slope of the Fourier spectrum of acceleration at near-source distances (Anderson and Hough, 1984; note Anderson and Hough originally referred to this parameter as kappa (0)). Kappa may be attributed to site effects ($f_{\text{max}}$, Hanks, 1982), source processes (Papageorgiou and Aki, 1983), or both.

Seismographic data place weak but significant constraints on kappa values. On average, there is no resolved kappa effect on spectra recorded at rock sites in eastern Canada, in the frequency range $f \leq 30$ Hz. Four firm-soil sites in southwestern Ontario also show no kappa effect. An implied upper bound for kappa is 0.004 (or lower bound of 30 Hz for $f_{\text{max}}$). By contrast, source spectra from earthquakes in the Cascadia region, recorded on hard-rock sites in southwestern British Columbia (B.C.), appear to be well described by a kappa of 0.011 ± 0.002. The B.C. spectra are thus intermediate to the eastern case, with zero apparent kappa, and the typical California case, for which kappa is about 0.04.

Introduction

High-frequency ground motions are controlled by the earthquake source spectrum, its attenuation with distance, and local site effects. For a reference “hard-rock” site condition, the general characteristics of the Fourier spectrum of ground acceleration are often described with reference to the Brune (1970) source model, accounting for its attenuation with distance using:

$$A(f) = C M_o (2\pi f)^2 \exp(-\pi R f (Q\beta))/(R[1 + (\pi f f_0)^2])$$

where $C$ is a constant; $M_o$ is seismic moment; $Q$ is the quality factor describing regional anelastic attenuation; $\beta$ is shearwave velocity; $R$ is hypocentral distance; and $f_0$ is the corner frequency of the spectrum, related to the seismic moment through the Brune stress parameter (e.g., Boore and Atkinson, 1987).

According to equation (1), the acceleration spectrum for a site near the source will be constant for frequencies greater than about two times the corner frequency [e.g., $A(f) = \text{constant}$ for $f \gg f_0$, when $\exp(-\pi R f (Q\beta)) \to 1$]. It has been observed, however, that for frequencies greater than about 5 to 10 Hz, spectral amplitudes decay as frequency increases, even at very short distances (Hanks, 1982; Anderson and Hough, 1984). This decay is often modeled by multiplying the right-hand side of equation (1) by a high-cut filter, $P(f)$.

This process is important because it controls the peak ground acceleration as well as response spectrum values at high frequencies (i.e., $>10$ Hz).

The high-cut filter process is often described by "kappa" (denoted $\kappa$), which is a decay slope imposed upon the Fourier spectrum of acceleration:

$$P(f) = \exp(-\pi \kappa f).$$

As used here, kappa is the zero-distance intercept of the parameter originally defined as kappa ($R$) by Anderson and Hough (1984). Anderson and Hough (1984) pointed out the exponential character of the earthquake spectrum at high frequencies and described it using a distance-dependent parameter kappa that included the effects of anelastic attenuation. In subsequent usage (and in this article), the parameter kappa has often been used to refer more specifically to the distance-independent high-frequency attenuation operator (kappa (0)), while the effects of anelastic attenuation are described separately through $Q$, as in equation (1).

For California ground-motion records on rock sites, Anderson and Hough (1984) reported an average kappa (0) value of 0.04. Boore et al. (1992) infer somewhat lower values, about 0.02.

Anderson and Hough’s description of high-frequency
spectral decay was proposed as an alternative to an earlier suggestion by Hanks (1982). Hanks noted that above some high-frequency cutoff, denoted \( f_{\text{max}} \), spectral amplitudes diminish rapidly. This behavior can be represented by a high-order Butterworth filter (Boore, 1986):

\[
P(f) = \left[ 1 + (f f_{\text{max}})^{8} \right]^{-1/2}.
\]

For California, typical \( f_{\text{max}} \) values are about 8 Hz (Boore et al., 1992). Both the kappa and \( f_{\text{max}} \) representations act to filter out high-frequency motions; the \( f_{\text{max}} \) filter is more abrupt than the kappa filter.

The physical origin of \( f_{\text{max}} \) or kappa is unresolved. Hanks (1982) suggested that it is a site effect, related to the competence of the near-surface materials. Papageorgiou and Aki (1983) have suggested that it is a source effect, related to the size of a minimum cohesive zone. It is also possible that kappa may have both source and site origins.

