Notes to myself (D.M. Boore)

`aa_pa_rv_pv_2.tex`

April 26, 2001

# Comparison of $AA$, $PA$ and $RV$, $PV$ for $M_L = 4.5$ and $\mathbf{M} = 5.0$ and 7.1 earthquakes

## INTRODUCTION

These notes are an update of my March 23, 2001 notes (which were distributed to a few people) and were prompted by the question of what response spectral measure to use: pseudo relative velocity, relative velocity, pseudo absolute acceleration, or absolute acceleration. Ken Campbell told me that Prof. Yamazaki at Tokyo University "as well as others in Japan" use absolute acceleration and relative velocity, unlike what Ken and BJF use— pseudo absolute acceleration or pseudo relative velocity, derived from the relative displacement spectrum $SD$ (I do not know about Sadigh *et al.* or Abrahamson and Silva— papers giving ground-motion prediction equations are usually hazy about what is actually being predicted). Charlie Kircher told me that engineers use relative displacement and absolute acceleration (they do not use any kind of velocity response spectrum, which is good, because I show here that relative velocity and pseudo relative velocity differ for both short-period and long-period oscillators). Charlie also said that as far as he is concerned, the difference between pseudo-absolute acceleration ($PA$) and absolute acceleration ($AA$) is so small that one may as well use $PA$, because then all that needs to be predicted is $SD$ (since $PA = (2\pi/T_o)^2 SD$, where $T_o$ is the oscillator period). I show in these notes that this statement is not true for longer period oscillators: $AA$ can be considerably higher than $PA$; the period at which the difference appears decreases with oscillator damping. With the newer higher quality instruments and the recent interest in displacement spectra, it is important to recognize the difference and decide on what is to be predicted. The problem comes if regression equations are derived in terms of $AA$ at all periods. I recommend that all predictions be in terms of relative displacements ($SD$) or quantities derived from relative displacement ($PV$ or $PA$). (A side note: CSMIP uses $SA$ and $SV$ to represent *absolute* acceleration and *relative* velocity, while Ken Campbell prefers to use these terms to represent *pseudo* absolute acceleration and *pseudo* relative velocity. I think $PA$ and $PV$ are preferable to represent *pseudo* absolute acceleration and *pseudo* relative velocity, and I will use these terms here, as well as $AA$ and $RV$ to represent the corresponding non-pseudo quantities.) The purpose of these notes is to compare the pseudo- and non-pseudo absolute acceleration and relative velocity spectra. I have chosen records from two small earthquakes (Loma Linda, $M_L = 4.5$ and Yountville, $\mathbf{M} = 5.0$) and one large earthquake (Hector Mine, $\mathbf{M} = 7.1$).

## PROPERTIES OF AN OSCILLATOR

Before showing computed response spectra, it is instructive to look into the expected asymptotic behavior of various definitions of response spectra. These can be derived from the equation for a single-degree-of-freedom damped oscillator coupled with the definitions of response spectra. The asymptotic properties of the oscillator are easy to predict from the equation for oscillator response:

$$\ddot{u} + 2\omega_o \eta \dot{u} + \omega_o^2 u = -\ddot{u}_g, \tag{1}$$

where $u$ is the relative oscillator displacement, $\omega_o = 2\pi/T_o$ is the oscillator natural frequency in radians ($T_o$ is the oscillator natural period), $\eta$ is the fractional damping, and $u_g$ is the ground displacement (more properly, the displacement time series of the oscillator support).

Two situations are of interest: $\omega_o >>$ ground motion frequencies (high frequencies or short periods) and $\omega_o <<$ ground motion frequencies (low frequencies or long periods). It is easy to see from equation (1) that

$$u \rightarrow -(1/\omega_o^2)\ddot{u}_g, \text{ as } \omega_o \rightarrow \infty \tag{2}$$

and

$$u \rightarrow -u_g, \text{ as } \omega_o \rightarrow 0.0. \tag{3}$$

With these asymptotic relations it is easy to predict the asymptotic behavior of various response spectral definitions.

## DEFINITIONS OF RESPONSE SPECTRA

Before giving the asymptotic forms of response spectra, here are the response spectral definitions that I am using.

**Displacement Spectra:**

$$SD \equiv |u|_{max}. \tag{4}$$

**Velocity Spectra:**

$$RV \equiv |\dot{u}|_{max}, \tag{5}$$

and

$$PV \equiv (2\pi/T_o)|u|_{max} = (2\pi/T_o)SD. \tag{6}$$

**Acceleration Spectra:**

$$AA \equiv |\ddot{u} + \ddot{u}_g|_{max} = |\omega_o^2 u + 2\omega_o \eta \dot{u}|_{max} \tag{7}$$

(where the second relation follows from equation (1)) and

$$PA \equiv (2\pi/T_o)^2 SD \tag{8}$$

from which, along with equations (1), (2), and (3), the asymptotic behavior can be found, as given in the next section. Note that from these equations, the following inequality holds

$$AA \leq \omega_o^2 |u|_{max} + 2\omega_o \eta |\dot{u}|_{max},$$

or

$$AA \leq PA + 2\omega_o \eta RV. \tag{9}$$

## ASYMPTOTIC BEHAVIOR OF RESPONSE SPECTRA

**Velocity Spectra:**

*For large $T_o$ (small $\omega_o$):*

From equations (3) and (5)

$$RV \rightarrow (v_g)_{max}, \tag{10}$$

where $v_g$ is the ground velocity, and

$$PV \rightarrow 0.0 \tag{11}$$

So $RV$ will approach a constant value, equal to the peak ground velocity, at long periods, whereas $PV$ approaches 0.0.

