Probabilistic Basis for 2000 SAC Federal Emergency Management Agency Steel Moment Frame Guidelines

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Abstract: This paper presents a formal probabilistic framework for seismic design and assessment of structures and its application to steel moment-resisting frame buildings. This is the probabilistic basis for the 2000 SAC Federal Emergency Management Agency (FEMA) steel moment frame guidelines. The framework is based on realizing a performance objective expressed as the probability of exceeding a specified performance level. Performance levels are quantified as expressions relating generic structural variables ''demand'' and ''capacity'' that are described by nonlinear, dynamic displacements of the structure. Common probabilistic analysis tools are used to convolve both the randomness and uncertainty characteristics of ground motion intensity, structural ''demand,'' and structural system ''capacity'' in order to derive an expression for the probability of achieving the specified performance level. Stemming from this probabilistic framework, a safety-checking format of the conventional ''load and resistance factor'' kind is developed with load and resistance terms being replaced by the more generic terms ''demand'' and ''capacity,'' respectively. This framework also allows for a format based on quantitative confidence statements regarding the likelihood of the performance objective being met. This format has been adopted in the SAC/FEMA guidelines.

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Introduction

This paper presents the formal probabilistic basis behind the new performance-based seismic design and assessment guidelines for steel moment frame buildings prepared by the SAC Federal Emergency Management Agency (FEMA) program (FEMA 2000). The reader is referred to the companion paper (Yun et al. 2002) for a general background, for a description of how the multiple demand and capacity factors in these guidelines appear to the typical user, and for the default numerical values assigned to various coefficients in the implementation for FEMA.

The framework rests on an explicitly nonlinear, dynamic, displacement-based representation of the seismic behavior of structures. In practical operations the format is, however, of the conventional ''load and resistance factor'' kind, with the more generic terms ''demand'' and ''capacity'' replacing the force-based terms ''load'' and ''resistance.'' Consistent with modern seismic assessment procedures in the nuclear community (DOE 1994), the probabilistic analysis separately characterizes both the randomness and the uncertainty in demand and capacity. Based on these assessments the engineer is provided in these guidelines with a confidence statement with respect to the likelihood of unacceptable behavior. A more detailed presentation of this and other such frameworks is provided by Jalayer and Cornell (1998, 2002).

Basic Approach: Probability Assessment Formulation

The objective is to show how the demand and capacity factors γ and ϕ, as well as λ, the confidence factor in the SAC Guidelines, have been derived by elementary probability theory from representations of the three random elements of the problem. These elements begin with the ground motion intensity, characterized here by the level of the spectral acceleration S_a at approximately the first natural period of the structure, and 5% or higher damping (Shome et al. 1998). (See the Appendix, where, in order to clarify the developments here, we shall collect all more detailed qualifications and justifications, generally by reference.) The spectral displacement S_D may be a more natural choice for this displacement-based scheme but we shall retain the more commonly available measure S_a for this presentation; the results and conclusions are the same. The other two random elements are the displacement demand D and the displacement capacity C. Both demand and capacity will be presumed here to be measured in terms of the maximum interstory drift angle, i.e., the largest such interstory drift over time over the structure. Here, we refer to this simply as ''drift.'' The likelihood of various levels of future intense ground motions at the site are represented in the standard way by the hazard function $H(s_a)$, which gives the annual probability that the (random) intensity S_a at the site will equal or

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exceed level s_a. This is provided by earth scientists on a site-specific or mapped regional basis. The prediction of the drift demand given any particular level of ground motion and the estimation of the capacities of various ''failure modes'' are the purview of the structural engineer. The developments here focus on these two elements and specifically on their probabilistic representations. Finally, it must be recognized that all such probabilistic predictions and representations are uncertain estimates; explicit quantification and analysis of these uncertainties will be addressed subsequently.

The goal is to provide criteria based on desired performance objectives which are defined as specified probabilities of exceeding the performance level, such as the collapse-prevention damage state (Yun et al. 2002) and life safety damage state. To do so one must fold together the probabilistic representations of the three elements above. In keeping with the general design approach of separately considering demand and capacity, comparison at the displacement or drift level (and not, for example, at the ground motion level; see the Appendix), this folding together is done in two steps. The first step couples the first two basic elements S_a hazard and drift demand (versus or conditional on S_a), to produce a (structure-specific) drift hazard curve $H_D(d)$. This curve provides the annual probability (or strictly speaking the mean annual frequency) that the drift demand D exceeds any specified value d. The second step combines this curve with the third element, the drift capacity representation, to produce P_{PL}, the (annual) probability of the performance level not being met (e.g., the annual probability of collapse or the annual probability of exceeding the life safety level).

Using the total probability theorem (Benjamim and Cornell 1970), $H_D(d)$ becomes, in discrete form

$$H_D(d)=P[D\geqslant d]=\sum_{\text{all } x_i} P[D\geqslant d|S_a=x_i]P[S_a=x_i] \tag{1}$$

To facilitate the computations, the probability of interest has been expanded by conditioning on all possible levels of the ground motion, as can be seen in Eq. (1).

