

## SEISMIC PROTECTIVE SYSTEMS: PASSIVE ENERGY DISSIPATION

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Virginia Polytechnic Institute & State University



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## Major Objectives

- Illustrate why use of passive energy dissipation systems may be beneficial
- Provide overview of types of energy dissipation systems available
- Describe behavior, modeling, and analysis of structures with energy dissipation systems
- Review developing building code requirements



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## Outline: Part I

- Objectives of Advanced Technology Systems and Effects on Seismic Response
- Distinction Between Natural and Added Damping
- Energy Distribution and Damage Reduction
- Classification of Passive Energy Dissipation Systems



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## Outline: Part II

- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
- Models for Velocity-Dependent Dampers
- Effects of Linkage Flexibility
- Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
- Concept of Equivalent Viscous Damping
- Modeling Considerations for Structures with Passive Energy Dissipation Systems



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## Outline: Part III

- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
- Representations of Damping
- Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
- Summary of MDOF Analysis Procedures



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## Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems



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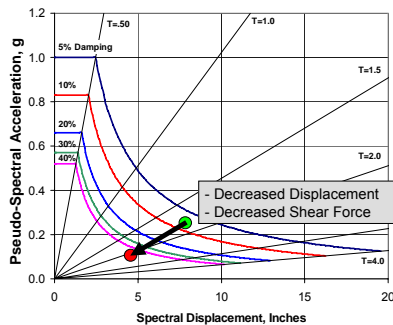


### Objectives of Energy Dissipation and Seismic Isolation Systems

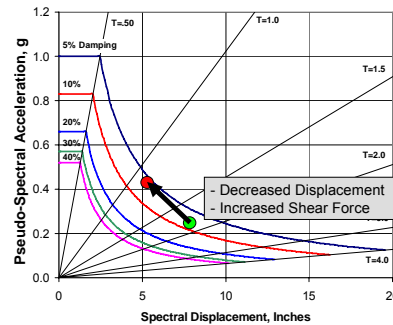
- Enhance performance of structures at all hazard levels by:
  - Minimizing interruption of use of facility (e.g., *Immediate Occupancy Performance Level*)
  - Reducing damaging deformations in structural and nonstructural components
  - Reducing acceleration response to minimize contents-related damage



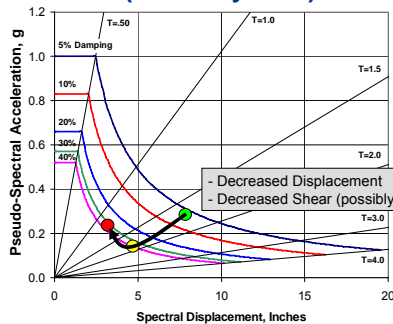
### Effect of Added Damping (Viscous Damper)



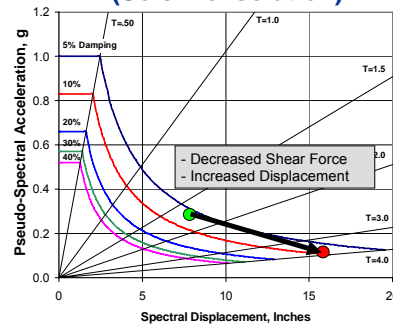
### Effect of Added Stiffness (Added Bracing)

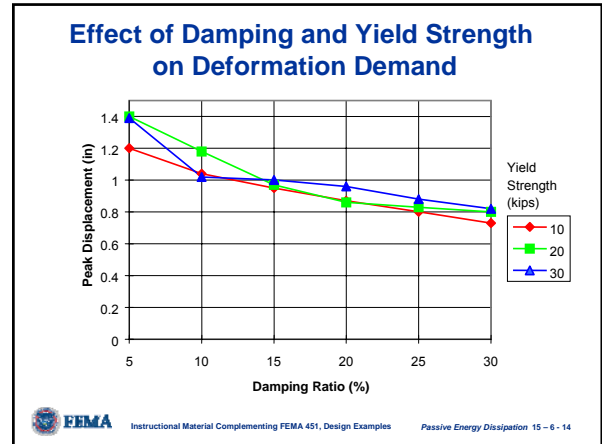
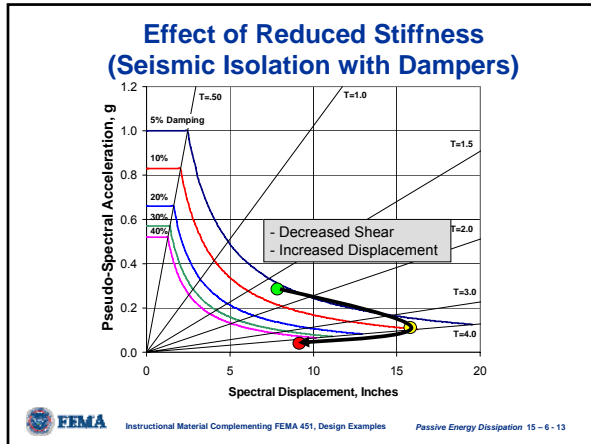


### Effect of Added Damping and Stiffness (ADAS System)



### Effect of Reduced Stiffness (Seismic Isolation)





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### Distinction Between Natural and Added Damping

**Natural (Inherent) Damping**

$\xi$  is a structural property, dependent on system mass, stiffness, and inherent energy dissipation mechanisms

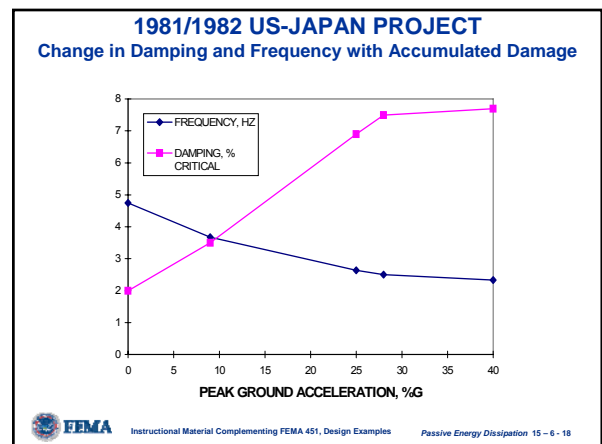
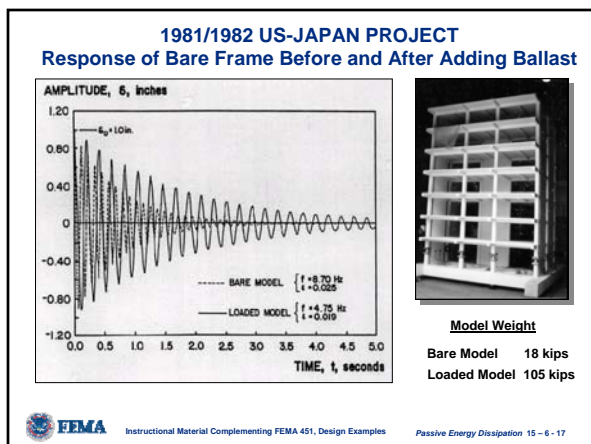
$\xi_{NATURAL} = 0.5 \text{ to } 7.0\%$

**Added Damping**

$\xi$  is a structural property, dependent on system mass, stiffness, and the added damping coefficient C

$\xi_{ADDED} = 10 \text{ to } 30\%$

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### Outline: Part I

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### Reduction in Seismic Damage

Energy Balance:

$$E_I = E_S + E_K + (E_{DI} + E_{DA}) + E_H$$

Inherent Damping

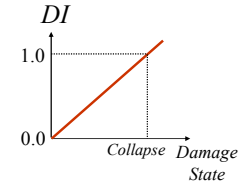
Hysteretic Energy

Added Damping

Damage Index:

$$DI(t) = \frac{u_{max}}{u_{ult}} + \rho \frac{E_H(t)}{F_y u_{ult}}$$

Source: Park and Ang (1985)



### Duration-Dependent Damage Index

$$DI(t) = \frac{u_{max}}{u_{ult}} + \rho \frac{E_H(t)}{F_y u_{ult}}$$

Source: Park and Ang (1985)

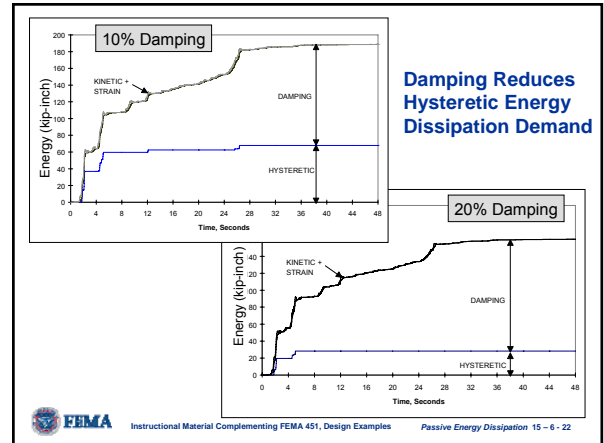
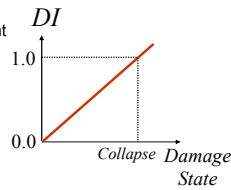
$u_{max}$  = maximum displacement