*For small $T_o$ (large $\omega_o$):* From equations (2) and (5)

$$RV \rightarrow (\dot{a}_g)_{max}/(2\pi/T_o)^2 \tag{12}$$

where $a_g$ is the ground acceleration. In other words, short-period $RV$ is controlled by the maximum of the first derivative of ground acceleration, but it approaches 0.0 as $T_o^2$. From equations (2) and (6)

$$PV \rightarrow (a_g)_{max}/(2\pi/T_o), \tag{13}$$

which approaches 0.0 as $T_o$ rather than $T_o^2$. Therefore at short periods (high frequencies), $RV$ will decay more rapidly than $PV$.

**Acceleration spectra:**

*For all $T_o$ and damping = 0.0:*

From equations (7) and (8), it follows that for damping of 0.0, $PA$ and $AA$ are exactly equal for all oscillator periods. The fun starts when damping is not equal to 0.0.

*For large $T_o$ (small $\omega_o$):* From equation (7)

$$AA \rightarrow 0.0 \tag{14}$$

and from equation (8)

$$PA \rightarrow 0.0. \tag{15}$$

Although both approach 0.0, however, they do not necessarily do so at the same rate. Notice from equations (7) and (9) that $AA$ includes an extra term involving the ground velocity. Although not obvious from the inequality in equation (9), all of the spectra I have computed obey the inequality

$$AA \geq PA \tag{16}$$

at long periods.

*For small $T$ (large $\omega_o$):* From equations (2) and (8)

$$PA \rightarrow (a_g)_{max} \tag{17}$$

and from equations (2) and (7)

$$AA \rightarrow (a_g + 2\eta \ddot{a}_g/(2\pi/T)^2)_{max} \rightarrow (a_g)_{max} \tag{18}$$

## Examples of Spectra

With this as background, I now show a bunch of figures. The first set (Figures 1 — 5) shows velocity spectra; the second set (Figures 6 — 10) shows acceleration spectra. In each set the figures are arranged by earthquake magnitude, smallest to largest. All data were obtained from digital instruments. Earthquake names and magnitudes, and station names are shown in each figure. Minimal processing was done for each accelerogram, although figures not shown here suggest that the results are not dependent on the low-cut filter, as long as the baseline correction, filtering has removed any obvious large drifts in the velocities and displacements. For all but two figures the damping was 5%; the exceptions are Figure 4 and 9, for which the damping was 20%. The spectra for both horizontal components at a given station are shown in each figure, with the exception of Figures 1 and 6, which contain spectra for only one horizontal component. Ratios of spectra are shown on each figure, using the scale on the right axis.

Bottom line:

The difference between $PA$ and $AA$ can be significant for large dampings, but for the most commonly used damping (5%) the difference is small except for periods longer than about 1 to 2 sec. Might this be a concern? As more data are acquired on high-quality digital instruments, it will be possible to obtain good signals at longer periods than before, even for small earthquakes, and the choice of whether to use $PA$ or $AA$ may be important. Does the design of structures whose resonant periods are long depend at all on acceleration spectra? I doubt it, in which case the real bottom line is to use $SD$ and $PA$ and be done with it.

It is fortunate that engineers do not use velocity spectra, because the difference between $RV$ and $PV$ can be significant for all periods and all magnitudes.

Here follow the figures, one page per figure.