The second factor within the sum, the likelihood of a given level of spectral acceleration $P[S_a=x]$, can easily be obtained from the standard hazard curve $H(s_a)$. In the first factor $P[D\geqslant d|S_a=x]$, one sees what the structural response analysis must be responsible for providing: the likelihood that the drift exceeds d given that the value of S_a is known. This factor is picked up again below.

In continuous, integral form Eq. (1) is

$$H_D(d)=\int P[D\geqslant d|S_a=x]|dH(x)| \tag{2}$$

in which the notation $|dH(x)|$ means the absolute value of the derivative of the site's spectral acceleration hazard curve times dx, i.e., loosely the likelihood that $S_a=x$. (The absolute value is needed only because the derivative is negative.)

Using the total probability theorem again P_{PL} itself becomes (in discrete form)

$$P_{\mathrm{PL}}=P[C\leqslant D]=\sum_{\text{all } d_i} P[C\leqslant D|D=d_i]P[D=d_i] \tag{3}$$

The second factor, the likelihood of a given displacement demand level $P[D=d]$, can be determined from the drift hazard curve derived in Eq. (2). The first factor, the likelihood that the drift capacity is less than a specified value d given that the drift demand equals that value, $P[C\leqslant D|D=d]$, can to a first approximation be assumed to be independent of the information about the drift level itself (see the Appendix), permitting this term to be simplified as below. The continuous form is

$$P_{\mathrm{PL}}=\int P[C\leqslant d]|dH_{\dot{D}}(d)| \tag{4}$$

The second factor $|dH_D(d)|$ is defined as above for the ground motion hazard curve: as the absolute value of the differential of the drift demand hazard curve.

Basic Approach: Probability Assessment in Closed Form

In principle, Eqs. (2) and (4) can be solved numerically for any assumptions about the form of the probabilistic representations of the three elements. In order to convert the conclusions to practical demand and capacity factors, however, these integrals should be tractable. This objective is achieved by three analytical approximations of the representations. These are shown in Fig. 1. First, assume that the site hazard curve can be approximated in the region around ${}^{P_{\mathrm{PL}}}s_a$, i.e., in the range of values in the region of hazard levels in the proximity of the limit state probability P_{PL}, by the form

$$H(s_a)=P[S_a\geqslant s_a]=k_o s_a^{-k} \tag{5}$$

implying that the hazard curve is linear on a log-log plot in the region of interest, i.e., where the contribution to the total probability integral is greatest. Typical values of the important log-log slope k are 1–4 (Kennedy and Short 1994; Yun and Foutch 2000). It tends to be larger (steeper) for western U.S. sites and for shorter periods.

Looking more closely at the two structure-related elements—drift demand and spectral acceleration—assume that, given the level of S_a, the predicted (i.e., the conditional median) drift demand $\hat{D}$ can be represented approximately (again in the region around ${}^{P_{\mathrm{PL}}}s_a$, at least) by the form

$$\hat{D}=a(S_a)^b \tag{6}$$

An example based on a regression analysis of nonlinear dynamic results is shown in Fig. 1. In order to complete this probabilistic representation of drift given S_a, assume, as experience suggests (e.g., Shome and Cornell 1999), that drift demands are distributed lognormally about the median with the standard deviation of the natural logarithm, $\beta_{D|S_a}$. We shall refer to this measure as ''dispersion.'' This SAC/FEMA use of the symbol β for dispersion follows the tradition of the nuclear industry where the early roots of some of these developments lie. It is unfortunate that it may be confused with the ''safety index'' of the first-order second-moment (FOSM) method, the first-order reliability method (FORM), and even the AISC LRFD Commentary (AISC 1994). The notation $\beta_{D|S_a}$ emphasizes that this is the (record-to-record) dispersion for drift D at a *given* S_a level. (Note: for moderate levels, e.g., less than 0.3, the dispersion as defined here and the coefficient of variation are about equal numerically.)

There are several practical ways to estimate the three parameters a, b, and $\beta_{D|S_a}$. The most direct, in principle, is to conduct a number of nonlinear analyses and then conduct a regression analysis of $\ln D$ on $\ln S_a$ (focusing on runs in the range of ${}^{P_{\mathrm{PL}}}s_a$). One may also use incremental dynamic analyses (Luco and Cornell 1998; Vamvatsikos and Cornell 2002). But still simpler options are available, as adopted by SAC/FEMA. Experience to date