$u_{ult}$  = monotonic ultimate displacement

$\rho$  = calibration factor

$E_H$  = hysteretic energy dissipated

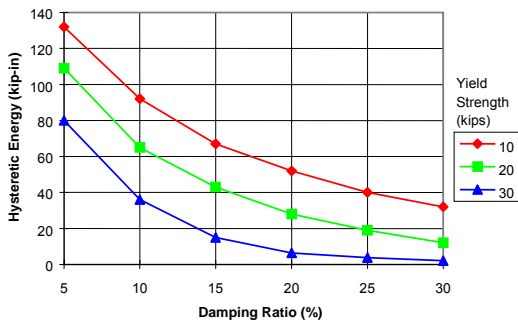
$F_y$  = monotonic yield force



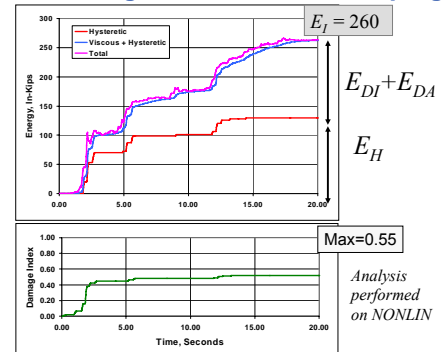
Damping Reduces Hysteretic Energy Dissipation Demand



### Effect of Damping and Yield Strength on Hysteretic Energy



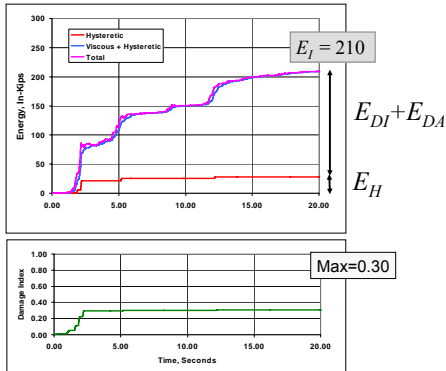
### Energy and Damage Histories, 5% Damping



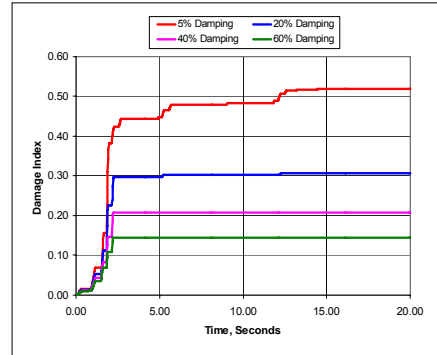
Analysis performed on NONLIN



## Energy and Damage Histories, 20% Damping



## Reduction in Damage with Increased Damping



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### Classification of Passive Energy Dissipation Systems

#### Velocity-Dependent Systems

- Viscous fluid or viscoelastic solid dampers
- May or may not add stiffness to structure

#### Displacement-Dependent Systems

- Metallic yielding or friction dampers
- Always adds stiffness to structure

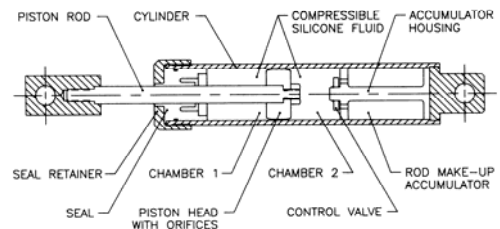
#### Other

- Re-centering devices (shape-memory alloys, etc.)
- Vibration absorbers (tuned mass dampers)

### Outline: Part II

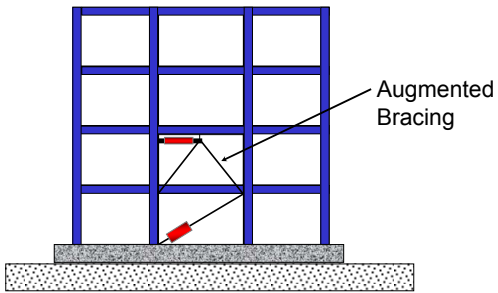
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### Cross-Section of Viscous Fluid Damper

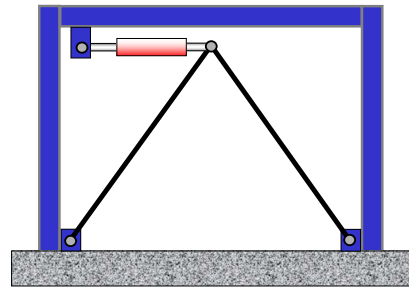


Source: Taylor Devices, Inc.

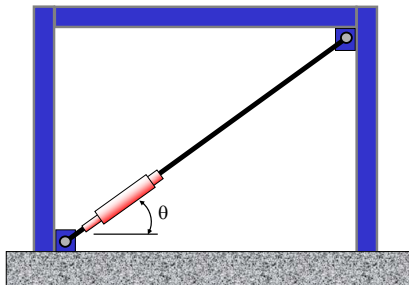
### Possible Damper Placement Within Structure



### Chevron Brace and Viscous Damper



### Diagonally Braced Damping System



### Fluid Dampers within Inverted Chevron Brace

Pacific Bell North Area Operation Center (911 Emergency Center)  
Sacramento, California  
(3-Story Steel-Framed Building Constructed in 1995)



62 Dampers: 30 Kip Capacity, +/-2 in. Stroke

### Fluid Damper within Diagonal Brace

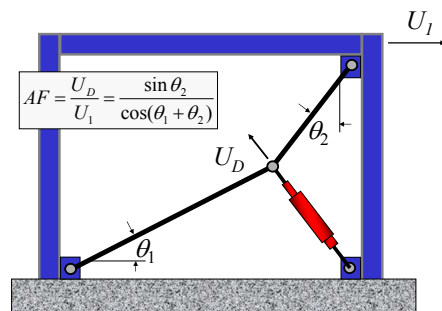


Huntington Tower  
Boston, MA



San Francisco State  
Office Building  
San Francisco, CA

### Toggle Brace Damping System



## Toggle Brace Deployment



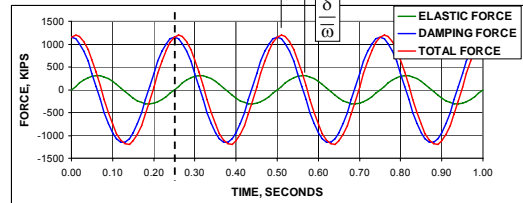
**Huntington Tower, Boston, MA**  
 - New 38-story steel-framed building  
 - 100 direct-acting and toggle-brace dampers  
 - 1300 kN (292 kips), +/- 101 mm (+/- 4 in.)  
 - Dampers suppress wind-induced vibration

## Harmonic Behavior of Fluid Damper

$$u(t) = u_0 \sin(\bar{\omega} t) \quad \text{Imposed Motion}$$

$$P(t) = P_0 \sin(\bar{\omega} t) \cos(\delta) + P_0 \cos(\bar{\omega} t) \sin(\delta) \quad \text{Total Force}$$

Phase Angle (Lag)

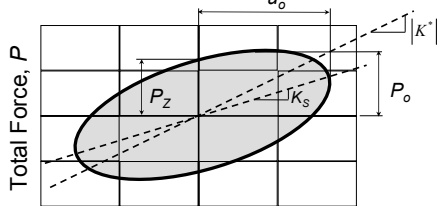


Note: Damping force 90° out-of-phase with elastic force.

$$P(t) = K_S u(t) + C \dot{u}(t)$$

$$K_S = \frac{P_0}{u_0} \cos(\delta) \quad K_L = \frac{P_0}{u_0} \sin(\delta) \quad C = \frac{K_L}{\bar{\omega}} \quad \delta = \sin^{-1}\left(\frac{P_Z}{P_0}\right)$$

Storage Stiffness      Loss Stiffness      Damping Coeff.      Phase Angle



$$P_Z = K_L u_0 = P_0 \sin(\delta) \quad \text{Damper Displacement, } u$$

$$E_D = \pi P_Z u_0 = \pi P_0 u_0 \sin(\delta)$$

## Frequency-Domain Force-Displacement Relation

$$P(t) = K_S u(t) + C \dot{u}(t)$$

Apply Fourier Transform:

