Some thoughts on relating density to velocity

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There are many occasions when density is needed for modeling of site amplification, but all that is available are values for shear-wave or compressional-wave velocity. For example, my program site_amp and Chuck Mueller's nrattle need input of shear-wave velocity and density. I've built in a default relation between the two in site_amp, but I've never been happy with it (it is shown in a figure below, and clearly my reservations were well founded), so these notes describe my attempt to replace that relation. I've made use of Tom Brocher's recent work (in particular, Brocher, 2005a, b).

Relating Density to Compressional-Wave Velocity:

A popular relation between density (ρ) and *P*-wave velocity (V_P) seems to be that of Gardner et al. (1974). The relation takes the following forms, depending on the units of V_P (in all cases the units of density are gm/cc):

ft/sec: $\rho = 0.23 V_P^{0.25}$ (1)

km/sec: $\rho = 1.74 V_P^{0.25}$ (2)

m/sec: $\rho = 0.31 V_P^{0.25}$ (3)

Their relation is simply an approximate average of the relations for a number of sedimentary rock types, weighted toward shales. The relation comes from Figure 1 in Gardner et al. (1974):

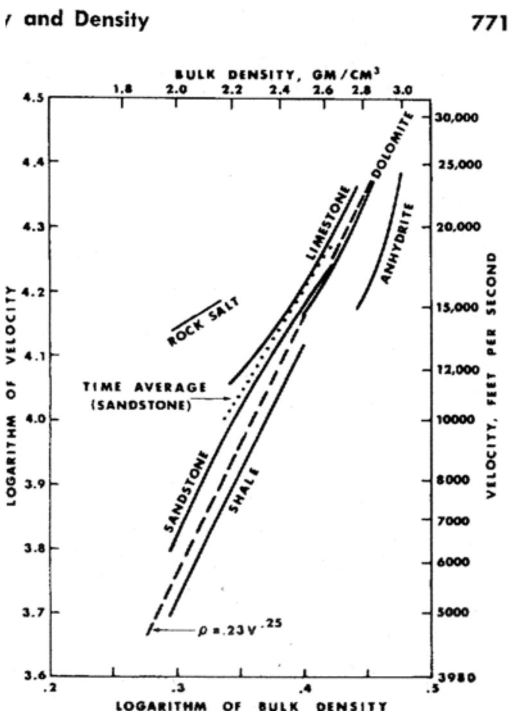

Figure 1. From Gardner et al. (1974)

Note that no data are shown and that the relation only applies for V_P above about 5000 ft/sec (1524 m/sec); consequently, there is no reason to think that the relation should hold for smaller values of V_P. Here is a plot of density and *P*-wave velocity, both measured in Quaternary sediments and from Gardner's relation. The values of V_P less than about 1500 m/sec are presumably in unsaturated sediments.