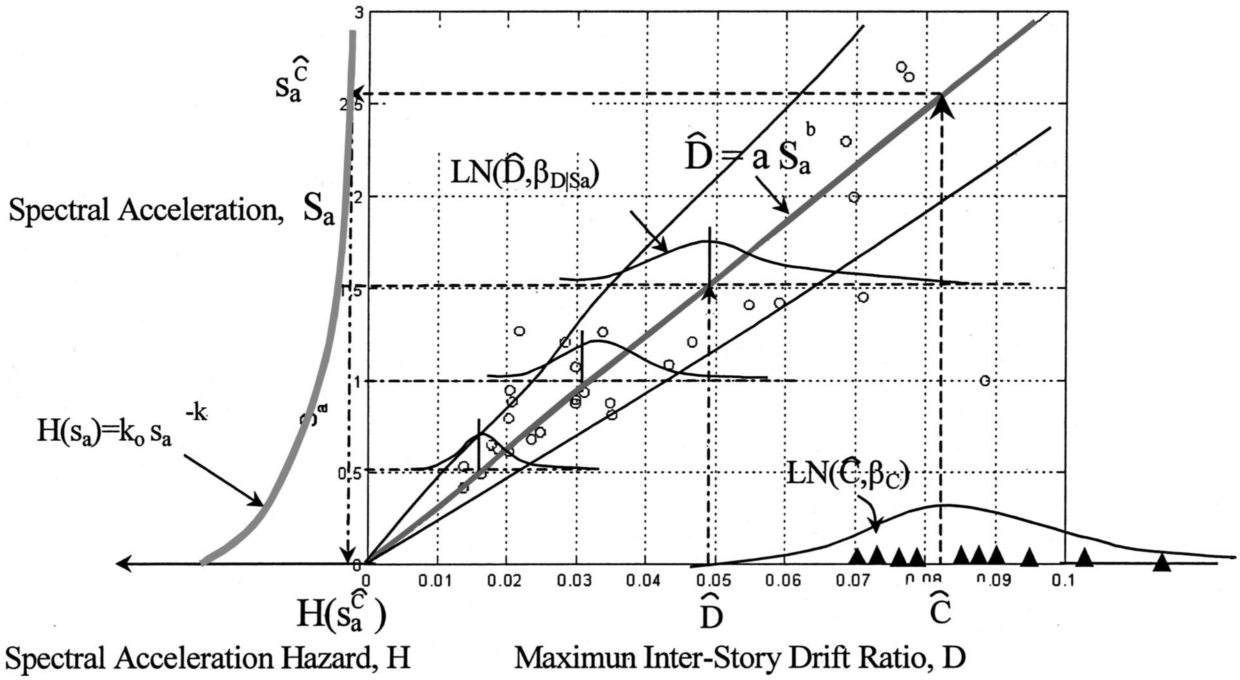

Fig. 1. Basic components: spectral acceleration hazard $H(s_a)$, lognormal (LN) distribution of drift demand D given S_a characterized by $\hat{D}$ and $\beta_{D|S_a}$, lognormal (LN) distribution of capacity variable C characterized by $\hat{C}$ and β_C. Dynamic response data points are for SAC three-story building (Luco and Cornell 1998).

(e.g., Luco and Cornell 1998, 2000) suggests that $b=1$ may be an effective default value for moment frames (see the Appendix). This assumption is consistent, for example, with the "equal displacement rule," which suggests that, for moderate-period structures without major strength degradation, inelastic displacements may be approximately equal to linear ones. The coefficient a can be estimated by simple, conventional methods [perhaps with bias correction factors, as in FEMA (2000)] or by nonlinear time history analyses. In the latter case, for accuracy in what follows, note that it is necessary only that the leading coefficient a be estimated from records with S_a levels near ${}^{P_{\mathrm{PL}}}s_a$. In fact we shall see below that records scaled to a particular common value of S_a will be sufficient. Values of $\beta_{D|S_a}$ are reported to be 0.3 or more (e.g., Luco and Cornell 2000; Yun and Foutch 2000).

With Eq. (6) and the lognormality assumption it follows that the first factor in Eq. (2) is

$$P[D\geqslant d|S_a=x]=1-\Phi(\ln[d/ax^b]/\beta_{D|S_a}) \tag{7}$$

in which $\Phi=$ widely tabulated "standardized" Gaussian distribution function. Using this result and Eq. (5), Eq. (2) for the drift hazard curve becomes, upon integration

$$H_D(d)=P[D\geqslant d]=H(s_a^d)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}\right] \tag{8}$$

in which s_a^d is defined as the spectral acceleration "corresponding to" the drift level d, that is, the inverse of Eq. (6)

$$s_a^d=(d/a)^{1/b} \tag{9}$$

Eq. (8) can be used to find the annual likelihood of exceeding any specified displacement demand recognizing that the dynamic behavior may be highly nonlinear and that for any specific ground motion there will typically be large variability ($\beta_{D|S_a}$) in the dynamic response. This implies that even ground motion intensity levels less than s_a^d may cause drift d or more. All possibilities have been included via the integral in Eq. (2).