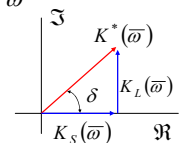
$$P(\bar{\omega}) = K_S u(\bar{\omega}) + K_L i \bar{\omega} u(\bar{\omega}) / \bar{\omega}$$

$$P(\bar{\omega}) = [K_S + i K_L] u(\bar{\omega})$$

Complex Stiffness:

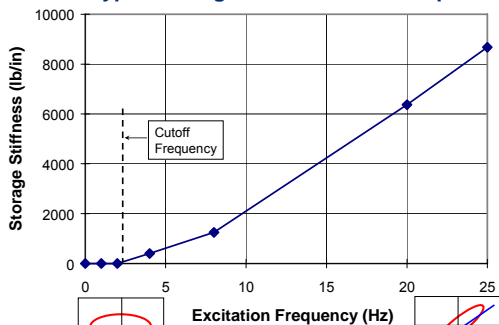
$$K^*(\bar{\omega}) = \frac{P(\bar{\omega})}{u(\bar{\omega})}$$

$$P(\bar{\omega}) = K^*(\bar{\omega}) u(\bar{\omega}) \quad \text{Compact Force-Displ. Relation for Viscoelastic Dampers}$$

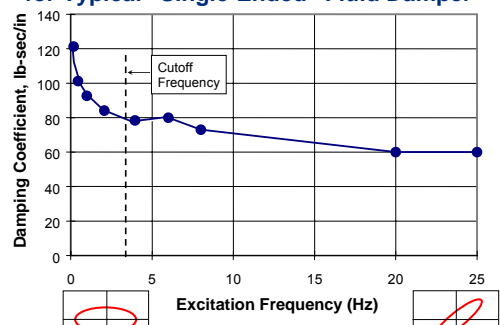


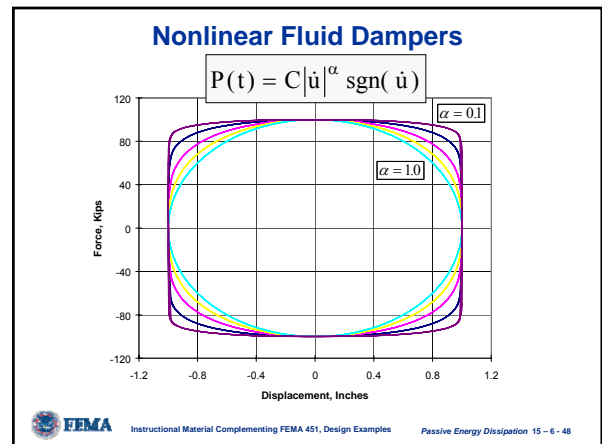
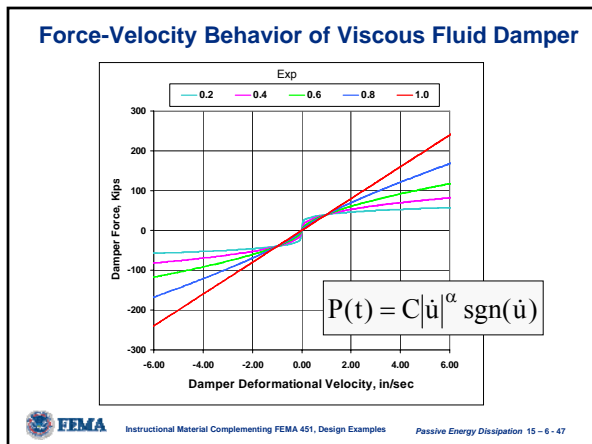
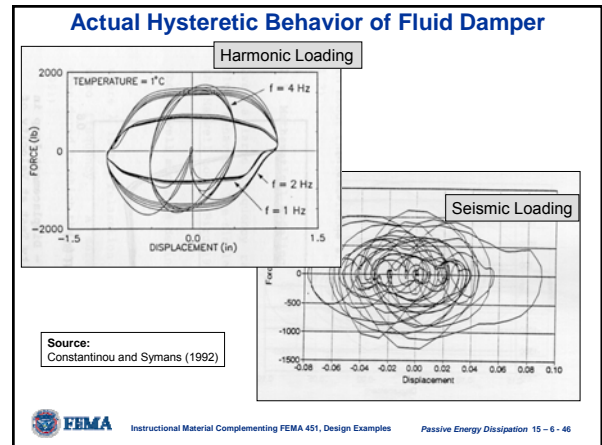
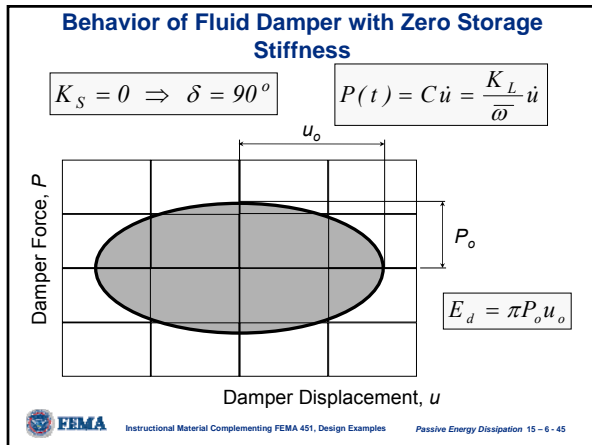
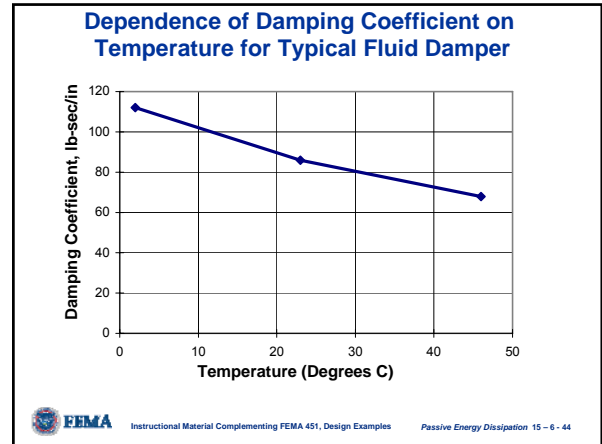
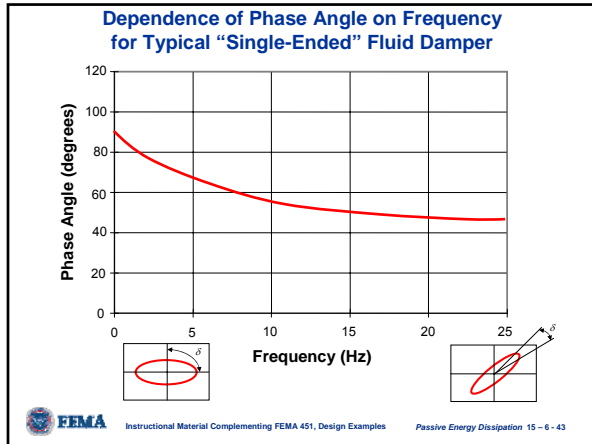
Note:  $\Re(K^*) = K_S$  and  $\Im(K^*) = K_L$

## Dependence of Storage Stiffness on Frequency for Typical "Single-Ended" Fluid Damper



## Dependence of Damping Coefficient on Frequency for Typical "Single-Ended" Fluid Damper







### Energy Dissipated Per Cycle for Linear and Nonlinear Viscous Fluid Dampers

Linear Damper:  $E_D = \pi P_o u_o$

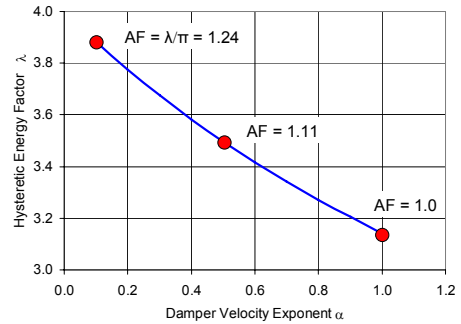
Hysteretic Energy Factor

Nonlinear Damper:  $E_D = \lambda P_o u_o$

$$\lambda = 4 \times 2^\alpha \frac{\Gamma^2\left(1 + \frac{\alpha}{2}\right)}{\Gamma(2 + \alpha)}$$

$\Gamma$  = Gamma Function

### Relationship Between $\lambda$ and $\alpha$ for Viscous Fluid Damper



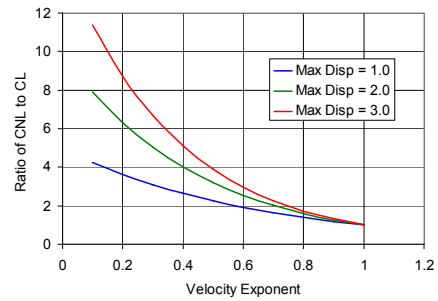
### Relationship Between Nonlinear and Linear Damping Coefficient for Equal Energy Dissipation Per Cycle

$$\frac{C_{NL}}{C_L} = \frac{\pi}{\lambda} (u_o \bar{\omega})^{1-\alpha}$$

Note: Ratio is frequency- and displacement-dependent and is therefore meaningful only for steady-state harmonic response.