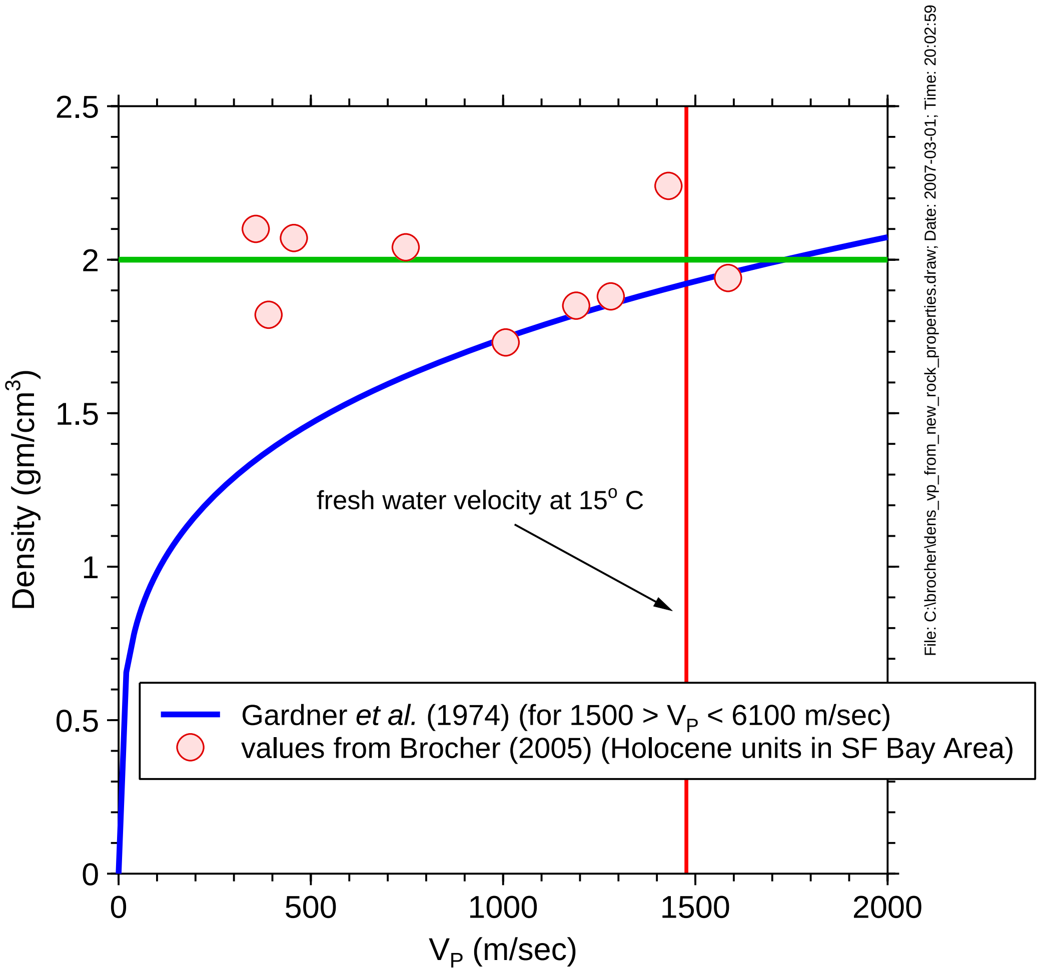

Figure 2. The vertical red line is the velocity of P-waves in fresh water at 15 degrees C (1477 m/s, Press, Table 9-6 in Clark, 1966); the velocity in salt water is above 1500 m/s.

The average densities of the Holocene units for $V_P < 1477\ \mathrm{m/s}$ is 2.0 gm/cc. The Gardner et al. (1974) relation clearly should not be used for unsaturated near-surface sediments (for which V_P is less than about 1500 m/sec). What values of density should be expected? The bulk density ($\bar{\rho}$) of a rock composed of solids and fluid-filled voids is:

$$\bar{\rho} = \rho_S(1-\phi) + \rho_V\phi \qquad (4)$$

where ρ_S, ρ_V, and ϕ are the densities of the solid material, the material filling the voids, and the porosity, respectively. For waterfilled voids, $\rho_V = 1$; for air-filled voids $\rho_V = 0$. Here are predicted bulk densities for a range of solid densities and porosities:

Table 1.

porosity	rho_saturated	rho_dry
0	2.65	2.65
0.1	2.49	2.39
0.2	2.32	2.12
0.3	2.16	1.86
0.4	1.99	1.59
0.5	1.83	1.33
0.6	1.66	1.06
0.7	1.50	0.80
0.8	1.33	0.53
0.9	1.17	0.27

where a value of $\rho_S = 2.65$ gm/cc was used ($\rho_S = 2.65$ for quartz and $\rho_S = 2.54 - 2.76$ for feldspars, according to Lambe and Whitman (1969, Table 3.1).

Here is a plot of the ratio of bulk density for dry and fully-saturated rocks, as a function of porosity.