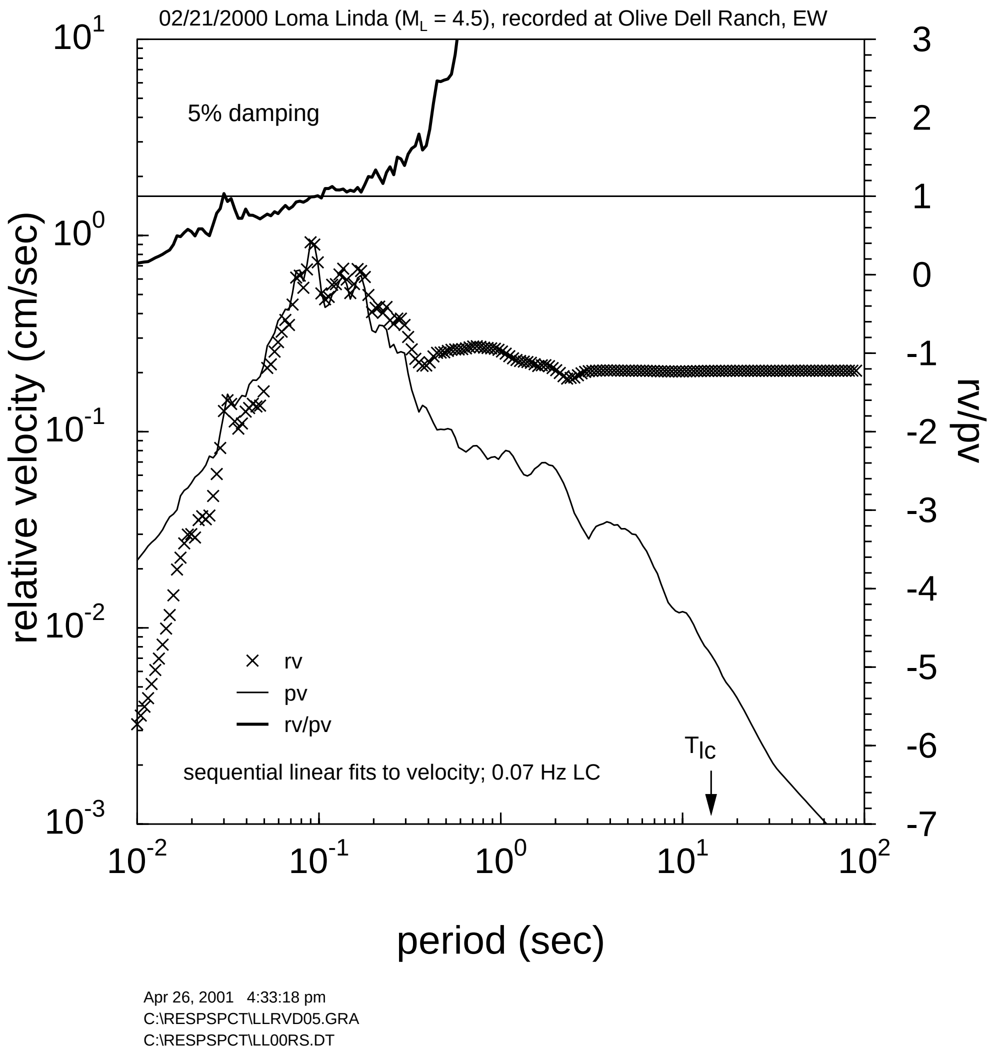


Figure 1. 5%-damped relative velocity spectra and ratios, Olive Dell Ranch recording of 2000 Loma Linda, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

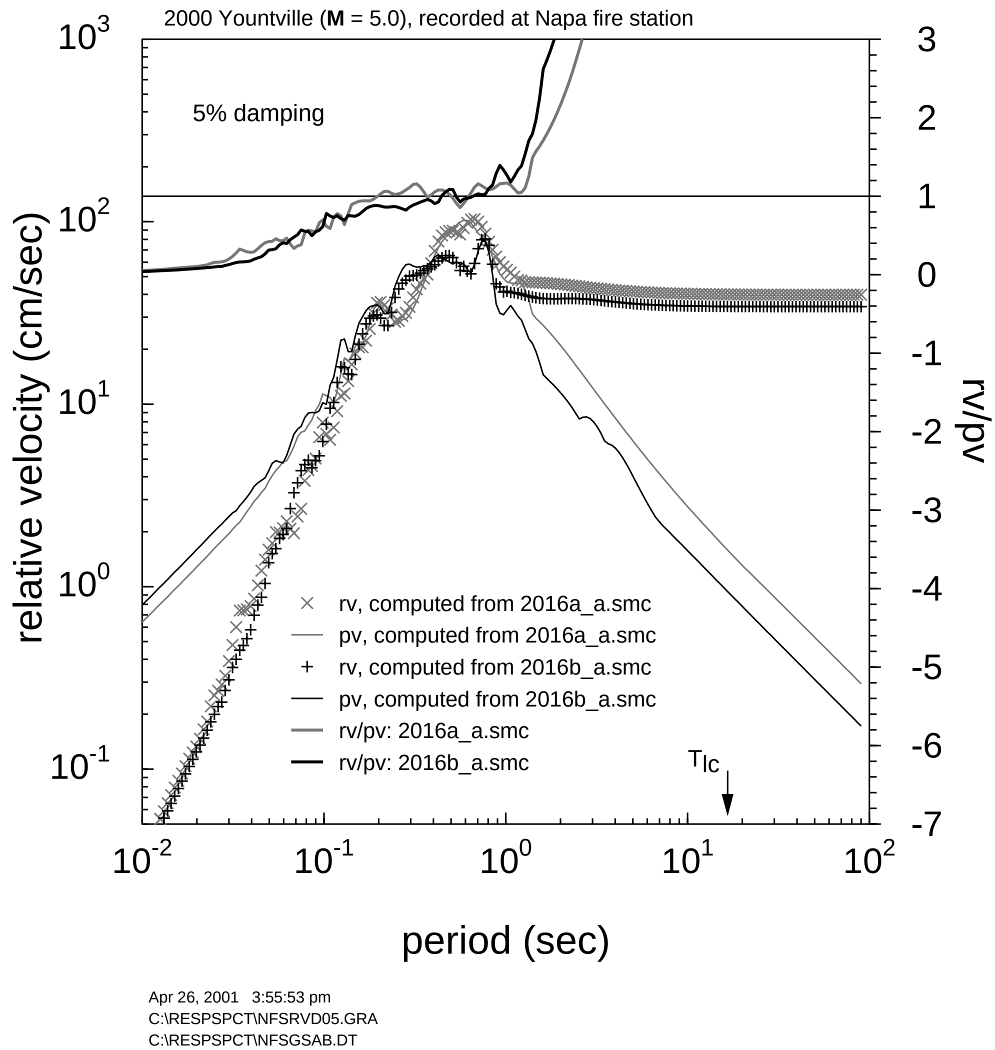


Figure 2. 5%-damped relative velocity spectra and ratios, Napa Fire Station recording of 2000 Yountville, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