It is well known that ground motions in eastern North America (ENA) are enriched in high-frequency energy relative to those from California earthquakes at similar magnitudes and distances (e.g., Atkinson and Boore, 1990). This is largely due to firmly quantified differences in crustal anelastic attenuation. In addition, it is inferred that there may be significant regional differences in the processes responsible for kappa or \( f_{\text{max}} \). It has previously been suggested, based on very limited information, that ENA has \( f_{\text{max}} \) values of the order of 50 Hz (Boore and Atkinson, 1987) or kappa values of about 0.006 (EPRI, 1993), at least for rock sites.

This article examines empirical evidence concerning ENA values of kappa (or \( f_{\text{max}} \)) using seismographic data from rock and soil sites in southeastern Canada. The shape of the high-frequency spectrum in this region is compared with that derived from similar data in the Vancouver Island region.

Dataset and Analysis Method

The dataset consists of approximately 1200 recordings from the Eastern Canada Telemetered Network (ECTN), previously analyzed by Atkinson and Mereu (1992), supplemented by recent recordings from the new Southern Ontario Network (SON). The ECTN database has been described in detail in previous articles (Atkinson and Mereu, 1992; Atkinson, 1993a, 1993b) and therefore will not be discussed here. The ECTN records form a good data base for regional attenuation but can only reliably recover frequencies less than 20 Hz. Furthermore, most ECTN stations record only vertical-component data. The SON, operated since 1990 by the University of Western Ontario in cooperation with Ontario Hydro, consists of four stations recording 3-component data at 100 samples/sec (Mereu and Asmis, 1993). Frequencies of up to 30 Hz can be reliably recovered. Figure 1 shows the locations of the SON stations relative to the ECTN; events recorded by both the ECTN and SON are also shown. The SON stations are located in seismic vaults 3-m deep, embedded in firm soil (hard clay or till), whereas all ECTN stations except WEO (near station WLV of the SON) are on rock. The new SON stations and recent upgrades to several of the ECTN stations extend the recoverable frequency band and allow comparisons between rock and soil recordings and between horizontal and vertical components.

In this study, 18 recent earthquakes (1990 through 1994) recorded by both the ECTN and SON (Fig. 1) are analyzed. Data processing procedures follow those described by Atkinson and Mereu (1992), by which the Fourier spectra of the shear phases of the signal are obtained and corrected for instrument response and noise. The new Fourier spectra data are added to the existing ECTN data base. This provides an addition of approximately 300 new records to the 1200-record data base studied in 1992.

The regression analyses of Atkinson and Mereu (1992) were repeated for the enhanced 1500-record dataset. There were several reasons to repeat the regressions: (1) to verify the applicability of the previous attenuation results to this study; (2) to extend the attenuation results to higher frequencies; (3) to determine source parameters for the new events; and (4) to determine the site responses of the soil SON stations relative to average eastern rock conditions.

Another comparison of interest concerns the high-frequency spectral shape for eastern versus western earthquakes. The high-frequency spectral shape has been inves-
tigated for California earthquakes (Hanks, 1982; Anderson and Hough, 1984; Boore et al., 1992), but little information is available for other western regions, notably the Cascadia region of southwestern British Columbia and the northwestern United States. Seismographic data from the Western Canada Telemeetered Network (WCTN) have recently been analyzed (Atkinson, 1995), following the same techniques used to study the ECTN data. These analyses are referenced here to derive the high-frequency shape of WCTN spectra for comparison purposes.

Results of Analyses

Selected Regression Results

The attenuation, source terms, site terms, and durations derived from the new regressions are consistent with those found in the earlier studies (Atkinson and Mereu, 1992; Atkinson, 1993a, 1993b; Atkinson and Boore, 1995). This is not surprising, given the large degree of overlap in the datasets. Regression residuals for the new SON data appear unbiased over all distance and frequency ranges; thus, attenuation in the southern Ontario region appears to be consistent with the rest of the eastern region.

