Comment on “Why Do Modern Probabilistic Seismic-Hazard Analyses Often Lead to Increased Hazard Estimates?” by Julian J. Bommer and Norman A. Abrahamson

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Introduction

In a recent paper, "Why Do Modern Probabilistic Seismic-Hazard Analyses Often Lead to Increased Hazard Estimates?", Bommer and Abrahamson (2006) provided an excellent review on probabilistic seismic hazard analysis (PSHA) and its key issue: how the ground-motion variability is treated. Bommer and Abrahamson (2006) stated that "although several factors may contribute to the higher estimates of seismic hazard in modern studies, the main reason for these increases is that in the earlier studies the ground-motion variability was either completely neglected or treated in a way that artificially reduced its influence on the hazard estimated." In other words, Bommer and Abrahamson (2006) argued that "the main reason for the increases in the modern estimates of seismic hazard is that the ground-motion variability in early application (and indeed formulations) of PSHA was not treated properly," and concluded "the increased hazard estimates resulting from modern probabilistic studies are entirely appropriate." We argue, however, that ground-motion variability may not be treated correctly in modern PSHA. This incorrect treatment of ground-motion variability perhaps leads to increased hazard estimates, at low annual frequency of exceedance (10^{-4} or lower) in particular.

Modern PSHA

As shown by Bommer and Abrahamson (2006), modern PSHA is often referred to as the Cornell-McGuire method (Cornell, 1968, 1971; McGuire, 1976). According to Cornell (1968, 1971) and McGuire (1976, 2004), modern PSHA is based on the following equation

$$\begin{aligned}\gamma(y) &= \sum vP[Y \geq y] \\ &= \sum v \iint \{1 - \int_0^y \frac{1}{\sqrt{2\pi}\sigma_{\ln,y}} \exp[-\frac{(\ln y - \ln y_{mr})^2}{2\sigma_{\ln,y}^2}] d(\ln y)\} f_M(m) f_R(r) dm dr\end{aligned}, \qquad (1)$$

where v is the activity rate, $f_M(m)$ and $f_R(r)$ are the probability density function (**PDF**) of earthquake magnitude M and epicentral or focal distance R, respectively, and y_{mr} and $\sigma_{ln,y}$ are the median and standard deviation at m and r. $f_M(m)$ and $f_R(r)$ were introduced to account for the variability of earthquake magnitude and epicentral or focal distance, respectively (Cornell, 1968, 1971; McGuire, 2004). y_{mr} and $\sigma_{ln,y}$ are determined by the ground-motion attenuation relationship (Campbell, 1981; Joyner and Boore, 1981; Abrahamson and Silva, 1997; Toro and others, 1997; EPRI, 2003; Atkinson and Boore, 2006; Akkar and Bommer, 2007). As demonstrated by Bommer and Abrahamson (2006), ground motion Y is generally modeled as a function of M and R with variability E (capital epsilon):

$$\ln(Y) = f(M, R) + E\,. \tag{2}$$

The variability E is modeled as a normal distribution with a zero mean and standard deviation $\sigma_{ln,Y}$ (Campbell, 1981; Joyner and Boore, 1981; Abrahamson and Silva, 1997; Toro and others, 1997; EPRI, 2003; Atkinson and Boore, 2006; Akkar and Bommer, 2007). In other words, the variability of ground motion Y is modeled as a log-normal distribution (Fig. 1). Therefore, equation (2) can be rewritten as

$$\ln(Y) = f(M, R) + n\sigma_{\ln,Y}\,, \tag{3}$$

where n (a constant) is a number of standard deviations (a variable) measured as the difference relative to the median ground motion $f(M,R)$ (Fig. 1) (**note**: n is equal to ε in equation [1] of Bommer and Abrahamson [2006]).