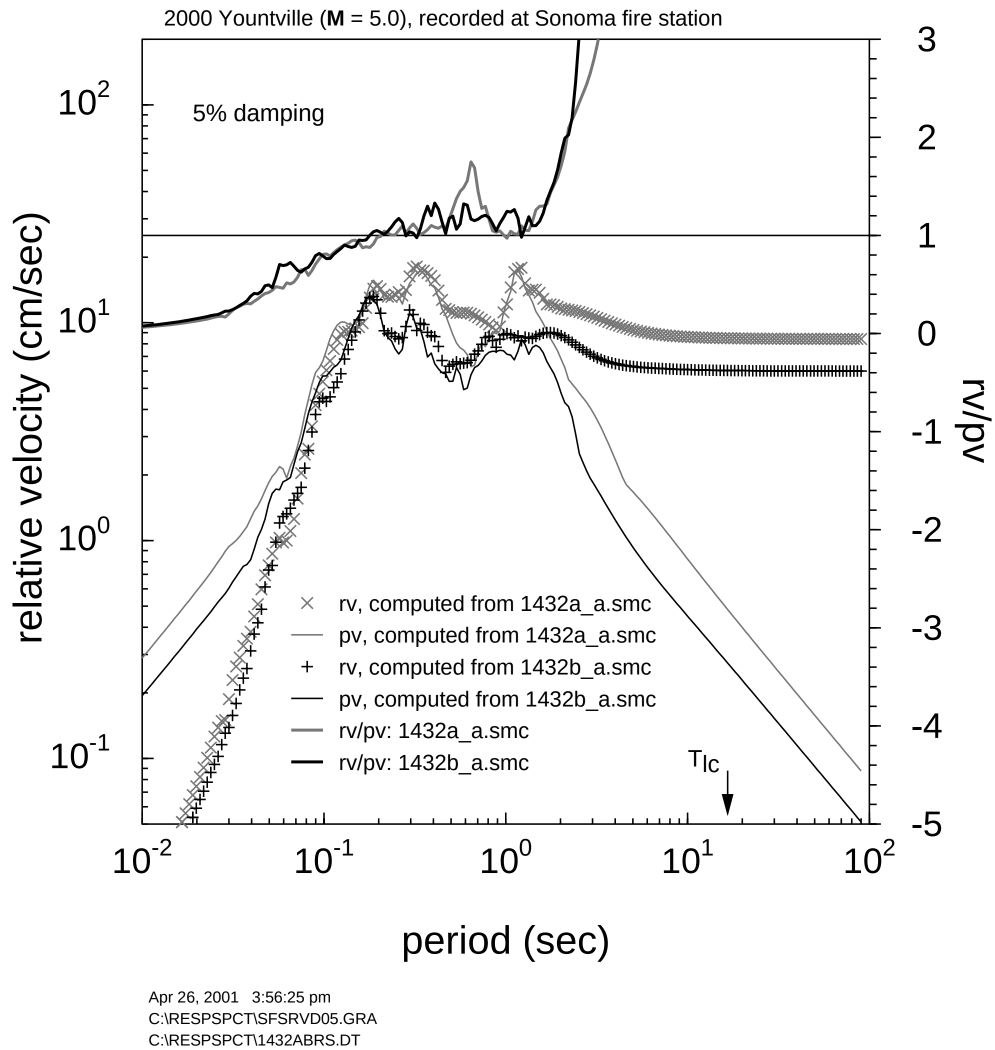


Figure 3. 5%-damped relative velocity spectra and ratios, Sonoma Fire Station recording of 2000 Yountville, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

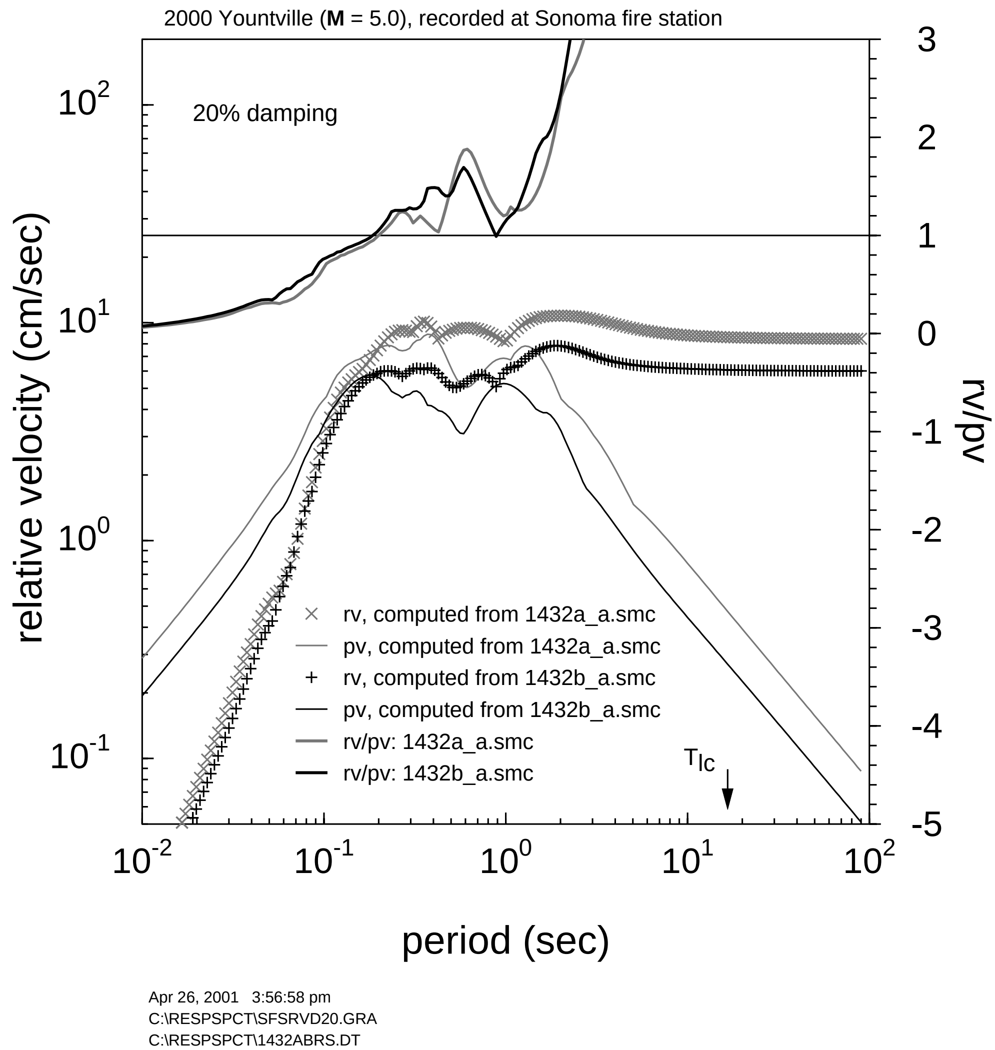


Figure 4. 20%-damped relative velocity spectra and ratios, Sonoma Fire Station recording of 2000 Yountville, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