The drift capacity C, i.e., the drift level at which the performance level will be exceeded (e.g., collapse will occur), is assumed to have a median value $\hat{C}$ and to be lognormally distributed with dispersion β_C. Estimation of these parameters is described by Yun and Foutch (2000) and Yun et al. (2002). With this assumption the first factor in Eq. (4) is

$$P[C\leqslant d]=\Phi(\ln[d/\hat{C}]/\beta_C) \tag{10}$$

Substituting and carrying out the integration one finds this *primary result*

$$P_{\mathrm{PL}}=H(s_a^{\hat{C}})\exp\left[\frac{1}{2}\frac{k^2}{b^2}(\beta^2_{D|S_a}+\beta^2_C)\right] \tag{11}$$

in which we have introduced $s_a^{\hat{C}}$ as the spectral acceleration "corresponding to" the median drift capacity

$$s_a^{\hat{C}}=(\hat{C}/a)^{1/b} \tag{12}$$

In words, this is the level of S_a that one "anticipates" will cause a drift demand equal to the median drift capacity $\hat{C}$; it is found by simply substituting $\hat{C}$ for $\hat{D}$ in Eq. (6) and solving for S_a. (It should be noted that $s_a^{\hat{C}}$ is not, however, strictly the median value of S_a given displacement $\hat{C}$.) Eq. (12) implies that, if there were no other dispersion (i.e., if the two β's were zero), P_{PL} would be found simply by substituting this "obvious" spectral acceleration value $s_a^{\hat{C}}$ into the hazard curve. The chains of steps getting from $\hat{C}$ to $H(s_a^{\hat{C}})$ are shown in Fig. 1. The effect of dispersion is to increase the P_{PL} by the exponential "correction factor" exp[(1/2)

$\times(k^2/b^2)(\beta^2_{D|S_a}+\beta^2_C)]$. It is so called because it corrects for the "total" additional randomness, both that in drift (given ground motion intensity) and that in capacity, and because the value for this "correction factor" is typically between 1.5 and 3, which is small relative to the randomness in the hazard H. Note that the exponent in the correction factor increases as the square of the net dispersion $(\beta^2_{D|S_a}+\beta^2_C)$ multiplied by the ratio k/b. This ratio is a "sensitivity factor;" a factor x change in drift leads to a change in S_a by a factor of $x^{1/b}$, which in turn implies a change of $x^{k/b}$ in the probability.

Basic Approach: Practical Format for Safety Checking

To transform this result [Eq. (11)] into a convenient, more conventional checking format, one sets the P_{PL} equal to the performance objective P_o e.g., 1/2,500 per year (or 2% in 50 years), and rearranges [making use of Eq. (5)], yielding

$$\left\{\exp\left[-\frac{1}{2}\frac{k}{b}\beta^2_C\right]\right\}\hat{C}\geqslant\left\{\exp\left[\frac{1}{2}\frac{k}{b}\beta^2_{D|S_a}\right]\right\}\hat{D}^{P_o}$$

or

$$\phi\hat{C}\geqslant\gamma\hat{D}^{P_o} \tag{13}$$

in which $\hat{D}^{P_o}$ = median drift demand under a given ground motion of intensity ${}^{P_o}s_a$, which in turn is defined as the S_a level with annual probability P_o of being exceeded, i.e., $\hat{D}^{P_o}=a({}^{P_o}s_a)^b$. We shall discuss the capacity and demand factors ϕ and γ below. Several observations about Eq. (11) are in order. Note that, if there were no dispersion in capacity and drift (given S_a), then $\hat{D}^{P_o}$ itself would be the drift demand with an annual probability P_o of being exceeded. Because these two sources of randomness (dispersion) are not zero, however, the annual probability of exceeding $\hat{D}^{P_o}$ is in fact greater than P_o, which is in turn why the capacity $\hat{C}$ must exceed $\hat{D}^{P_o}$ to ensure a probability as low as P_o. Note, too, that it is merely a happy "coincidence" of the mathematics (but sometimes misleading to first readers) that by this scheme one can establish this demand level $\hat{D}^{P_o}$ by using records with just *one* S_a intensity level, and, furthermore, that the appropriate level is just that associated with the performance objective P_o. (One uses records of this intensity in order to establish the median demand $\hat{D}^{P_o}$, which is, recall, the median demand *given* ground motion intensity ${}^{P_o}s_a$.) But this fortunate outcome of the mathematics does *not* imply that one is just "designing for the earthquake with probability P_o" or just for the ground motion level ${}^{P_{\mathrm{PL}}}s_a$. Rather, the whole range of possible levels of S_a has been considered in the integration of Eq. (4), including records with intensity levels both lower and higher than ${}^{P_{\mathrm{PL}}}s_a$, all of which are weighted by their relative likelihoods of being felt at the site.

In the second line of Eq. (13), the exponential forms have been replaced by a (drift) capacity reduction factor ϕ and a demand factor γ, which are defined and calculated simply as