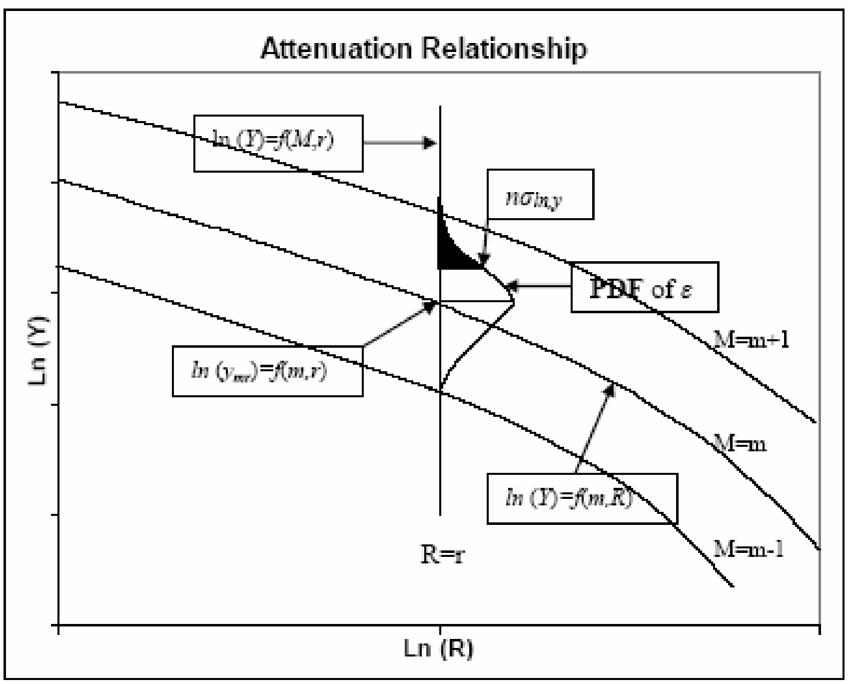

Figure 1. Ground-motion attenuation relationship.

According to Benjamin and Cornell (1970) and Mendenhall and others (1986), if and only if M, R, and E are independent random variables, the joint probability density function of M, R, and E is

$$f_{M,R,E}(m,r,\varepsilon) = f_M(m) f_R(r) f_E(\varepsilon), \tag{4}$$

where $f_E(\varepsilon)$ is the **PDF** of E. The exceedance probability $P[Y \geq y]$ is

$$\begin{aligned} P[Y \geq y] &= \iiint f_{M,R,E}(m,r,\varepsilon) H[\ln Y(m,r,\varepsilon) - \ln y] dm dr d\varepsilon \\ &= \iiint f_M(m) f_R(r) f_E(\varepsilon) H[\ln Y(m,r,\varepsilon) - \ln y] dm dr d\varepsilon \end{aligned}, \tag{5}$$

where $H[\ln Y(m,r,\varepsilon) - \ln y]$ is the Heaviside step function, which is zero if $\ln Y(m,r,\varepsilon)$ is less than $\ln y$, and 1 otherwise (McGuire, 1995). Because E follows a normal distribution (Fig. 1), equation (5) can be rewritten as

$$P[Y \geq y] = \iint \{1 - \int_0^y \frac{1}{\sqrt{2\pi}\sigma_{\ln,y}} \exp[-\frac{(\ln y - \ln y_{mr})^2}{2\sigma_{\ln,y}^2}] d(\ln y)\} f_M(m) f_R(r) dm dr, \tag{6}$$

where $\ln y_{mr} = f(m,r)$. Therefore, we have equation (1), the heart of modern PSHA (Cornell, 1968, 1971; McGuire, 1976, 2004).