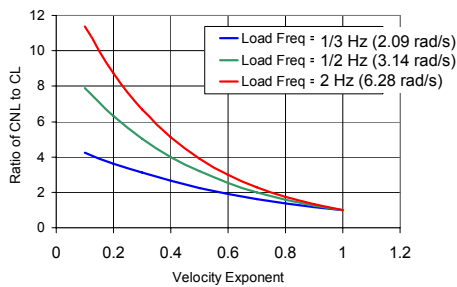
### Ratio of Nonlinear Damping Coefficient to Linear Damping Coefficient (For a Given Loading Frequency)

Loading Frequency = 1 Hz (6.28 rad/sec)

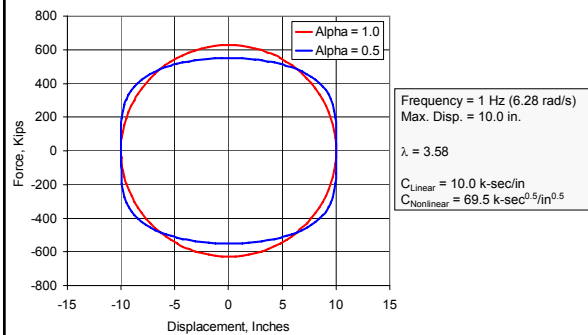


### Ratio of Nonlinear Damping Constant to Linear Damping Constant (For a Given Maximum Displacement)

Maximum Displacement = 1



### Example of Linear vs Nonlinear Damping



### Recommendations Related to Nonlinear Viscous Dampers

- Do NOT attempt to linearize the problem when nonlinear viscous dampers are used. Perform the analysis with discrete nonlinear viscous dampers.
- Do NOT attempt to calculate effective damping in terms of a damping ratio ( $\xi$ ) when using nonlinear viscous dampers.
- DO NOT attempt to use a free vibration analysis to determine equivalent viscous damping when nonlinear viscous dampers are used.



### Advantages of Fluid Dampers

- High reliability
- High force and displacement capacity
- Force Limited when velocity exponent < 1.0
- Available through several manufacturers
- No added stiffness at lower frequencies
- Damping force (possibly) out of phase with structure elastic forces
- Moderate temperature dependency
- May be able to use linear analysis

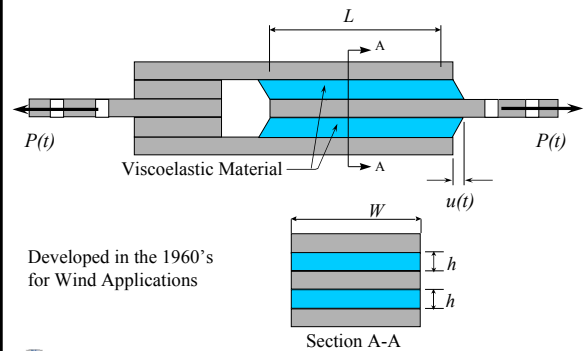


### Disadvantages of Fluid Dampers

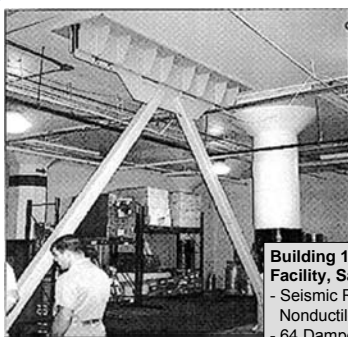
- Somewhat higher cost
- Not force limited (particularly when exponent = 1.0)
- Necessity for nonlinear analysis in most practical cases (as it has been shown that it is generally not possible to add enough damping to eliminate all inelastic response)



### Viscoelastic Dampers



### Implementation of Viscoelastic Dampers



**Building 116, US Naval Supply Facility, San Diego, CA**  
 - Seismic Retrofit of 3-Story Nonductile RC Building  
 - 64 Dampers Within Chevron Bracing Installed in 1996

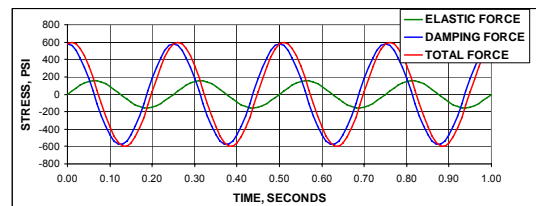


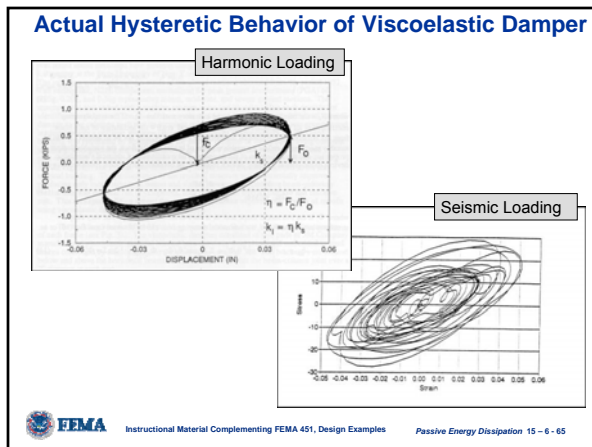
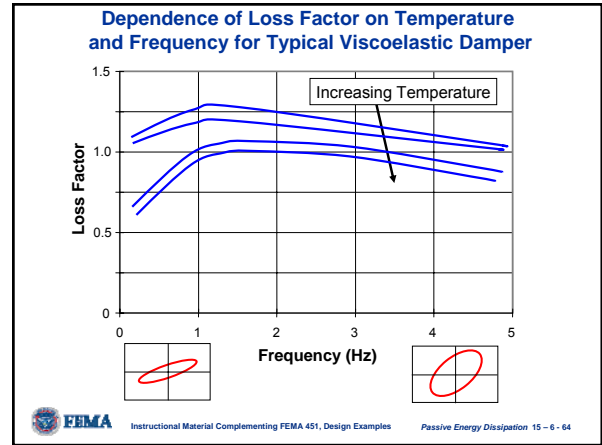
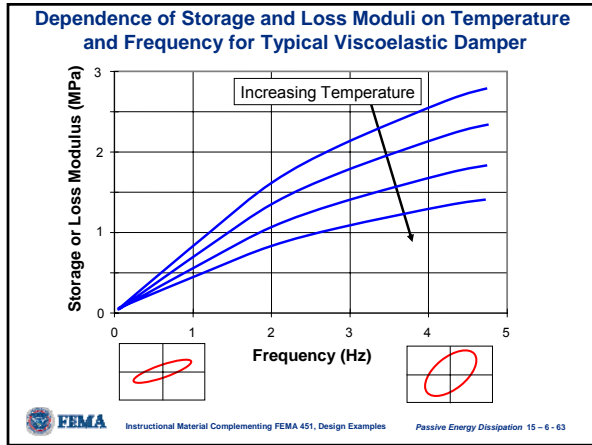
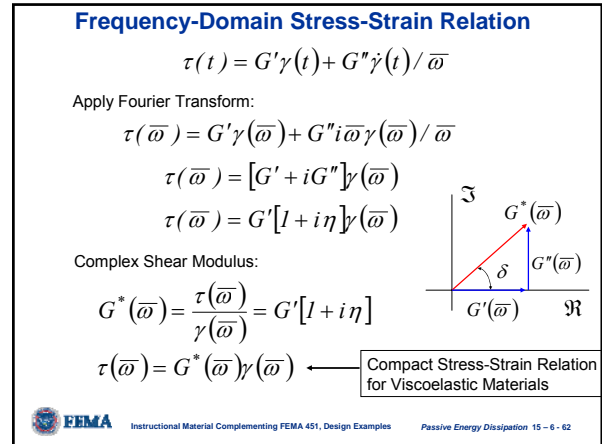
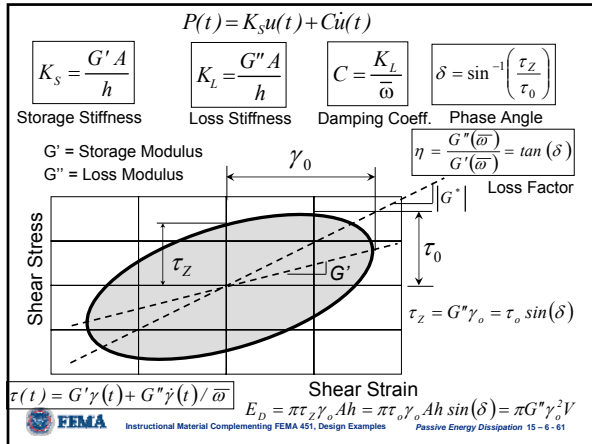
### Harmonic Behavior of Viscoelastic Damper

$$u(t) = u_0 \sin(\bar{\omega}t) \leftarrow \text{Imposed Motion}$$

$$P(t) = P_0 \sin(\bar{\omega}t) \cos(\delta) + P_0 \cos(\bar{\omega}t) \sin(\delta)$$