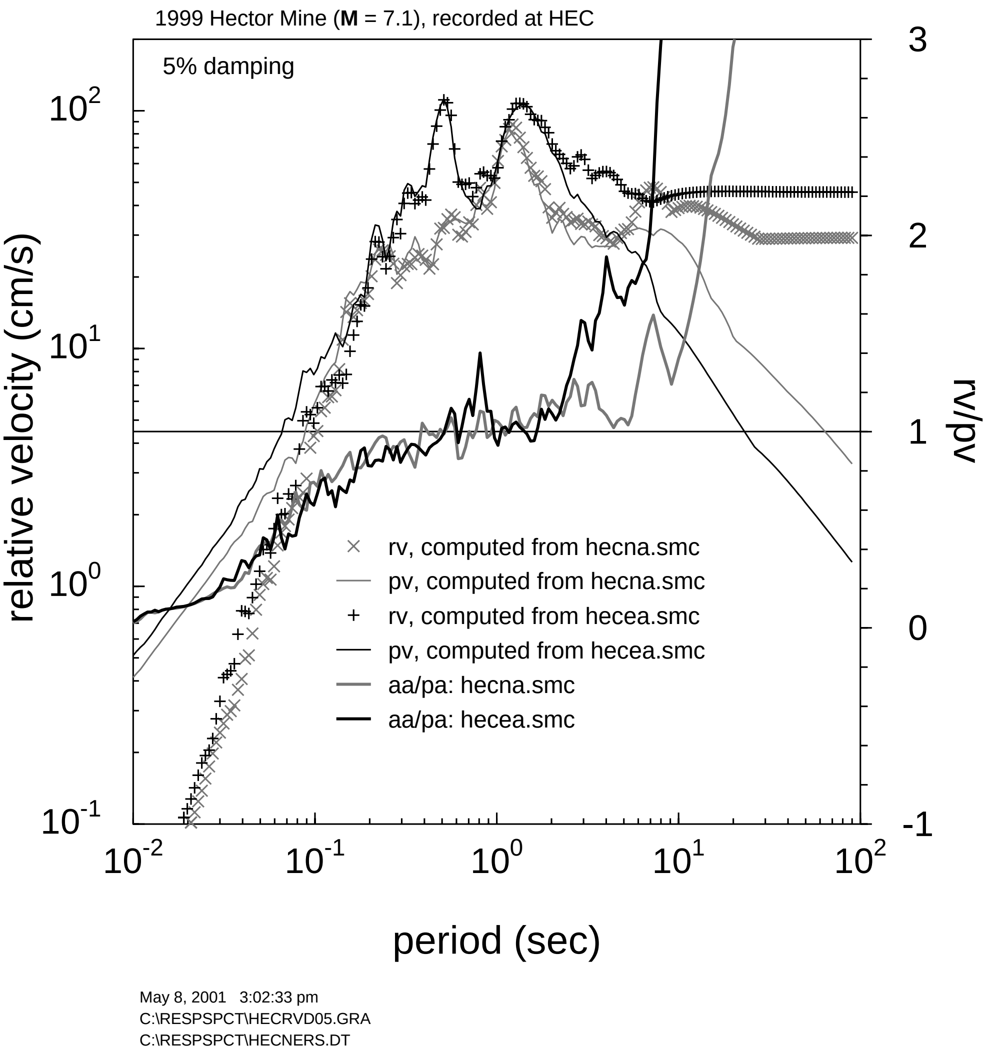


Figure 5. 5%-damped relative velocity spectra and ratios, HEC recording of 1999 Hector Mine earthquake. A baseline correction was applied, but no low-cut filtering.