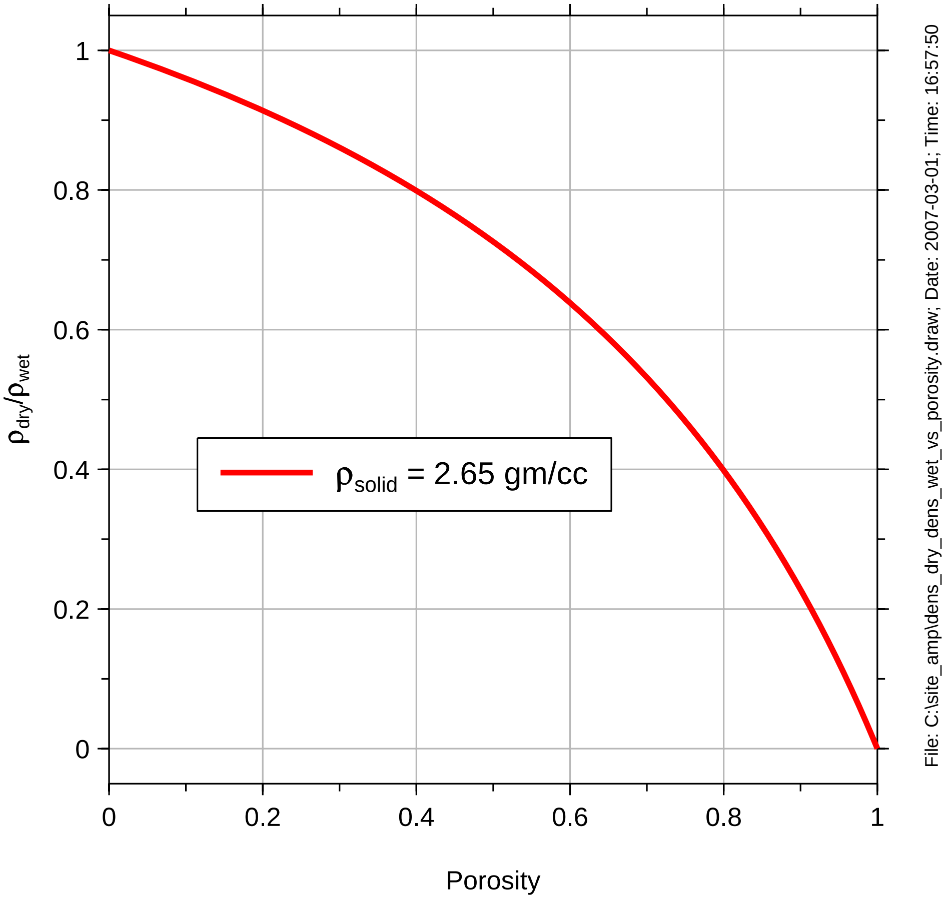

Figure 3.

So the important question is: “what is the porosity”? Porosity depends on material, consolidation, size distribution, and so on. For materials of most relevance, the porosities are generally less than 0.5 (Lambe and Whitman, 1969, Table 3.2).

According to the Wikipedia entry (http://en.wikipedia.org/wiki/Porosity):

Porosity of soil

Porosity of surface soil typically decreases as particle size increases. This is due to soil aggregate formation in finer textured surface soils when subject to soil biological processes. Aggregation involves particulate adhesion and higher resistance to compaction. Typical bulk density of sandy soil is between 1.5 and 1.7 g/cm^3. This calculates to a porosity between 0.43 and 0.36. Typical bulk density of clay soil is between 1.1 and 1.3 g/cm^3. This calculates to a porosity between 0.58 and 0.51. This seems counterintuitive because clay soils are termed *heavy*, implying *lower* porosity.

Heavy apparently refers to a gravitational moisture content effect in combination with terminology that harkens back to the relative force required to pull a tillage implement through the clayey soil at field moisture content as compared to sand.

Porosity of subsurface soil is lower than in surface soil due to compaction by gravity. Porosity of 0.20 is considered normal for unsorted gravel size material at depths below the biomantle. Porosity in finer material below the aggregating influence of pedogenesis can be expected to approximate this value.

Soil porosity is complex. Traditional models regard porosity as continuous. This fails to account for anomalous features and produces only approximate results. Furthermore it cannot help model the influence of environmental factors which affect pore geometry. A number of more complex models have been proposed, including fractals, bubble theory, cracking theory, Boolean grain process, packed sphere, and numerous other models.

Using the spreadsheet table above with porosities of 0.2 to 0.35 gives wet and dry densities in good agreement with the values in Figure 2.

Here is another plot of velocity vs density. The base is a graph I scanned out of a report by Nafe and Drake (I think) that I found on my computer. I've superimposed data from Brocher for Quaternary sediments (references indicated in the graph). There is a lot on this graph, because it is a working plot. Note that the Nafe and Drake plot included both *P*- and *S*-wave velocities. Let's look at the *P*-wave velocities first. Based on the graph below and the considerations above, I propose the following model relating density (units of gm/cc) and *P*-wave velocity (units of km/sec):

$$V_P < 1.50 \text{ km/sec:} \quad \rho = 1.93 \text{ gm/cm}^3 \qquad (5)$$

$$1.50 \text{ km/sec} \le V_P < 6.0 \text{ km/sec:} \quad \rho = 1.74 V_P^{0.25} \quad \text{(Gardner et al., 1974)} \qquad (6)$$

$$6.0 \text{ km/sec} \le V_P:$$

$$\rho = 1.6612 V_P - 0.4721 V_P^2 + 0.0671 V_P^3 - 0.0043 V_P^4 + 0.000106 V_P^5 \quad \text{(Brocher, 2005b, eq. 1)} \qquad (7)$$