Labels: Total Force, Loading Frequency, Phase Angle (Lag)





- ### Advantages of Viscoelastic Dampers
- High reliability
  - May be able to use linear analysis
  - Somewhat lower cost
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## Disadvantages of Viscoelastic Dampers

- Strong Temperature Dependence
- Lower Force and Displacement Capacity
- Not Force Limited
- Necessity for nonlinear analysis in most practical cases (as it has been shown that it is generally not possible to add enough damping to eliminate all inelastic response)

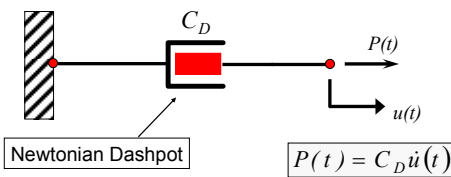


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## Modeling Viscous Dampers: Simple Dashpot

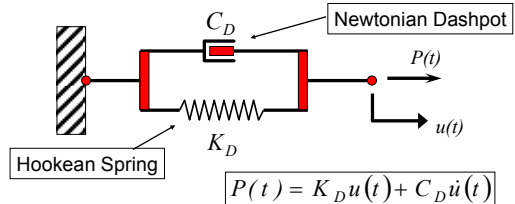


Useful For :  
Fluid Dampers with Zero Storage Stiffness

This Model Ignores Temperature Dependence



## Modeling Linear Viscous/Viscoelastic Dampers: Kelvin Model



Useful For :  
Viscoelastic Dampers and Fluid Dampers with Storage Stiffness and Weak Frequency Dependence.

This Model Ignores Temperature Dependence



## Kelvin Model (Continued)

$$P(t) = K_D u(t) + C_d \dot{u}(t)$$

Apply Fourier Transform:

$$P(\bar{\omega}) = [K_D + i\bar{\omega}C_d]u(\bar{\omega}) \quad K_S(\bar{\omega})$$

Complex Stiffness:

$$K^*(\bar{\omega}) = K_D + i\bar{\omega}C_d$$

Storage Stiffness:

$$K_S(\bar{\omega}) = \Re[K^*(\bar{\omega})] = K_D$$

Loss Stiffness:

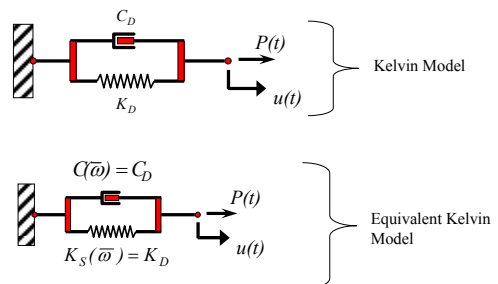
$$K_L(\bar{\omega}) = \Im[K^*(\bar{\omega})] = C_D \bar{\omega}$$

Damping Coefficient:

$$C(\bar{\omega}) = \frac{K_L(\bar{\omega})}{\bar{\omega}} = C_D$$



## Kelvin Model (Continued)



### Modeling Linear Viscous/Viscoelastic Dampers: Maxwell Model

Newtonian Dashpot;  
Hookean Spring

$$P(t) + \frac{C_D}{K_D} \dot{P}(t) = C_D \dot{u}(t)$$

Useful For :  
Viscoelastic Dampers and Fluid Dampers with Strong Frequency Dependence.  
This Model Ignores Temperature Dependence

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### Maxwell Model (Continued)

$$P(t) + \frac{C_D}{K_D} \dot{P}(t) = C_D \dot{u}(t)$$

Apply Fourier Transform:  

$$P(\bar{\omega}) + i\bar{\omega} \frac{C_D}{K_D} P(\bar{\omega}) = i\bar{\omega} C_D u(\bar{\omega})$$

Complex Stiffness:  

$$K^*(\bar{\omega}) = \frac{C_D \lambda \bar{\omega}^{-2}}{1 + \lambda^2 \bar{\omega}^{-2}} + i \frac{C_D \bar{\omega}}{1 + \lambda^2 \bar{\omega}^{-2}}$$

Relaxation Time:  $\lambda = C_D / K_D$

Storage Stiffness:  

$$K_S(\bar{\omega}) = \Re[K^*(\bar{\omega})] = \frac{K_D \lambda^2 \bar{\omega}^{-2}}{1 + \lambda^2 \bar{\omega}^{-2}}$$

Loss Stiffness:  

$$K_L(\bar{\omega}) = \Im[K^*(\bar{\omega})] = \frac{C_D \bar{\omega}}{1 + \lambda^2 \bar{\omega}^{-2}}$$

$K_S(\bar{\omega})$

$K_D$

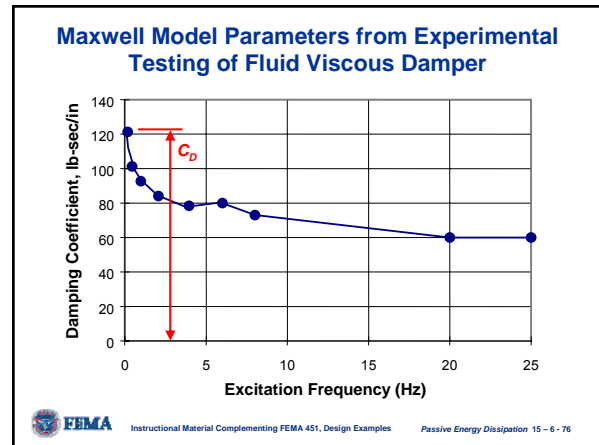
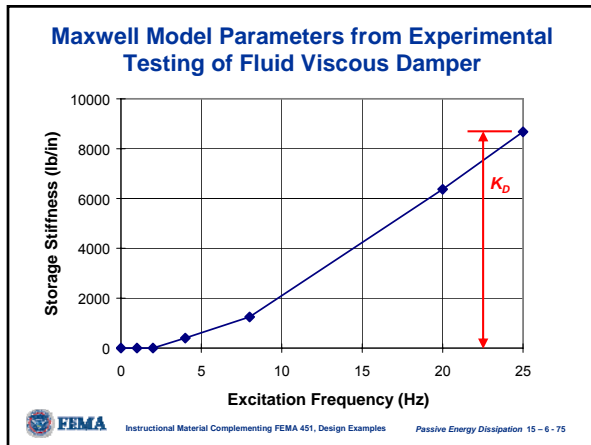
$C(\bar{\omega})$

$C_D$

Damping Coefficient:  

$$C(\bar{\omega}) = \frac{K_L(\bar{\omega})}{\bar{\omega}} = \frac{C_D}{1 + \lambda^2 \bar{\omega}^{-2}}$$

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### Maxwell Model (Continued)

Maxwell Model

Equivalent Kelvin Model

$$C(\bar{\omega}) = \frac{C_D}{1 + \lambda^2 \bar{\omega}^2}$$

$$K_S(\bar{\omega}) = \frac{K_D \lambda^2 \bar{\omega}^2}{1 + \lambda^2 \bar{\omega}^2}$$

Note:

- If  $K_D$  is very large,  $\lambda$  is very small,  $K_S$  is small and  $C = C_D$
- If  $C_D$  is very small,  $\lambda$  is very small,  $K_S$  is small and  $C = C_D$
- If  $K_D$  is very small,  $\lambda$  is very large,  $C$  is small and  $K_S = K_D$

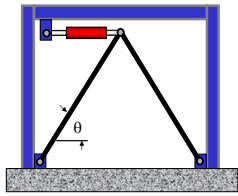
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### Outline: Part II

- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
- Models for Velocity-Dependent Dampers
- Effects of Linkage Flexibility
- Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
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### Effect of Linkage Flexibility on Damper Effectiveness



Because the damper is always in series with the linkage, the damper-brace assembly acts like a Maxwell model.