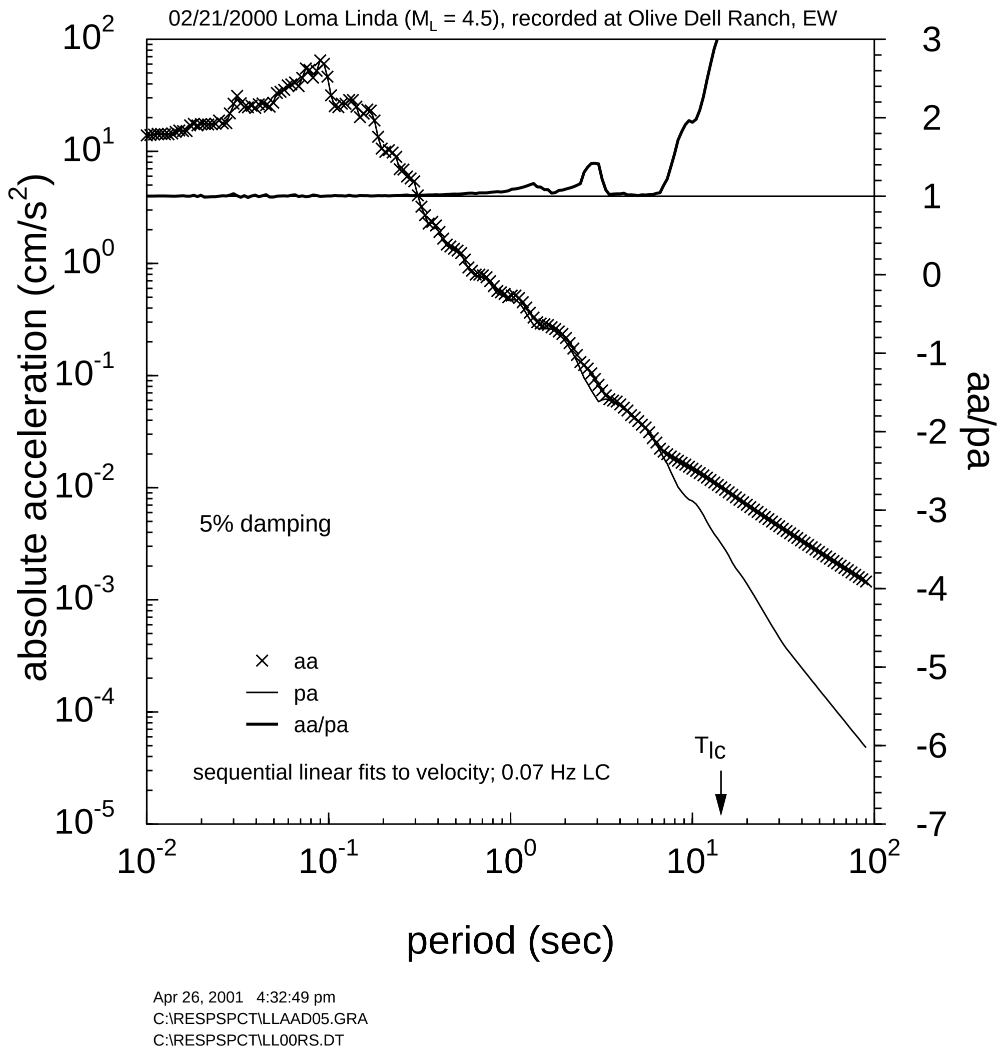


Figure 6. 5%-damped absolute acceleration spectra and ratios, Olive Dell Ranch recording of 2000 Loma Linda, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

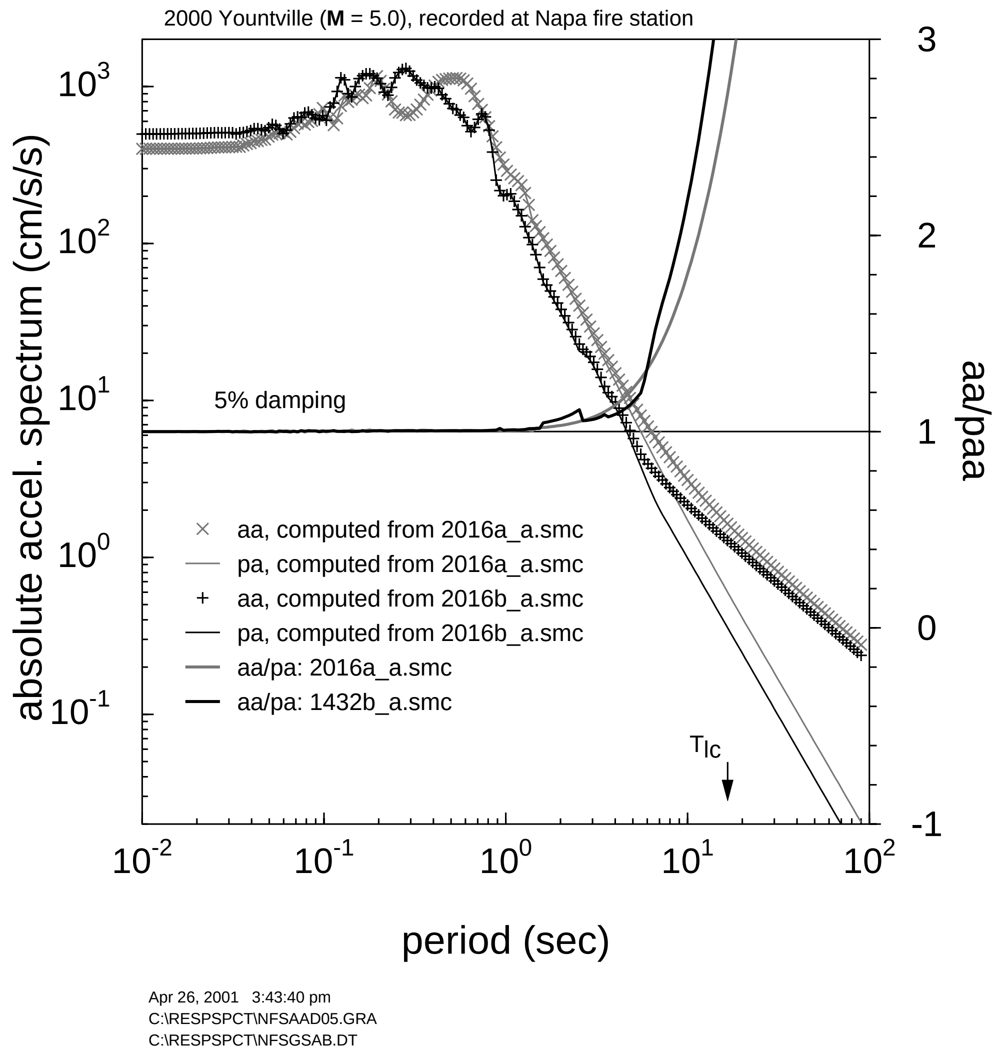


Figure 7. 5%-damped absolute acceleration spectra and ratios, Napa Fire Station recording of 2000 Yountville, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