I chose the Gardner et al. curve because it does a somewhat better job of fitting the Quaternary data than Nafe and Drake (as given by Brocher's eq. 1) for intermediate values of V_P. In addition, the Gardner et al. relation seems to be the standard in the exploration geophysics, where it is referred to as "Gardner's Rule" (Sheriff and Geldart, 1995). I chose the break point at $V_P = 1.5$ km/s because it is between the velocities in fresh and salt water and because it yields a density somewhat below 2.0 gm/cc (I think it is better to err slightly on the side of densities that are too small, as that will increase the amplification; in addition, many soils near the surface in dry materials probably have densities somewhat less than 2.0 gm/cc; coincidentally, I'm sure, the average density of the data in Figure 2 for velocities less than 1430 m/s is 1.93 gm/cc). Conversely, Brocher (2005b) does a better job of fitting Nafe and Drake for larger values of V_P (as it should).

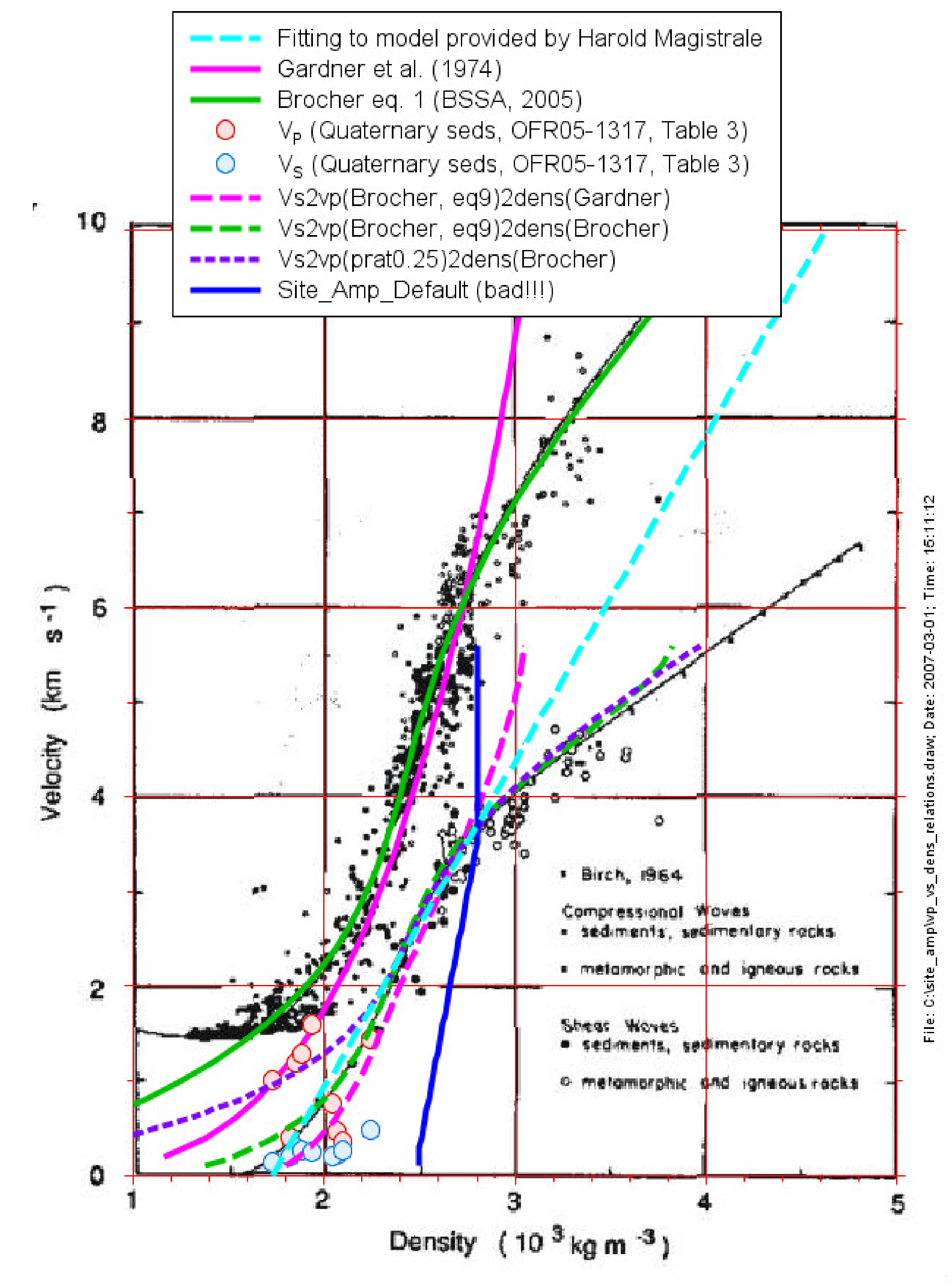
Fitting to model provided by Harold Magistrale
Gardner et al. (1974)
Brocher eq. 1 (BSSA, 2005)
V_P (Quaternary seds, OFR05-1317, Table 3)
V_S (Quaternary seds, OFR05-1317, Table 3)
Vs2vp(Brocher, eq9)2dens(Gardner)
Vs2vp(Brocher, eq9)2dens(Brocher)
Vs2vp(prat0.25)2dens(Brocher)
Site_Amp_Default (bad!!!)
Velocity (km s^{-1})
0
2
4
6
8
10
Density (10^3 kg m^{-3})
1
2
3
4
5
Birch, 1964
Compressional Waves
sediments, sedimentary rocks
metamorphic and igneous rocks
Shear Waves
sediments, sedimentary rocks
metamorphic and igneous rocks
File: C:\site_amp\vp_vs_dens_relations.draw; Date: 2007-03-01; Time: 15:11:12