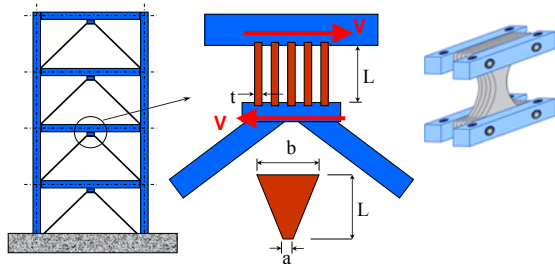
Hence, the effectiveness of the damper is reduced. The degree of lost effectiveness is a function of the structural properties and the loading frequency.

$$K_{\text{Brace, Effective}} = 2 \frac{AE}{L} \cos^2 \theta$$

### Outline: Part II

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### Steel Plate Dampers (Added Damping and Stiffness System - ADAS)

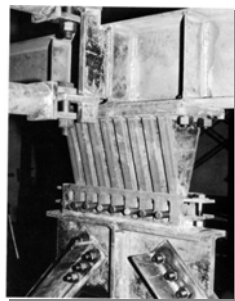


### Implementation of ADAS System

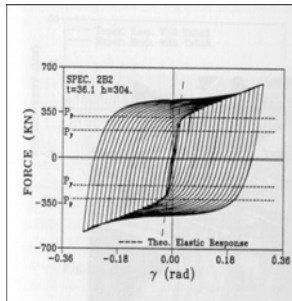


**Wells Fargo Bank, San Francisco, CA**  
 - Seismic Retrofit of Two-Story Nonductile Concrete Frame; Constructed in 1967  
 - 7 Dampers Within Chevron Bracing Installed in 1992  
 - Yield Force Per Damper: 150 kips

### Hysteretic Behavior of ADAS Device



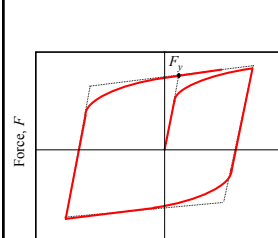
ADAS Device (Tsai et al. 1993)



Experimental Response (Static) (Source: Tsai et al. 1993)

### Ideal Hysteretic Behavior of ADAS Damper

(SAP2000 and ETABS Implementation)



Displacement, D.

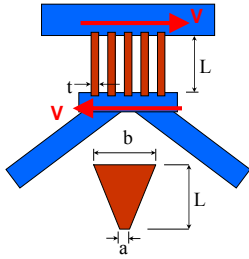
$$F = \beta k D + (1 - \beta) F_y Z$$

$$\dot{Z} = \frac{k}{F_y} \begin{cases} \dot{D} (1 - |Z|^\alpha) & \text{if } \dot{D} \cdot Z > 0 \\ \dot{D} & \text{otherwise} \end{cases}$$

Yield Sharpness

Z is a Path Dependency Parameter

### Parameters of Mathematical Model of ADAS Damper

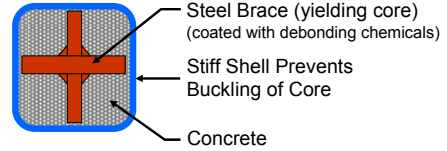


$$k = \frac{n(2 + a/b)EI_b}{L^3}$$

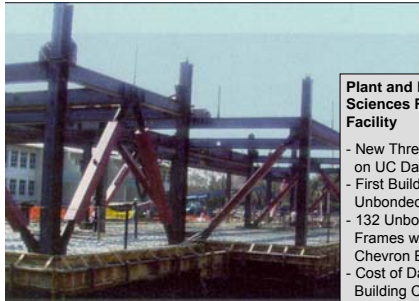
$$F_y = \frac{nf_ybt^3}{4L}$$

$n$  = Number of plates  
 $f_y$  = Yield force of each plate  
 $I_b$  = Second moment of area of each plate at  $b$  (i.e., at top of plate)

### Unbonded Brace Damper



### Implementation of Unbonded Brace Damper

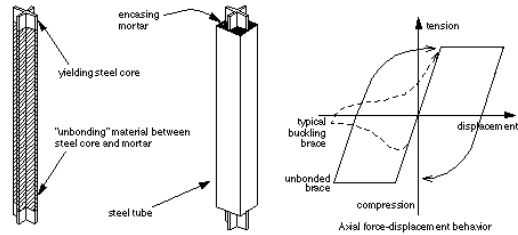


**Plant and Environmental Sciences Replacement Facility**

- New Three-Story Building on UC Davis Campus
- First Building in USA to Use Unbonded Brace Damper
- 132 Unbonded Braced Frames with Diagonal or Chevron Brace Installation
- Cost of Dampers = 0.5% of Building Cost

Source: ASCE Civil Engineering Magazine, March 2000.

### Hysteretic Behavior of Unbonded Brace Damper



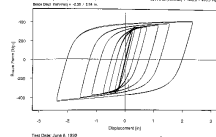
### Testing of Unbonded Brace Damper



Testing Performed at UC Berkeley



Typical Hysteresis Loops from Cyclic Testing



### Advantages of ADAS System and Unbonded Brace Damper

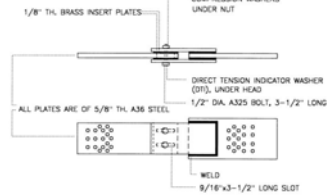
- Force-Limited
- Easy to construct
- Relatively Inexpensive
- Adds both "Damping" and Stiffness

## Disadvantages of ADAS System and Unbonded Brace Damper

- Must be Replaced after Major Earthquake
- Highly Nonlinear Behavior
- Adds Stiffness to System
- Undesirable Residual Deformations Possible



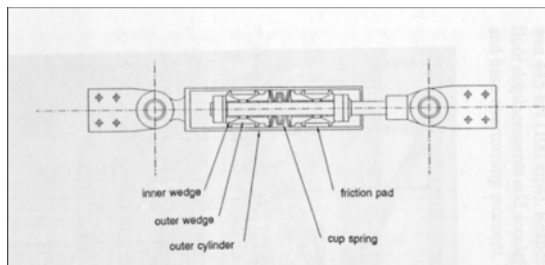
## Friction Dampers: Slotted-Bolted Damper



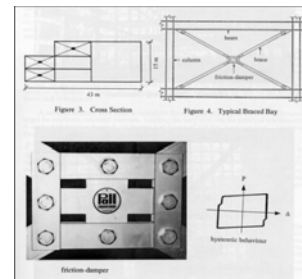
Pall Friction Damper



## Sumitomo Friction Damper (Sumitomo Metal Industries, Japan)



## Pall Cross-Bracing Friction Damper



Interior of Webster Library at Concordia University, Montreal, Canada



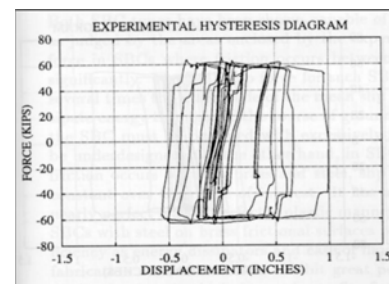
## Implementation of Pall Friction Damper



**McConnel Library at Concordia University, Montreal, Canada**  
 - Two Interconnected Buildings of 6 and 10 Stories  
 - RC Frames with Flat Slabs  
 - 143 Cross-Bracing Friction Dampers Installed in 1987  
 - 60 Dampers Exposed for Aesthetics

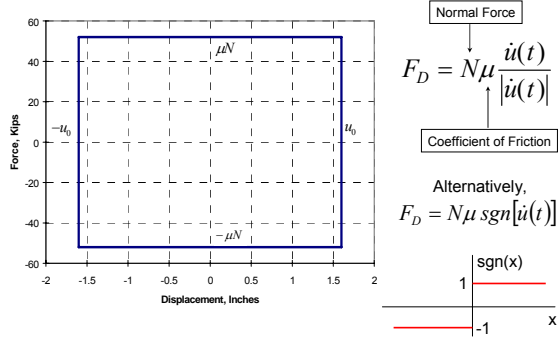


## Hysteretic Behavior of Slotted-Bolted Friction Damper





### Ideal Hysteretic Behavior of Friction Damper



### Advantages of Friction Dampers

- Force-Limited
- Easy to construct
- Relatively Inexpensive

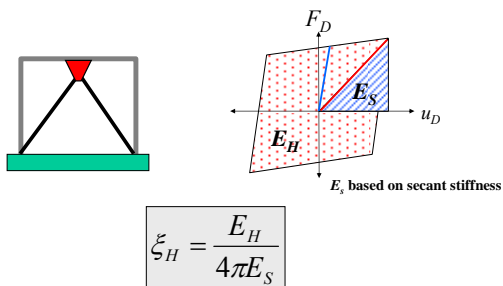
### Disadvantages of Friction Dampers

- May be Difficult to Maintain over Time
- Highly Nonlinear Behavior
- Adds Large Initial Stiffness to System
- Undesirable Residual Deformations Possible

