Complex Notation

Consider a function of the form:

$$x(t) = Xe^{i\Omega t} \tag{1}$$

where X is complex valued:

$$X = a + bi \tag{2a}$$

This can also be written as:

$$X = |X|e^{i\theta} \tag{2b}$$

where $|X| = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}(b/a)$.

Substituting Eq. 2b into 1 we obtain:

$$x(t) = |X|e^{i(\Omega t+\theta)} \tag{3a}$$

which may be written as:

$$x(t) = |X|(\cos(\Omega t + \theta) + i\sin(\Omega t + \theta)) \tag{3b}$$

Taking the real part of Eq. 3b yields:

$$\mathrm{Re}(x(t)) = |X|\cos(\Omega t + \theta) \tag{4a}$$

or the imaginary part yields:

$$\mathrm{Im}(x(t)) = |X|\sin(\Omega t + \theta) \tag{4a}$$

Thus, using $x(t) = Xe^{i\Omega t}$ and taking either the real or imaginary component is equivalent to using either $x(t) = |X|\cos(\Omega t + \theta)$ or $x(t) = |X|\sin(\Omega t + \theta)$, respectively. The primary advantage of using the exponential form is that derivatives with respect to time are conveniently manipulated.