$$\phi=\exp\left[-\frac{1}{2}\frac{k}{b}\beta^2_C\right] \tag{14}$$

$$\gamma=\exp\left[\frac{1}{2}\frac{k}{b}\beta^2_{D|S_a}\right] \tag{15}$$

By obtaining these explicit relationships we can ensure that we achieve the probabilistic performance objective, but we do so by way of a conventional "load-and-resistance-factor" (LRFD) format. This format is, however, now for explicitly nonlinear and dynamic behavior, and it is based on displacements rather than forces. Note that each factor, capacity and demand (given S_a), depends directly on the corresponding dispersion measure. It should be recalled, however, that there were *three* random elements identified at the beginning, the ground motion intensity measure S_a, drift demand D, and drift capacity C. The ground motion randomness enters the demand factor γ through the fact that the median drift in Eq. (13) is conditional on the ground motion intensity (i.e., spectral acceleration) level associated with the probability of exceedance P_o. The effects of randomness in all three elements have been coupled together. Note, for example, that even the capacity factor ϕ contains the sensitivity factor k/b, because, as explained above, this ratio reflects the sensitivity of probability to a change in drift, either demand or capacity. As an aside, those familiar with FOSM or FORM probabilistic bases for current static LRFD formats, e.g., AISC (1994), will appreciate that one can also interpret the exponents in the capacity and demand factors as the product of the corresponding dispersion β and a factor $k\cdot\beta/b=\beta/(b/k)$ that reflects the *relative* (probabilistic) importance of this variable. The factor roughly represents the ratio of the dispersion of the capacity to that of the intensity S_a, as measured by $1/k$, the "flatness" of the hazard curve. Again, the b parameter enters to reflect the power form of the drift versus S_a relationship.

Preliminary Practical Conclusion

Eq. (13) implies the following three steps to confirm, in practice, whether an existing building or a design of a new building meets the performance objective P_o. One (1) finds from the hazard curve the ground motion with the corresponding intensity ${}^{P_o}s_a$, (2) determines the (median) drift demand $\hat{D}$ for this S_a, and (3) compares the factored (median) capacity $\hat{C}$ against the factored $\hat{D}$. But to be complete one needs an additional consideration, uncertainty.

Uncertainty Treatment: Probability Assessment

Because scientific and professional information will always be limited, the representations above of the three elements can only be estimates. Hence the predictions based on them, such as P_{PL} in Eq. (11), are also only estimates. Because this estimation uncertainty can never be completely eliminated the best strategy is to quantify it and to allow for it in the performance objective assessment. The approach to be followed is (1) to introduce representations of this so-called "epistemic" uncertainty (as distinct from the "aleatory" randomness captured above) in each of the three elements, (2) to deduce from them the implied uncertainty representation of P_{PL}, and then (3) to reflect this result in the performance objective checking format. The uncertainty representations and the analysis will again use elementary probability, but there is a type of double bookkeeping that must go on and with a resultant notational complexity.

As above the uncertainty in the ground motion hazard curve need not be dealt with in detail here because it is common probabilistic seismic hazard analysis (PSHA) practice to represent the uncertainty in the inputs and thence in the resulting hazard curve. The latter is in the form of information such as "confidence bands" on the annual probability of exceeding any intensity level $S_a=s_a$. These indicators include the 50% confidence level [or *median estimate* $\hat{H}(s_a)$] and other upper confidence band levels (e.g., the 84%, the 95%, etc.) from which one can deduce a dispersion β_H and a *mean estimate* $\bar{H}(s_a)$. It is customary, for example, to plot both $\hat{H}$ and $\bar{H}$ hazards curves, the latter exceeding the former by $\exp[(1/2)\beta_H^2]$. As above, it is sufficient here to assume that β_H is constant in the region of interest, i.e., around ${}^{P_o}s_a$. It is assumed too that the lognormal distribution is an adequate representation of this uncertainty.

To represent the uncertainty in the drift demand representation, it is assumed that a in Eq. (6) is a (lognormally distributed) uncertain quantity with median estimate $\hat{a}$ and dispersion β_a. The implication is that (always given $S_a = s_a$) the median drift $\hat{D}$ is uncertain with median (''best'') estimate $\hat{a}(s_a)^b$ and (uncertainty) dispersion $\beta_{\hat{D}} = \beta_a$. For future notational simplicity we shall typically use $\hat{D}$ both for the (uncertain) median and for this median estimate of the median. This uncertainty dispersion reflects the degree of information available to estimate $\hat{D}$, e.g., the accuracy of the estimation method, and, in the case of time history analysis, the number of records run to estimate $\hat{D}$. For further notational simplicity and clarity we shall use, as many SAC/FEMA references do, β_{DU} for this *uncertainty* in (median) drift demand, and β_{DR} for, in contrast, the (record-to-record) *randomness* in drift. The latter dispersion was denoted $\beta_{D|S_a}$ in the section above. In order to avoid notational complexity we have dropped in subsequent sections the reference to the fact that β_{DR} and β_{DU} are associated with a given level of spectral acceleration S_a.

Finally, to represent the uncertainty in drift capacity it is assumed that the median drift capacity, denoted $\hat{C}$ in the section above, is (lognormally) uncertain with ''best estimate'' (median estimate) $\hat{\hat{C}}$ (or simply again just $\hat{C}$) and dispersion β_{CU}; the latter uncertainty dispersion is in contrast to randomness in drift capacity measured by β_{CR} (which we now use in place of the notation β_C used in the section above).