### Outline: Part II

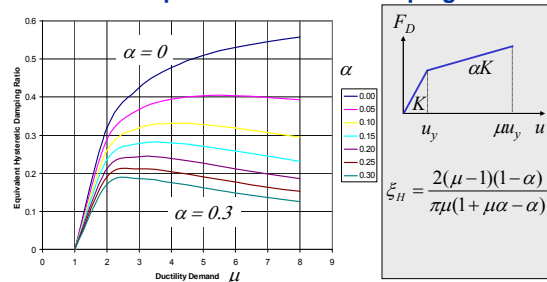
- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
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### Equivalent Viscous Damping: Damping System with Inelastic or Friction Behavior



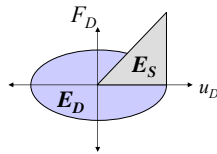
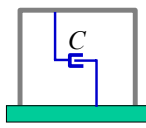
Note: Computed damping ratio is displacement-dependent

### Effect of Inelastic System Post-Yielding Stiffness on Equivalent Viscous Damping



Note: May be Modified (✗) for Other (less Robust) Hysteretic Behavior

### Equivalent Viscous Damping: “Equivalent” System with Linear Viscous Damper



$$C = 2m\omega\zeta_H$$

$E_s$  and  $\omega$  are based on Secant Stiffness of **Inelastic System**

### Equivalent Viscous Damping: Caution!

- It is not possible, on a device level, to “replace” a displacement-dependent device (e.g. a Friction Damper) with a velocity-dependent device (e.g. a Fluid Damper).
- Some simplified procedures allow such replacement on a structural level, wherein a “smeared” equivalent viscous damping ratio is found for the whole structure. This approach is marginally useful for preliminary design, and should not be used under any circumstances for final design.

### Outline: Part II

- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
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### Modeling Considerations for Structures with Passive Energy Dissipation Devices

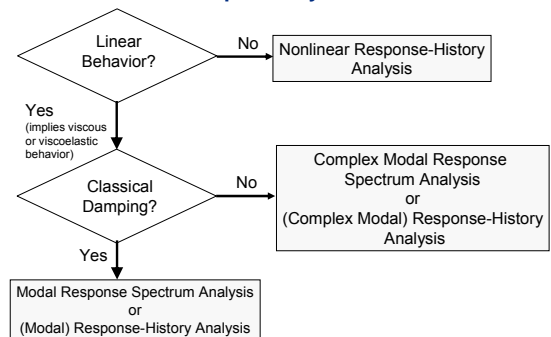
- Damping is almost always nonclassical (Damping matrix is not proportional to stiffness and/or mass)
- For seismic applications, system response is usually partially inelastic
- For seismic applications, viscous damper behavior is typically nonlinear (velocity exponents in the range of 0.5 to 0.8)

Conclusion: This is a **NONLINEAR** analysis problem!

### Outline: Part III

- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
- Representations of Damping
- Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
- Summary of MDOF Analysis Procedures

### Seismic Analysis of Structures with Passive Energy Dissipation Systems



### Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t)$$

Inherent Damping:  
Linear

Added Viscous Damping:  
Linear or **Nonlinear**

Restoring Force:  
(May include Added Devices)  
Linear or **Nonlinear**

$C_A \neq f(\omega)$

Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 109

### MDOF Solution Techniques

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t)$$

**Explicit integration of fully coupled equations:**

- Treat  $C_I$  as Rayleigh damping and model  $C_A$  explicitly.
- Use Newmark solver (with iteration) to solve full set of coupled equations.

System may be linear or nonlinear.

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### MDOF Solution Techniques

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t)$$

**Fast Nonlinear Analysis:**  
Treat  $C_I$  as modal damping and model  $C_A$  explicitly. Move  $C_A$  (and any other nonlinear terms) to right-hand side. Left-hand side may be uncoupled by Ritz Vectors. Iterate on unbalanced right-hand side forces.

System may be linear or nonlinear.

Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 111

### Fast Nonlinear Analysis

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + K_E v(t) + F_H(t) = -MR\ddot{v}_g(t)$$

Move Added Damper Forces and Nonlinear Forces to RHS:

$M\ddot{v}(t) + C_I\dot{v}(t) + K_E v(t)$   
Linear Terms

$= -MR\ddot{v}_g(t) - F_H(t) - C_A\dot{v}(t)$   
Nonlinear Terms

Transform Coordinates:  $v(t) = \Phi y(t)$

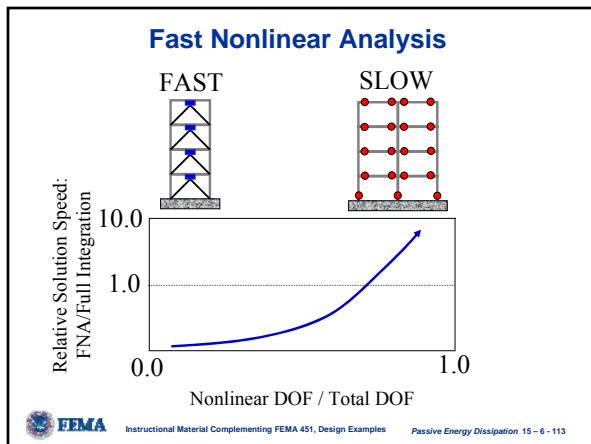
Apply Transformation:

Orthogonal basis of Ritz vectors:  
Number of vectors  $\ll N$

$\tilde{M}\ddot{y}(t) + \tilde{C}_I\dot{y}(t) + \tilde{K}_E y(t)$   
Uncoupled

$= -\Phi^T MR\ddot{v}_g(t) - \Phi^T F_H(t) - \tilde{C}_A\dot{y}(t)$   
Coupled

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### MDOF Solution Techniques

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t)$$

**Explicit integration or response spectrum analysis of first few uncoupled modal equations:**

- Treat  $C_I$  as modal damping or Rayleigh damping
- Use Modal Strain Energy method to represent  $C_A$  as modal damping ratios.

System must be linear.  
Applicable only to viscous (or viscoelastic) damping systems.

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### Outline: Part III

- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
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### Modal Damping Ratios

$$M\ddot{v} + C\dot{v} + Kv = -MR\ddot{v}_g$$

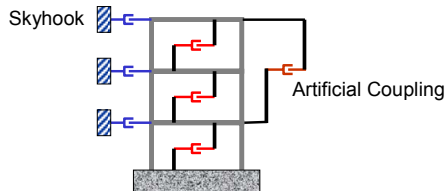
$$v = \Phi y$$

$$\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega_i^2 y_i = \Gamma_i\ddot{v}_g$$

Specify modal damping values directly

### Modal Superposition Damping

$$C = M \left[ \sum_{i=1}^n \frac{2\xi_i\omega_i}{\phi_i^T M \phi_i} \phi_i^T \phi_i \right] M$$

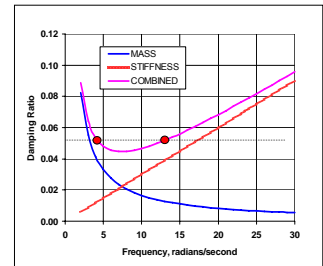
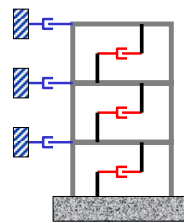


Note: There is no need to develop  $C$  explicitly.

### Rayleigh Proportional Damping

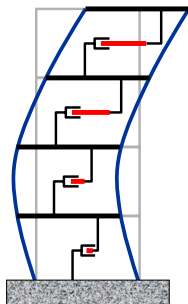
$$C_R = \alpha M + \beta K$$

Skyhook



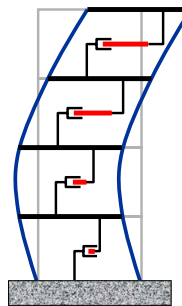
### Derivation of Modal Strain Energy Method

Deformation of Structure in its Second Mode



Floor Displacement	Damper Deformation
$\phi_{2,1}$	$\phi_{2,1} - \phi_{2,2}$
$\phi_{2,2}$	$\phi_{2,2} - \phi_{2,3}$
$\phi_{2,3}$	$\phi_{2,3} - \phi_{2,4}$
$\phi_{2,4}$	$\phi_{2,4}$

### Derivation of Modal Strain Energy Method

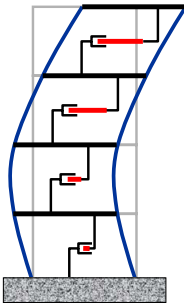


$$E_{D,i,storyk} = \pi\omega_i C_k (\phi_{i,k} - \phi_{i,k-1})^2$$

$$E_{S,i} = \frac{1}{2} \phi_i^T K \phi_i = \frac{1}{2} \omega_i^2 \phi_i^T M \phi_i$$

$$\xi_i = \frac{\sum_{k=1}^{n \text{ stories}} E_{D,i,storyk}}{4\pi E_{S,i}}$$

### Derivation of Modal Strain Energy Method



$$\xi_i = \frac{\sum_{k=1}^{N \text{ stories}} C_k (\phi_{i,k} - \phi_{i,k-1})^2}{2\omega_i \phi_i^T M \phi_i}$$

$$\xi_i = \frac{\phi_i^T C_A \phi_i}{2\omega_i \phi_i^T M \phi_i} = \frac{\phi_i^T C_A \phi_i}{2m_i^* \omega_i}$$

