Stress Paths
Stress Point

\[ \frac{1}{2}(\sigma_1 + \sigma_3) \]

\[ \frac{1}{2}(\sigma_1 - \sigma_3) \]
\[ p = \frac{\sigma_1 + \sigma_3}{2}, \quad q = \frac{\sigma_1 - \sigma_3}{2} \]
\[
\sigma_h = K \sigma_v
\]

Stress Path

\[
K = \frac{\sigma_h}{\sigma_v} = \frac{1 - \tan \beta}{1 + \tan \beta} \quad \Rightarrow \quad \tan \beta = \frac{q}{p} = \frac{1 - K}{1 + K}
\]
Failure Envelope

\[ \sigma_{hf} = K_f \sigma_{vf} \]

\[ K_f = \frac{\sigma_{hf}}{\sigma_{vf}} = \frac{\sigma_{3f}}{\sigma_{1f}} \Rightarrow \tan \psi = \frac{1 - K_f}{1 + K_f} \]
Not tangent!

Fig. 10.23 Relationship between the $K_f$ line and the Mohr-Coulomb failure envelope.
Direct Shear Test
\[ \tau = \frac{T}{A} \]

\( e = \text{constant (dense sand)} \)

\[ \sigma_{n3} = \frac{P_3}{A} \]
\[ \sigma_{n2} = \frac{P_2}{A} \]
\[ \sigma_{n1} = \frac{P_1}{A} \]

(b) Test results
(c) Mohr diagram
Element on the failure plane

Initial conditions:

\[ \sigma_n = \sigma_{10} \]

\[ \sigma_h = K_0 \sigma_{10} = \sigma_{30} \]

Mohr diagrams

Pole

\[ \sigma_{30} \]

\[ \sigma_n = \sigma_{10} \]
Example

• A direct shear test is run on a medium dense sandy silt with $\sigma_n = 65$ kPa. At failure the shear stress is 41 kPa.

• Draw the Mohr circles for the initial and failure conditions and determine:
  – The principal stresses at failure
  – The orientation of the failure plane
  – The orientation of the plane of maximum normal stress at failure
  – The orientation of the plane of maximum shear stress at failure
Fig. Ex. 10.7
Triaxial Shear Test
To volume change or pore water pressure measurement device ($\Delta V_{ol}$ or $\Delta u$)
\[ \sigma_{\text{axial}} = (\sigma_1 - \sigma_3) \]

\[ \sigma_{\text{cell}} = \sigma_2 = \sigma_3 \]

\[ \sigma_1 \]
Example

- A conventional triaxial compression (CTC) test is run on a loose sand with a friction angle of 20. The cell pressure is 30 psi. Determine the following:
  - The orientation of the failure plane
  - The stresses acting on the failure plane at failure
  - The axial stress at failure
  - The stress path corresponding to this test
CTC or AC
LE or RTC
\[ \sigma_{\text{cell}} \]

45°

CTE or LC
\[ q \]

AE or RTE

45°

\[ \text{cell} \]

\[ p \]
Symbol | Geotechnical Engineering example
--- | ---
AC: Axial Compression | Foundation loading — increase $\sigma_v$, $\sigma_h$ constant
LE: Lateral Extension | Active earth pressure — decrease $\sigma_h$, $\sigma_v$ constant
AE: Axial Extension | Unloading (excavation) — decrease $\sigma_v$, $\sigma_h$ constant
LC: Lateral Compression | Passive earth pressure — increase $\sigma_h$, $\sigma_v$ constant