The $Q$ model quoted by Atkinson and Boore (1995) for frequencies less than 20 Hz appears to be applicable for frequencies up to 30 Hz. This model gives $Q = 680 f^{0.36}$, associated with a trilinear attenuation with apparent geometric spreading coefficients of 1.0, 0.0, and 0.5 for the distance ranges $R \leq 70$ km, $70 < R \leq 130$ km, and $R > 130$ km, respectively. The differing apparent geometric spreading rates in different distance ranges account for the transition from direct waves to the $Lg$ phase. As discussed by Atkinson (1989), Frankel et al. (1990), and Atkinson and Mereu (1992), there are unresolved trade-offs between the anelastic and geometric attenuation terms. These lead to uncertainties in the near-source amplitudes (and hence in the high-frequency shape) of about 0.1 log units for most events (Atkinson and Mereu, 1992); this uncertainty is relatively low due to the good distribution of data with distance.

Table 1 lists the source parameters of the new events of $M \geq 3.5$. These were determined from the regression source terms of the vertical-component data, because they are so much more abundant than horizontal-component data. Source terms obtained from regressions of the horizontal-component data do not differ systematically from the vertical-component results, but they are less stable. Interestingly, Table 1 suggests the occurrence of two new “high-stress” earthquakes (i.e., equivalent Brune stress parameter much higher than the regional average of 180 bars), similar to the 1988 Saguenay and 1990 Mont Laurier events.

Figures 2 and 3 show the inferred site responses of the firm-soil SON sites, for vertical and horizontal components, respectively. These responses are, by definition, relative to an average level of zero log units for the portfolio of 30 or so hard-rock ECTN sites. The responses shown on the figures were obtained from the site terms of independent regressions (but with the same attenuation) of the vertical- and horizontal-component datasets, for the shear-wave phases. Regressions of the $P$-wave phases (not shown) yielded similar results.

Response of the SON stations to the vertical component of ground motion is apparently not significantly different from rock; site terms are generally within one standard deviation of the average level of zero obtained for the rock sites. The horizontal responses, by contrast, tend to be more peaked. The Tyneside (TYN) and St. Catherine’s (STC) stations both have a resonant peak near 4 Hz, which could be interpreted in terms of the thickness of the soil layer and its shear-wave velocity, if these parameters were known. Station TYN is founded on hard overconsolidated clay; STC is founded on hard clay or till (Mereu and Asmis, 1993). The Wesleyville (WLV) station, founded on hard clay, appears to amplify 20-Hz motions. The Acton (ACT) station, which is believed to be on a very thin hard till layer over sedimen-

Table 1

<table>
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<th>Date</th>
<th>Location</th>
<th>MN M m f, (Hz)</th>
<th>Stress (bars)</th>
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<td>72.59 4.5 3.9 3.7 4.3 101</td>
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</tbody>
</table>

MN = Nuttli magnitude.
$M =$ moment magnitude.
$m =$ high-frequency magnitude (Atkinson and Hanks, 1995).
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Figure 3. Site response for SON soil stations relative to ECTN average rock response: horizontal-component data. Solid lines show the standard deviation of response terms for rock stations.

Figure 4. Horizontal-to-vertical component ratio for SON soil sites. Solid line shows mean H/V ratio for rock sites.

tary rock (Mereu, personal comm., 1995), shows a modest amplification of frequencies near 10 Hz.

There is ambiguity in the interpretation of the horizontal site-response terms at high frequencies due to limitations of the dataset. The only horizontal-component rock records are from the stations in the Charlevoix area. The Charlevoix stations cannot recover frequencies above 10 Hz, resulting in a loss of the rock reference level at high frequencies. Consequently, the horizontal-component SON site-response terms for $f > 10$ Hz are relative only to each other, not to rock. This means that any horizontal site-response features for the SON stations at $f > 10$ Hz will appear in the source terms. The vertical-component site-response terms are better constrained because there are rock stations against which to compare them at all frequencies.

The different shape of the site responses for the vertical and horizontal components for the SON sites also manifests itself in the horizontal/vertical (H/V) component ratios obtained for the SON data, shown in Figure 4. The ratios shown are obtained directly from the recorded spectral amplitude data for each station; inferred ratios obtained by division of site terms from the regressions are similar. As a result of peaks in the horizontal-component responses for the soil sites, compared to the relatively flat vertical-component response, the H/V ratios are peaked. Consequently, the H/V ratios of the SON sites are poorly described by the average H/V relationship obtained by Atkinson (1993b) for ECTN rock sites. This supports the conclusions of previous studies (Gupta and McLaughlin, 1987; Gupta et al., 1989), which have found that H/V is a site-specific parameter for soil sites.