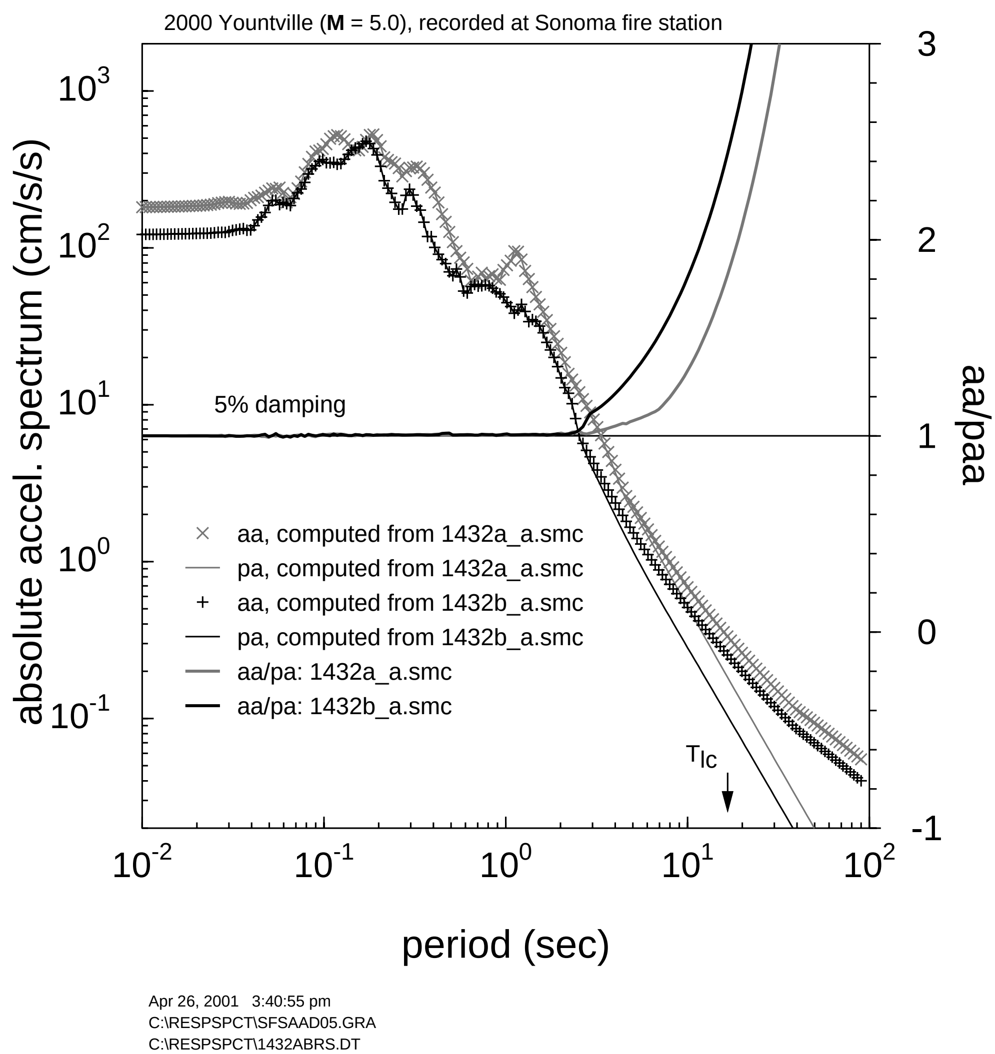


Figure 8. 5%-damped absolute acceleration spectra and ratios, Sonoma Fire Station recording of 2000 Yountville, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