Next it must be recognized that performance level probability P_{PL} is now itself an uncertain quantity because it is a function [Eq. (11)] of the uncertain quantities $H(s_a)$, $\hat{D}$, and $\hat{C}$ just described. It is a straightforward application of probability theory (Jalayer and Cornell 2002) to deduce that, because of this uncertainty, the probability P_{PL} is lognormally distributed with parameters to be given. For cost-benefit-risk assessments it is useful to know the *mean* value (or mean estimate) of P_{PL}

$$\bar{P}_{\mathrm{PL}} = \hat{H}(s_a^{\hat{C}})\exp\left[\frac{1}{2}\beta_H^2\right]\times\exp\left[\frac{1}{2}\frac{k^2}{b^2}(\beta_{DR}^2+\beta_{DU}^2+\beta_{CR}^2+\beta_{CU}^2)\right]$$
$$=\bar{H}(s_a^{\hat{C}})\exp\left[\frac{1}{2}\frac{k^2}{b^2}(\beta_{DR}^2+\beta_{DU}^2+\beta_{CR}^2+\beta_{CU}^2)\right] \quad (16)$$

One can see that the second version of Eq. (16) looks much like Eq. (11) except it is now specified that it is the *mean* estimate of the hazard curve into which one must substitute $s_a^{\hat{C}}$, and now four dispersion contributions appear, two representing randomness and two representing uncertainty. The effect of the uncertainty in the hazard curve has been ''captured'' by using this mean (rather than the median) estimate. It is a form analogous to this latter equation (without the b) that serves as the basis for the DOE-1020 seismic guidelines (DOE 1994; Kennedy and Short 1994); those authors have chosen to set the performance objective in terms of the mean estimate of the probability of the limit state.

Uncertainty Treatment: Practical Formats

In the development of the following *structural* checking procedure, a decision was made by the SAC/FEMA project to focus on the uncertainty in the two structural elements of the problem, namely, the drift capacity and the drift demand (given the ground motion level). Subsequent analyses will bear on P_{PL} and its uncertainty, strictly speaking *given the mean hazard curve*. In other words, the uncertainty in the hazard will be presumed to have been dealt with as per the second version of Eq. (16) above, i.e., by using the *mean* estimate of the hazard curve, which reflects directly the uncertainty β_H. Consequently, confidence statements made below should strictly speaking be preceded by the phrase ''given the mean hazard curve,'' but further reference to this condition will be suppressed for simplicity.

Under this condition, the median (50% confidence) estimate of P_{PL} is

$$\hat{P}_{\mathrm{PL}} = \bar{H}(s_a^{\hat{C}})\exp\left[\frac{1}{2}\frac{k^2}{b^2}(\beta_{DR}^2+\beta_{CR}^2)\right] \quad (17)$$

which is just Eq. (11) using the *mean* estimate of the hazard curve. And the epistemic uncertainty in P_{PL} is measured by the dispersion

$$\beta_{P_{\mathrm{PL}}} = \sqrt{(k^2/b^2)(\beta_{DU}^2+\beta_{CU}^2)} \quad (18)$$

The implication of these two equations is that one can define any particular confidence level estimate of P_{PL}, call it P_{PL}^x, by

$$P_{\mathrm{PL}}^x = \hat{P}_{\mathrm{PL}}\exp[K_x\beta_{P_{\mathrm{PL}}}] \quad (19)$$

in which K_x = standardized Gaussian variate associated with probability x of not being exceeded. For example, $K_x = 1$ is associated with an 84% confidence level.

Finally, the following safety or performance checking schemes can be developed from the information above. The simplest form results from using the mean estimate of the probability P_{PL} as the objective. To do this, substitute the performance objective P_o in the second form of Eq. (16), and rearrange as done in the section above (when uncertainty was not recognized), obtaining now

$$\left\{\exp\left[-\frac{1}{2}\frac{k}{b}(\beta_{CR}^2+\beta_{CU}^2)\right]\right\}\hat{C} \geq \left\{\exp\left[\frac{1}{2}\frac{k}{b}(\beta_{DR}^2+\beta_{DU}^2)\right]\right\}\hat{D}^{P_o}$$

or

$$\phi\hat{C} \geq \gamma\hat{D}^{P_o} \quad (20)$$

in which the capacity and demand factors are defined by the obvious two exponential terms. It is sometimes useful to replace the sum of the two squared dispersions by the ''total'' squared dispersion, e.g., $\beta_{CT}^2 = \beta_{CR}^2 + \beta_{CU}^2$, and similarly for the demand term, yielding

$$\phi = \exp\left[-\frac{1}{2}\frac{k}{b}\beta_{CT}^2\right] \quad (21)$$

and

$$\gamma = \exp\left[\frac{1}{2}\frac{k}{b}\beta_{DT}^2\right] \quad (22)$$

A structure or design satisfying the condition above [Eq. (20)] can be said to have a *mean* estimate of P_{PL} less than or equal to the performance objective P_o. Further, because the mean estimate is always larger than the median estimate, it is known that satisfying the condition in Eq. (20) also implies that the confidence is somewhat greater than 50% that the true (but uncertain) P_{PL} is less than the objective.