Figure 4.

Relating Density to Shear-Wave Velocity:

At first thought it would seem that the relations given in equations (5), (6), and (7), in combination with equations relating V_S and V_P could be used to relate density and shear-wave velocity. Brocher (2005b, eq. 9) gives the following relation between V_S and V_P:

$$V_P\,(\text{km/sec}) = 0.9409 + 2.0947V_S - 0.8206V_S{}^2 + 0.2683V_S{}^3 - 0.0251V_S{}^4 \quad (8)$$

A potential problem is that near-surface (largely at depths less than 100 m) data indicate that the relation should be multivalued, because below the water table V_P is controlled by the water velocity and therefore the Poisson's ratio jumps to a large value. The multivalued relationship is shown in the graph below. The data come from velocity models fit to many borehole measurements, as given in Boore (2003). The data in the graph below were extracted from the results provided by Boore (2003) as follows: (1) copy the Poisson's ratio files for all boreholes into a folder "pr_out" (the program that produced these files subdivided layers if necessary so that both the *P*-wave and the *S*-wave velocity models had the same stack of constant velocity layers; this in general is not the case for the individual *P*-wave and *S*-wave model files); 2) use a Fortran program to read each Poisson's ratio file and write a summary file containing all *P*- and *S*-values at the same depths for all boreholes.

