# **Review of Foundation Vibrations**

## Philosophy

Recall that our objective is to determine the characteristics (i.e. displacement, natural frequency, etc.) of the machine-foundation system shown below. There are two approaches which we could adopt:

- model the machine-foundation system using simple models with closed-form solutions (e.g. Wolf, 1994).
- model the machine-foundation system directly using numerical modeling (e.g. finite element, boundary element, finite differences)
- model the machine-foundation system as a SDOF system

We have adopted the third approach because the analysis of SDOF systems is relatively simple.

## **SDOF Systems**

Our model is a SDOF system loaded either actively or passively. Thus far we have modeled the attenuation in the system using dashpots which produce linear viscoelastic damping.

We describe the response of the system to some external excitation using transfer functions which are defined as:

$$H(\Omega) = \frac{Output}{Input}$$
(1)

For active loading in which we are interested in the absolute displacement of the mass due to an input force, the transfer function has the form:

$$H(\Omega) = \frac{U(\Omega)}{F(\Omega)}$$
(2a)

$$H(\Omega) = \frac{1}{k - \Omega^2 m + i\Omega c}$$
(2b)

or

$$H(\Omega) = \frac{1}{(k + i\Omega c) - \Omega^2 m}$$
(2c)

or

$$H(\Omega) = \frac{1}{\kappa - \Omega^2 m}$$
(2d)

Combining Equations 2a and 2d yields:

$$U(\Omega) = \frac{F(\Omega)}{\kappa - \Omega^2 m}$$
(2d)

Thus we can use transfer functions to calculate the displacement as a function of frequency (i.e. in the frequency domain).

Qualitative analysis of the transfer function provids insight into which of the system parameters (k,m,c) control in different frequency ranges.

## **Fourier Analysis**

Fourier analysis is a tool we can use to transform between the time and frequency domains. We take an arbitrary time history of force or displacement and decompose it into a finite number of harmonic functions (sines and cosines), each with a different frequency, amplitude, and phase shift. We then use the transfer functions to determine the response of the system to each harmonic function independently (Note: we must assume a linear system for this to be valid). We then use the inverse Fourier transform to re-combine the individual responses and calculate the response of the system in the time domain.

## **Foundation Vibrations**

Using the approach developed by Gazetas (1991) allows us to calculate the complex dynamic stiffness,  $\kappa$ , of shallow and deep foundations. Typically these are calculated in the following manner:

$$\kappa = \mathbf{K} \cdot \mathbf{k} (\mathbf{a}_0) + \mathbf{i} \mathbf{\Omega} \mathbf{C} (\mathbf{a}_0)$$
(3a)

or

$$\kappa = \mathbf{K} \cdot \mathbf{k} (\mathbf{a}_0) + 2\mathbf{i}\mathbf{D}\mathbf{K}$$
(3b)

### **Observations on Foundation Vibrations**

- There are, in general, 6 modes of vibration:
  - 1. Vertical (displacement in z direction)
  - 2. Torsion (rotation about z axis)
  - 3. Horizontal (displacement in x direction)
  - 4. Rocking (rotation about y axis)
  - 5. Horizontal (displacement in y direction)
  - 6. Rocking (rotation about x axis)

Modes 3 and 4 and Modes 5 and 6 are coupled. Vertical and torsional modes are uncoupled.

- Rotational modes (rocking and torsion) have less damping that translational modes.
- Embedment increases the stiffness and damping of foundation systems, but care must be taken to assure good contact along the sides of the foundation.
- Foundations on a layer over bedrock are different than foundations on a homogeneous half space in 3 ways:
  - 1. static stiffness increases
  - 2. the dynamic stiffness coefficient decreases near the natural frequency of the soil layer
  - 3. radiation damping is reduced to zero at frequencies less than the natural frequency of the soil layer.
- Group interaction is important for pile groups.

### **Dynamic Response of Shallow Foundations**

Surface Foundations - All Modes of Vibration

- Dobry, R., and Gazetas, G., (1986), "Dynamic Response of Arbitrarily Shaped Foundations," *Journal of Geotechnical Engineering*, ASCE Vol. 112, No. 2, pp. 109-135.
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Embedded Foundations - Vertical Vibration

Gazetas, G., Dobry, R., and Tassoulas, J.L., (1985), "Vertical Response of Arbitrarily Shaped Embedded Foundations," *Journal of Geotechnical Engineering*, ASCE Vol. 111, No. 6, pp. 750-771.

Embedded Foundations - Horizontal Vibration

- Gazetas, G., and Tassoulas, J.L., (1987), "Horizontal Stiffness of Arbitrarily Shaped Embedded Foundations," *Journal of Geotechnical Engineering*, ASCE Vol. 113, No. 5, pp. 440-457.
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Embedded Foundations - Rocking Vibration

- Hatzikonstantinou, E., Tassoulas, J.L., Gazetas, G., Kotsanopoulos, P., and Fotopoulou, M., (1989), "Rocking Stiffness of Arbitrarily Shaped Embedded Foundations," *Journal* of Geotechnical Engineering, ASCE Vol. 115, No. 4, pp. 457-472.
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Embedded Foundations - Torsional Vibration

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#### Summary

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## **Dynamic Response of Deep Foundations**

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- Dynamic Response of Pile Foundations: Experiment, Analysis, and Observation, (1987), ASCE Geotechnical Special Publication No. 11, T. Nogami, Ed. 183 pp.

Gazetas, G., and Dobry, R., (1984), "Horizontal Response of Piles in Layered Soils," *Journal of Geotechnical Engineering*, ASCE, Vol. 110, No 1, pp. 20-40.

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