As demonstrated above, equation (1) is derived from the pre-condition that *if* and *only if* M, R, and E are independent random variables (Benjamin and Cornell, 1970; Mendenhall and others, 1986). In other words, the ground-motion variability E must be an independent random variable. However, the ground-motion variability E is not an independent random variable. In modern ground-motion attenuation relationships, the ground-motion variability E is modeled implicitly or explicitly as a dependence of M or R or both (Youngs and others, 1995; Abrahamson and Silva, 1997; Boore and others, 1997; EPRI, 2003; Akkar and Bommer, 2007). This is clearly shown in Figure 1 of Bommer and Abrahamson (2006). Bommer and Abrahamson (2006) stated that "this large variability is not due to the stations having significantly different site conditions but rather reflects the large variability of ground motions when the wave propagation from a finite fault is characterized only by the distance from the station to the closest point on the fault rupture." In other words, the large variability of ground motions reflects the distance (R) being characterized for a finite fault. Youngs and others (1995) found "a statistically significant dependence of the standard error on earthquake magnitude" from the large California strong-motion data set. Akkar and Bommer (2007) also showed the dependency of the standard error on earthquake magnitude. The dependency of ground-motion variability on M and R in the central and eastern United States was summarized by EPRI (2003) as

$$\sigma_{\ln,y} = \sqrt{\sigma_{source}^2 + \sigma_{path}^2 + \sigma_{\mathrm{mod}eling}^2}\ , \tag{7}$$

where σ_{source} is the variability related to M, and σ_{path} is the variability related to R. Therefore, equation (6) is not valid. Neither is equation (1).

Concluding Remarks

Modern PSHA (i.e., the Cornell-McGuire method) was developed in the early 1970's (Cornell, 1968, 1971; McGuire, 1976), whereas modern ground-motion attenuation relationships were developed in the 1980's (Campbell, 1981; Joyner and Boore, 1981). In the early 1970's, an earthquake was generally considered as a point source, and epicentral or focal distance was modeled in the ground-motion attenuation relationship (Cornell, 1968, 1971). The ground-motion variability was not well understood and was treated as an independent random variable in the formulation of modern PSHA (Cornell, 1968, 1971). However, in modern ground-motion attenuation relationships, an earthquake is considered a finite fault, and fault distance, not epicentral or focal distance, was modeled in the ground-motion attenuation relationship (Campbell, 1981; Joyner and Boore, 1981; Youngs and others, 1995; Abrahamson and Silva, 1997; Boore and others, 1997; EPRI, 2003; Akkar and Bommer, 2007). Ground-motion variability is modeled implicitly or explicitly as a dependence of earthquake magnitude or distance, or both. Therefore, ground-motion variability is not treated correctly in modern PSHA.

This incorrect treatment of ground-motion variability results in extrapolation of the return period for ground motion from the recurrence interval of earthquakes (temporal measurement) and the variability of ground motion (spatial measurement) (Wang and others, 2003, 2005; Wang, 2005, 2006), or the so-called *ergodic* assumption, "treating spatial uncertainty (variability) of ground motions as an uncertainty (variability) over time at a single point" (Anderson and Brune, 1999). Modern PSHA mixes the temporal measurement (occurrence of an earthquake and its consequence [ground motion] at a site) with spatial measurement (ground-motion variability due to the source, path, and site effects) (Wang and others, 2003, 2005; Wang, 2005, 2006). The temporal and spatial measurements are two intrinsic and independent characteristics of an earthquake and its consequence (ground motion) at a site, and must be treated separately.

This incorrect treatment of ground-motion variability also results in variability in earthquake magnitude and distance being counted twice. As shown in equation (1), $f_M(m)$ and $f_R(r)$ are the **PDF** for earthquake magnitude and distance, and are designed to account for the variability in earthquake magnitude and distance, respectively (Cornell, 1968, 1971; McGuire, 2004), whereas the integration over y (shaded area in Fig. 1) also includes the variability in earthquake magnitude and distance, because $\sigma_{ln,y}$ is a dependence of earthquake magnitude and distance. Therefore, variability, ground-motion variability in particular, becomes a controlling factor in PSHA. This can be seen clearly in Figures 3 to 7 of Bommer and Abrahamson (2006), at low annual frequency of exceedance (less than 10^{-4}) in particular.

As it is modeled in modern ground-motion attenuation relationships, ground-motion variability is an implicit or explicit dependence of earthquake magnitude and distance. However, ground-motion variability is treated as an independent random variable in modern PSHA. This incorrect treatment of ground-motion variability perhaps leads to increased hazard estimates and causes confusion and difficulty in understanding and applying modern PSHA.

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