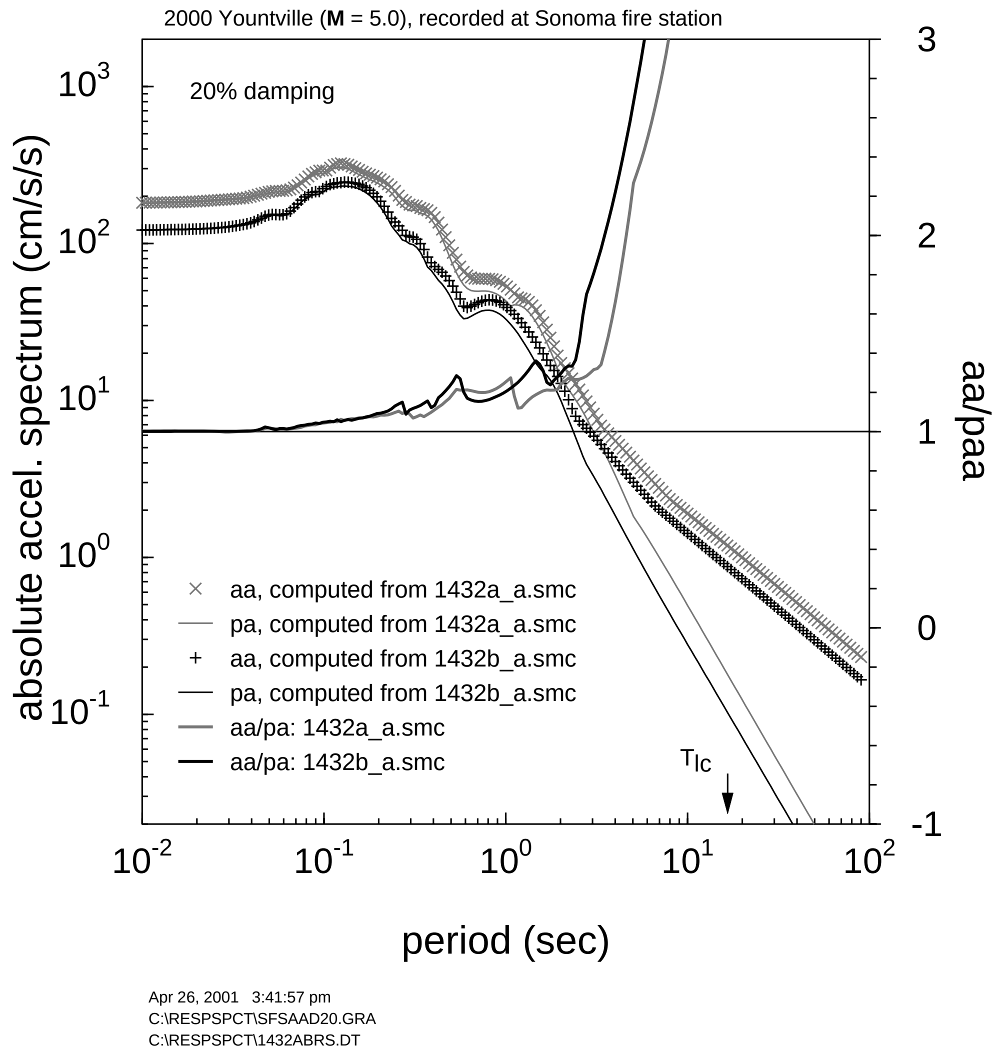


Figure 9. 20%-damped absolute acceleration spectra and ratios, Sonoma Fire Station recording of 2000 Yountville, CA, earthquake. $T_{lc}$ is the corner frequency of the low-cut filter applied to the records during the processing.

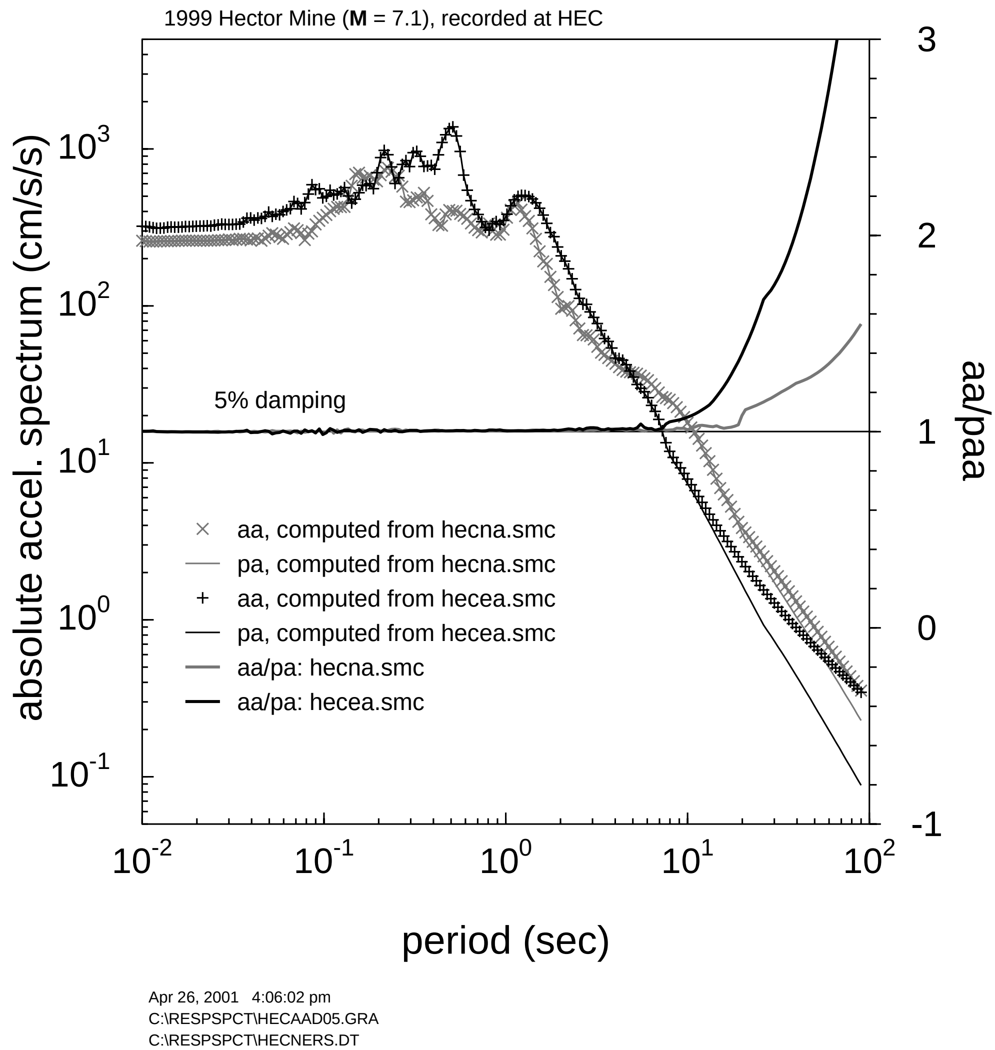


Figure 10. 5%-damped absolute acceleration spectra and ratios, HEC recording of 1999 Hector Mine earthquake. A baseline correction was applied, but no low-cut filtering.