High-Frequency Spectral Shape in ENA

I now examine the evidence that the data provide regarding the high-frequency shape of the Fourier spectrum at near-source distances. The most straightforward way to do this is to correct the recorded Fourier spectra for the effects of anelastic attenuation ($Q$), to obtain the near-source shape of each record. Several restrictive criteria are applied to the data used for this purpose. First, to avoid having the importance of the $Q$ correction on the shape overwhelm the underlying source/site effects that I would like to examine, I restrict the investigation to data within 300 km of the source. For this distance range, uncertainty in $Q$ for the trilinear attenuation model (approximately ±100) implies uncertainty in near-source shape, as defined by the level of the $Q$-corrected 30-Hz spectral amplitude relative to the $Q$-corrected 5-Hz spectral amplitude, of less than 0.1 log units. Second, for each earthquake, I look only at frequencies greater than twice the event’s corner frequency, to ensure examination of only the high-frequency portion of the spectrum. Finally, I wish to consider only frequencies ≥4 Hz to simplify comparisons between vertical- and horizontal-component data. The H/V data from both ECTN records (Atkinson, 1993b) and strong-motion records from the Saguenay earthquake (Boore and Atkinson, 1992) support a nearly constant H/V ratio for $f > 3$ Hz: by restricting attention to this frequency range, it is reasonable to assume that no frequency-dependent H/V corrections are required.

The above criteria greatly restrict the number of records for study, particularly since the instruments impose upper frequency limits on each record (15 Hz for the ECTN and 32 Hz for the SON). For many small events, the corner frequency of the earthquake is too close to the instrument’s upper frequency limit, and the records thus contain no information on shape. In total, there are 65 vertical-component records and 12 horizontal-component records containing useful shape information.

The shapes of the spectra are generalized as follows. For each Fourier spectrum meeting the selection criteria, the
distance-dependent shape modification caused by anelastic attenuation is removed, by multiplying by \( \exp(\pi fR/(Q\beta)) \) (see equation 1). This is the only correction that alters the shape of the recorded spectrum. For each record, the spectral amplitudes are averaged over frequency bins 2 Hz in width; thus, amplitudes are obtained for \( f = 4 \pm 1 \text{ Hz}, 6 \pm 1 \text{ Hz}, \) etc. Each spectrum is then normalized to a common reference amplitude level. This normalization shifts the spectrum up or down in amplitude but does not change its shape.

The normalization is based on the high-frequency level of the record. It is a cumulative process, working as illustrated schematically on Figure 5. It begins with the subset of data containing the largest events, for which \( f_0 \leq 2 \text{ Hz} \). For this subset, the high-frequency shape of the spectrum may be examined using the frequency range \( f \geq 4 \text{ Hz} \). Spectral amplitudes of all records in this first subset are normalized to a reference high-frequency amplitude level of zero at 4 Hz \( (A_4 = 0) \). The average normalized amplitudes of the spectra in this subset at the next frequency, \( f = 6 \text{ Hz} \), denoted \( A_6 \), may then be somewhat less than zero due to the average effects of kappa. Therefore, \( A_6 \) is used as the reference level for the second subset of data. The second subset is comprised of events with \( 2 < f_0 \leq 3 \text{ Hz} \); for this subset, we may examine the high-frequency spectrum using the frequency range \( f \geq 6 \text{ Hz} \). Similarly, the average normalized data of both the first and second subset are used to set the reference level \( (A_{10}) \) used for events of \( 3 < f_0 \leq 4 \text{ Hz} \), and so on. In this way, I ensure that any gradual decay of amplitudes with increasing frequency is not masked by “adding in” higher-frequency events at a zero reference level.

The underlying assumption that motivates this analysis is that the high-frequency shape may have some average regional characteristics, attributable either to gross regional source properties, or to the similarity of typical rock site conditions throughout the region. This assumption appears reasonable for most ECTN sites: the regression site terms do not exhibit the frequency-dependent effects that would be expected if kappa values differed from site to site. Two exceptions are the sites near Charlevoix and Sudbury, which will be discussed further later.