To determine the level of confidence associated with *any* particular factored-capacity and factored-demand ratio $\phi\hat{C}/\gamma\hat{D}^{P_o}$, it can easily be shown by simple rearrangements of the results above that the relationship between this ratio, denoted λ_{con}

$$\lambda_{con}=\gamma\hat{D}^{P_o}/\phi\hat{C} \tag{23}$$

and the confidence-measuring parameter K_x is

$$\lambda_{con}=\exp\left[-K_x\beta_{UT}+\frac{1}{2}\frac{k}{b}\beta_{UT}^2\right] \tag{24}$$

in which $\beta_{UT}^2=\beta_{CU}^2+\beta_{DU}^2$ is the "total" uncertainty. Solving

$$K_x=\left[\ln(\lambda_{con})+\frac{1}{2}\frac{k}{b}\beta_{UT}^2\right]\Big/\ \beta_{UT} \tag{25}$$

Thus, determining the factored-capacity to factored-demand ratio λ_{con} [Eq. (23)] and substituting it in the previous equation [Eq. (25)] will produce K_x, from which the confidence level follows from any standard Gaussian table. This approach is used in the SAC/FEMA guidelines as an *evaluation* methodology (Yun et al. (2002).

Conversely, if, as SAC/FEMA has chosen for design requirements, one wants to *set* the criterion that there must be a confidence of at least 90% that the actual (but uncertain) probability of the limit state is less than the objective P_o, then the checking procedure or format becomes the following: ensure that the ratio of factored capacity to factored demand, $\lambda_{con}=\gamma\hat{D}^{P_o}/\phi\hat{C}$, is greater than a certain critical value λ_{con} found by substituting the appropriate value of K_x into Eq. (23). For example, for 90% confidence K_x needs to be 1.28, and λ_{con} may need to be 1.3–1.7, say, depending primarily [Eq. (23)] on the level of β_{UT}, which measures the "total uncertainty" in capacity and demand (given, strictly speaking, recall, the mean hazard curve).

This completes only the formal, probabilistic basis of the SAC/FEMA guidelines. As described by Yun et al. (2002), the generic development here has been expanded for clarity in application into a larger set of dispersions, such as $\beta_{damping}$ associated with uncertainty in estimating the damping value of the structure, $\beta_{live\ load}$ associated with uncertainty in the live load, and $\beta_{material\ property}$ associated with uncertainty in material properties. This leads to the calculation of demand and capacity factors γ and ϕ each covering various elements of the entire problem. Their relationship to the general dispersions and factors presented here should be readily apparent. The practical implementation of this formal basis has required a major effort.

Summary

A probabilistic framework was developed for seismic design and assessment of structures and applied to steel moment-resisting frame buildings. This framework was based on realizing a performance objective, expressed as the probability of exceeding a specified performance level for the structure in question. Performance levels described the desired level of structural behavior in terms of generic structural variables, demand and capacity. Demand and capacity were represented by an explicitly nonlinear, dynamic, and displacement-based structural response, the maximum interstory drift ratio. The framework development involved introduction of a ground motion intensity measure into the problem. This ground motion intensity measure was characterized by the level of the spectral acceleration S_a at the approximate first natural period of the structure, and 5% or higher damping. Separate probabilistic models (distributions) were used to describe the randomness and uncertainty in the structural demand given the ground motion level, and the structural capacity. Demand and capacity may be defined at the local level or at the system level. A common probabilistic tool (the total probability theorem) was used to convolve the probability distributions for demand, capacity, and ground motion intensity hazard. This provided an analytical expression for the probability of exceeding the performance level as the primary product of framework development. Consideration of uncertainty in the probabilistic modeling of demand and capacity allowed for the definition of confidence statements for the likelihood performance objective being achieved. The framework was rearranged into a LRFD-like format with "load" and "resistance" being replaced by "demand" and "capacity." One such format, which is adopted by the SAC/FEMA guidelines, includes an explicit quantification of the confidence level at which the objective has been achieved.

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Appendix

This Appendix discusses in more detail certain items in the body of the text. The properties of first-mode spectral acceleration as an effective scalar measure of ground motion intensity for purposes of prediction of nonlinear drift demand of steel moment-resisting frames has been studied by Shome et al. (1998). It gives rise to comparatively small dispersion values and relatively small (conditional) sensitivity to magnitude provided the structure is first-mode dominated and of moderate period [see Shome and Cornell (1999) for a discussion of first-mode dominated and/or moderate-period structures]. For other structures, specifically for tall, long-period structures one must choose the drift estimation method and/or the recordings used in nonlinear time history analyses with some care to ensure unbiased and low-variance estimates (Shome and Cornell 1999).