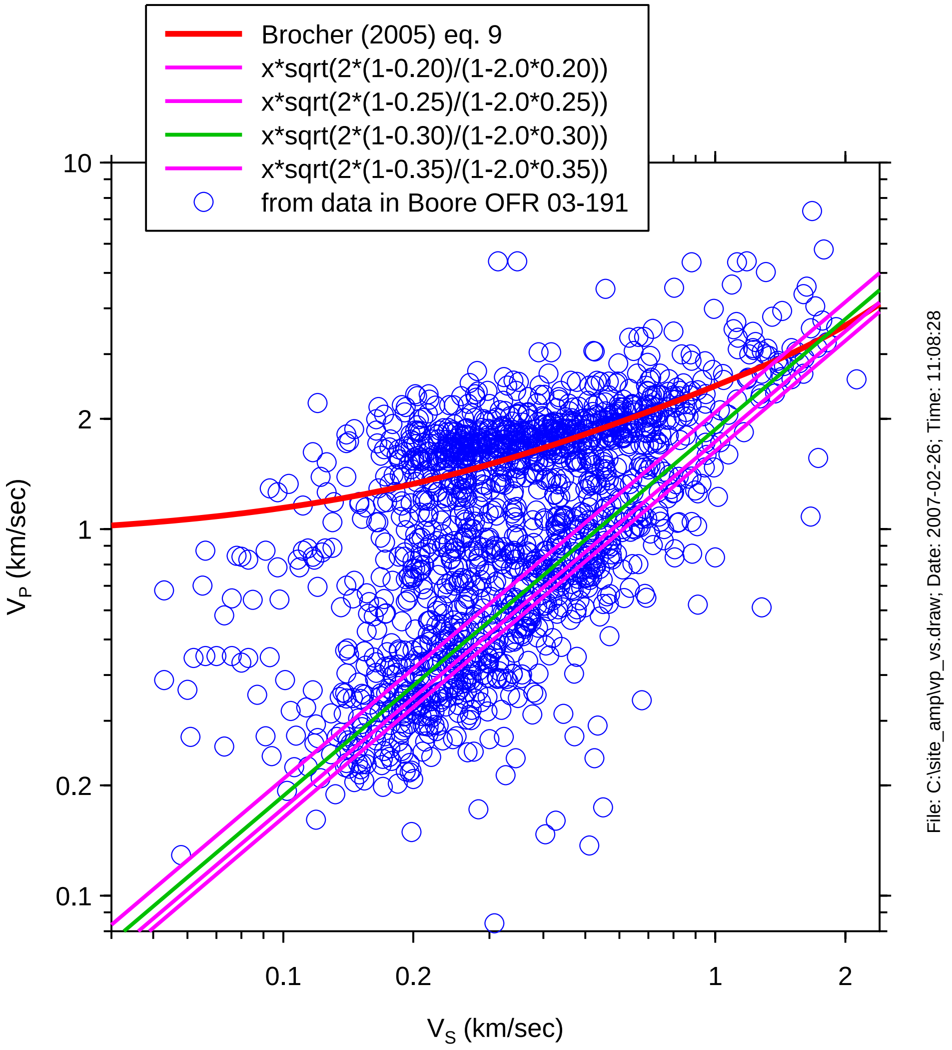

Figure 5.

The straight lines in the figure above assume Poisson ratio (σ) values of 0.20, 0.25, 0.30, and 0.35. The green line (for $\sigma = 0.30$) seems like a good representation of the average value.

I will ignore the complication due to the multivalued nature of the relation between V_S and V_P, which may be less important than it would first appear: values of density for soils with relatively low-shear wave velocities may be similar (and close to 2.0 gm/cc) for both dry and saturated soils. With this assumption, Figure 4 suggests that the following procedure, which is parallel to that proposed above for the $\rho - V_P$ correlation, gives a reasonable fit to the scant data:

$V_S < 0.30$ km/sec: ρ=1.93 gm/cm^3

$0.30\ \mathrm{km/s} < V_S < 3.55\ \mathrm{km/s}$: Use Brocher's (2005b) relation (equation 8) between *S*-wave velocity and *P*-wave velocity, in combination with Gardner et al.'s (1974) relation between *P*-wave velocity and density (equation 2). Note that using this procedure for *S*-wave velocities less than 0.3 km/s gives reasonable values of density (1.8 gm/cc for 0.1 km/s), but I've kept the break to a constant density to have an algorithm parallel to that for *P*-wave velocities.

$3.55\ \mathrm{km/s} \le V_S$: Use Brocher's (2005b) relation (equation 8) between *S*-wave velocity and *P*-wave velocity, in combination with Brocher's (2005b) relation between *P*-wave velocity and density (equation 7).

Acknowledgments

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Grant, F. S. and G. F. West (1965). *Interpretation Theory in Applied Geophysics*, McGraw-Hill Book Company, New York, NY, 584 pp. {pp. 192--200 contain an excellent discussion of rock densities, including graphs from Birch indicating that soils and alluvia have densities from about 1.65 to 2.2, with peaks in the distribution at 1.7 and 1.9 (the latter being larger); also included is a plot of the Nafe and Drake P-wave vs density data.}

Mavko, G., T. Mukerji, and J. Dvorkin (1998). The Rock Physics Handbook: Tools for Seismic Analysis in Porous Media, Cambridge University Press, Cambridge, UK, 329 pp. {contains a chapter on velocity—density relations; the chapter makes heavy use of Castagna et al., 1993---referenced above. Note that most of the figures in the Mavko book are available in a pdf file from http://pangea.stanford.edu/courses/gp262/Notes/16.avo.pdf.}

Santamarina, J. C., K. A. Klein, and M. A. Fam (2001). *Soils and Waves*, John Wiley & Sons, Ltd, Chichester, UK, 488 pp. {excellent discussion of porosity and wave speeds in particulate media}.