Figure 6 shows the mean normalized amplitudes, and their 90% confidence levels, for all vertical-component data, including both the ECTN stations and the SON soil stations. The slight amplification of amplitudes at high frequencies reflects the characteristics of some of the SON stations. As shown on Figure 7, this amplification disappears when only rock data are considered. Figure 8 shows the shape derived from the limited horizontal-component data; note that only small events recorded on the SON stations provided data meeting the selection criteria.

There are no discernible kappa or \( f_{\text{max}} \) effects in Figures 6 through 8. From this, I infer an upper bound of about 0.004 for kappa, which would correspond to a maximum spectral decay of about 0.2 log units for \( f = 30 \text{ Hz} \). Similarly, a lower bound of 30 Hz is inferred for \( f_{\text{max}} \), for typical eastern sites. Interestingly, the SON soil sites do not appear to attenuate high-frequency motions; if anything, they are amplified, at least on the vertical component. The depths of the soil columns underlying the SON stations are unknown; these responses suggest that the soils may be shallow.

High-Frequency Spectral Shape in Western Canada

To compare the eastern spectral shape with that obtained from similar instruments in western Canada, the above shape analyses were repeated using the WCTN data analyzed by Atkinson (1995). Only records within 200 km of well-re-
corded shallow crustal events were considered; with this constraint, the uncertainty in shape caused by the $Q$ corrections is comparable with the corresponding uncertainty for ENA (<0.1 log units for the trilinear attenuation model). There were 90 records meeting these criteria.

As shown on Figure 9, the high-frequency spectrum from the WCTN events decays with frequency in a manner that is consistent with a kappa value of $0.011 \pm 0.002$. This behavior is intermediate to that of the flat eastern spectrum and the California kappa of about 0.04 (Anderson and Hough, 1984).

Discussion

The kappa values obtained in this study provide some evidence, albeit weak, that there are regional differences in the average high-frequency spectral shape, even for hard-rock sites. Both the ECTN and WCTN stations are founded on hard rock, whereas the SON stations are on soil. For both the ECTN and WCTN sites, near-surface shear-wave velocities are believed to exceed 2 km/sec (Boore and Atkinson, 1987; Atkinson, 1995). This has not been verified by measurements at the seismograph sites, so it is possible that there are systematic differences in shallow shear-wave velocity structure between the sites in the two regions. Nevertheless, if kappa is entirely a site effect, I would have expected low kappa values for both ECTN and WCTN hard-rock data, with a significant kappa for the eastern SON soil sites. In contrast to this expectation, the rock WCTN kappa values differ markedly from the rock ECTN values, while the eastern soil sites behave very much like eastern rock sites.

On the other hand, the results of regression analyses of the ECTN data provide some support for a site-effect kappa for at least some stations. The site terms reported by Atkinson and Mereu (1992), and also by Boatwright (1994), indicate a high-frequency decay of amplitudes for stations in the Charlevoix and Sudbury regions, relative to the other rock sites. The site terms for these stations are consistent with kappa values in the range of 0.02 to 0.04, although the shape of the decay does not closely match the log-linear shape of the kappa model. Atkinson and Boore (1992) also noted that several of the strong-motion recordings of the Saguenay earthquake from stations in the Charlevoix region showed a pronounced $f_{max}$ or kappa effect, although other stations in the same region did not. Both Charlevoix and Sudbury are within the boundaries of large meteor-impact

Understanding the factors that influence the high-frequency shape of the spectrum is important to the prediction of high-frequency ground motion. For example, Boore and Atkinson (1987) investigated the sensitivity of peak ground acceleration to the shape of the high-frequency spectrum. They showed that for $f_{\text{max}} = 100 \text{ Hz}$, corresponding roughly to kappa = 0.002, near-source peak accelerations are twice as large as for the case of $f_{\text{max}} = 25 \text{ Hz}$, corresponding to kappa = 0.006. In view of this sensitivity, it would be highly desirable to measure the shear-wave velocity as a function of depth at all seismograph stations, even those founded on hard rock. This would allow meaningful interpretation of the origin of observed high-frequency spectral shapes.

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References


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