The possibility of correlation between the random (record-to-record variability) aspects of drift demand and capacity has received comparatively little attention to date. As yet undocumented studies by Cornell and Jalayer indicate that the correlation between random drift demand and random capacity is not large. If it is identified subsequently to be significant, it can be included in the formulation without undue difficulty (Jalayer and Cornell 2002) as will be seen below. In deliberations for the SAC/FEMA project it was concluded that, in contrast, such correlation was strong with respect to the (epistemic) uncertainty in the estimates of the median global (collapse) drift capacity and of the median drift demand at larger ground motion levels where significant nonlinear behavior would be involved. In particular, uncertainties (in log medians) due to nonlinear modeling issues such as P-Δ effects were deemed likely to be effectively *negatively* correlated. If, for example, the stiffness with respect to large drifts were underestimated, then the drift demands would be overestimated and the drift capacity would be underestimated to comparable degrees. The implication is that, conditional on the spectral acceleration level, the uncertainty in the critical ratio of (median) demand to (median) capacity due to this so-called nonlinear-time-history (NTH) uncertainty issue would have total (squared) dispersion $\beta_{UT}^2=\beta_{UD}^2+\beta_{UC}^2-2\rho\beta_{UD}\beta_{UC}\approx\beta_{UD}^2+3\beta_{UC}^2$, in which it has been assumed that $\beta_{UC}\approx\beta_{UD}$ and that the correlation coefficient ρ is -1. In the implementation of SAC/FEMA, therefore, the NTH uncertainty in capacity was simply treated as if it were three times its assigned (marginal) value (Yun et al. 2002).

As discussed in the text, in the implementation of SAC/FEMA it was assumed that the power b in Eq. (6) was approximately unity. Experience with steel frame drift estimates in the SAC project and elsewhere has shown that the nonlinear drifts are typically approximately equal to (or less than) the drifts of a linear model under the same records for ground motions up to and even beyond the 2% in 50 years level of prime interest in the project. At larger motions the effects of P-Δ and/or connection-degradation cause the (median of) incremental dynamic analyses to ''soften,'' implying that a local fit to Eq. (6) would produce $b>1$. The use of $b>1$ would make the values of the demand and capacity factors closer to unity. For all these reasons the simplifying assumption of $b=1$ was deemed appropriate.

It is also possible to construct a format in which the global stability limit or capacity, as determined from incremental dynamic analyses (Yun et al. 2002), is represented directly in terms of the (random) spectral acceleration S_a required to induce the structural instability. This random capacity is then ''compared'' to the S_a demand as represented by the hazard curve $H(s_a)$, bypassing the need for an explicit incorporation of the drift per se. The resulting format (Jalayer and Cornell 2002) involves comparing a (factored) S_a capacity versus the value ${}^{P_o}s_a$ associated with the performance objective probability level. This format may have advantages for the engineer who is conducting an evaluation that includes finding the global stability by independent analysis rather than from the default tables of the SAC/FEMA guidelines. The results are typically comparable (Jalayer and Cornell 2002).

Notation

The following symbols are used in this paper:

a, b = regression coefficients for linear regression of drift demand D on intensity S_a in logarithmic space;
C = capacity variable for structural demand D;
$\hat{C}$ = median drift capacity;
D = generic displacement-based structural demand variable;
$\hat{D}$ = median drift demand;
$\hat{D}^{P_o}$ = median drift demand under ground motion of intensity ${}^{P_o}s_a$;
$H(s_a)$ = hazard function of spectral acceleration, annual probability that intensity S_a at site will equal or exceed s_a;
$\hat{H}(s_a)$ = median estimate of spectral acceleration hazard;
$\overline{H}(s_a)$ = mean estimate of spectral acceleration hazard;
$H_D(d)$ = hazard function of drift, mean annual probability that drift demand D exceeds any specific value d;
K_x = standardized Gaussian variate associated with probability x of not being exceeded;
k_o, k = coefficients for linear regression of hazard $H(s_a)$ on intensity S_a in proximity of limit state probability $P_{\rm PL}$ (region of interest) in logarithmic space;
$P_{\rm PL}$ = annual probability of performance level not being met;
$\hat{P}_{\rm PL}$ = median estimate of $P_{\rm PL}$;
$\overline{P}_{\rm PL}$ = mean estimate of $P_{\rm PL}$;
$P_{\rm PL}^x$ = x confidence level estimate of $P_{\rm PL}$;
P_o = specific value for annual probability of performance level not being met;
S_a = elastic spectral acceleration (measure of ground motion intensity);
S_D = elastic spectral displacement (measure of ground motion intensity);
s_a^d = spectral acceleration ''corresponding to'' drift demand level d;
${}^{P_{\rm PL}}s_a$ = spectral acceleration at a hazard level equal to the limit state probability $P_{\rm PL}$;
β_C = dispersion measure for drift capacity C (standard deviation of natural logarithm);
β_{CR} = dispersion measure for randomness in drift capacity;
β_{CU} = dispersion measure for uncertainty in drift capacity;
$\beta_{D|S_a}$ = dispersion measure for drift demand D at given S_a level;
β_{DR} = dispersion measure for randomness in drift demand (given S_a);
β_{DU} = dispersion measure for uncertainty in drift demand (given S_a);
β_H = dispersion measure for hazard;
$\beta_{P_{\rm PL}}$ = dispersion measure for uncertainty in $P_{\rm PL}$;
β_{UT} = total uncertainty dispersion measure, measure of total uncertainty in demand (given S_a) and capacity;
γ = drift demand factor;
$\lambda_{\rm con}$ = factored-demand to factored-capacity ratio (measure of level of confidence associated with likelihood of performance objective being achieved); and
ϕ = (drift) capacity reduction factor.

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