Note: IF  $C_A$  is diagonalized by  $\Phi$ ,  
THEN

$$\xi_i = \frac{c_i^*}{2m_i^* \omega_i}$$

### Modal Strain Energy Damping Ratio

$$\xi_i = \frac{\phi_i^T C_A \phi_i}{2m_i^* \omega_i}$$

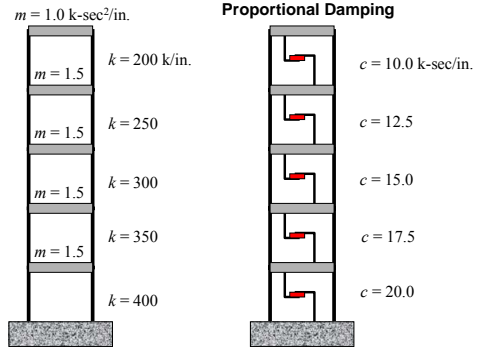
Note:  $\phi$  is the *Undamped* Mode Shape

The Modal Strain Energy Method is **approximate** if the structure is non-classically damped since the undamped and damped mode shapes are different.

### Outline: Part III

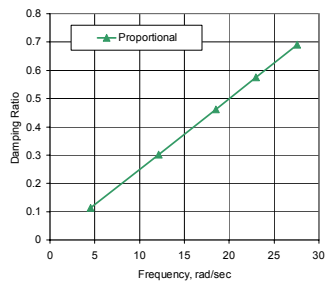
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### Example: Application of Modal Strain Energy Method

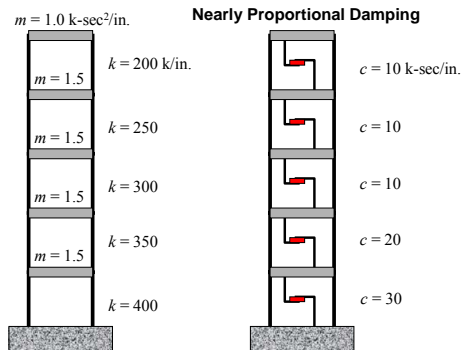


### Modal Damping Ratios from Modal Strain Energy Method for Proportional Damping Distribution

Frequency (rad/sec)	Damping Ratio, $\xi$
4.54	0.113
12.1	0.302
18.5	0.462
23.0	0.575
27.6	0.690

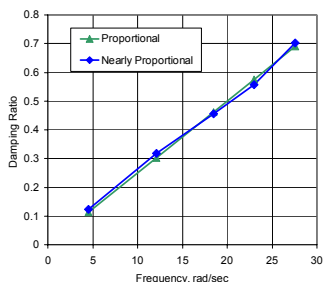


### Example: Application of Modal Strain Energy Method

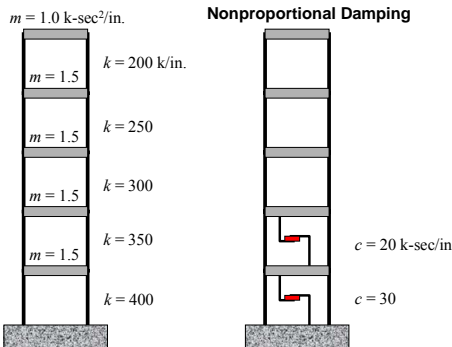


### Modal Damping Ratios from Modal Strain Energy Method for Nearly Proportional Damping Distribution

Frequency (rad/sec)	Damping Ratio, $\xi$
4.54	0.123
12.1	0.318
18.5	0.455
23.0	0.557
27.6	0.702

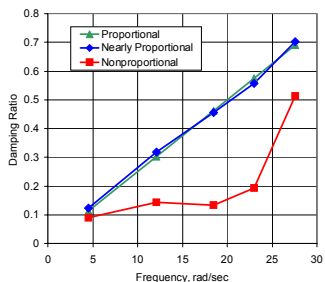


### Example: Application of Modal Strain Energy Method



### Modal Damping Ratios from Modal Strain Energy Method for Nonproportional Damping Distribution

Frequency (rad/sec)	Damping Ratio, $\xi$
4.54	0.089
12.1	0.144
18.5	0.134
23.0	0.194
27.6	0.514



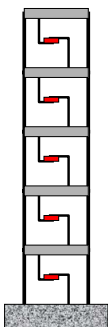
### Modal Superposition Damping

$$C = M \left[ \sum_{i=1}^n \frac{2\xi_i \omega_i}{\phi_i^T M \phi_i} \phi_i^T \phi_i \right] M$$

Modal Superposition Damping can be used to construct the damping matrix from the modal damping ratios obtained via the Modal Strain Energy Method

### Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Proportional Damping



Actual Damping Matrix

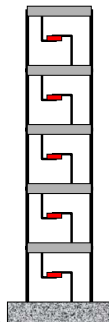
$$C_A = \begin{bmatrix} 10.0 & -10.0 & 0 & 0 & 0 \\ -10.0 & 22.5 & -12.5 & 0 & 0 \\ 0 & -12.5 & 27.5 & -15.0 & 0 \\ 0 & 0 & -15.0 & 32.5 & -17.5 \\ 0 & 0 & 0 & -17.5 & 37.5 \end{bmatrix}$$

Modal Superposition Damping Matrix Using MSE Damping Ratios

$$C = \begin{bmatrix} 10.0 & -10.0 & 0 & 0 & 0 \\ -10.0 & 22.5 & -12.5 & 0 & 0 \\ 0 & -12.5 & 27.5 & -15.0 & 0 \\ 0 & 0 & -15.0 & 32.5 & -17.5 \\ 0 & 0 & 0 & -17.5 & 37.5 \end{bmatrix}$$

### Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Nearly Proportional Damping

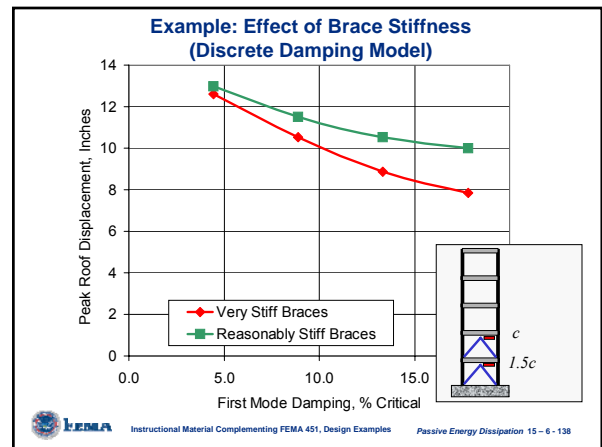
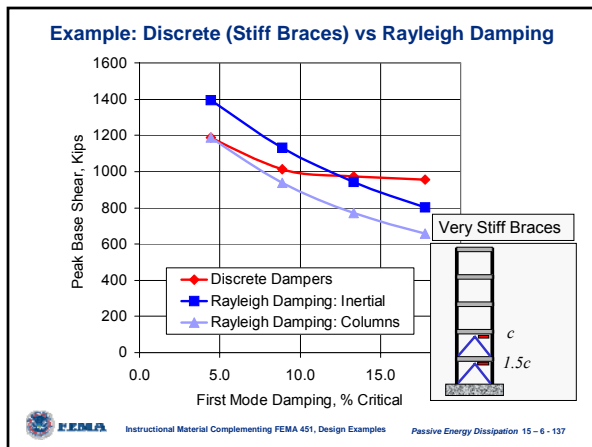
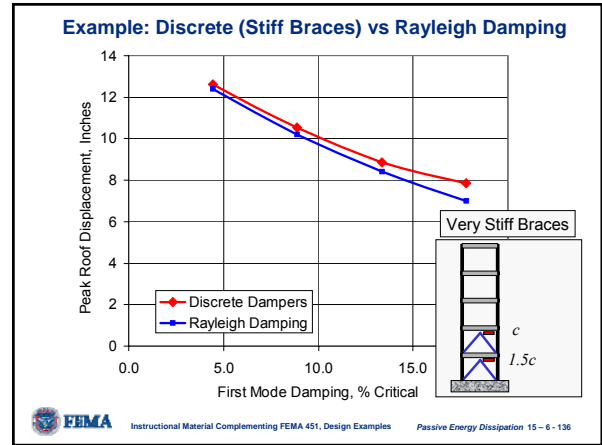
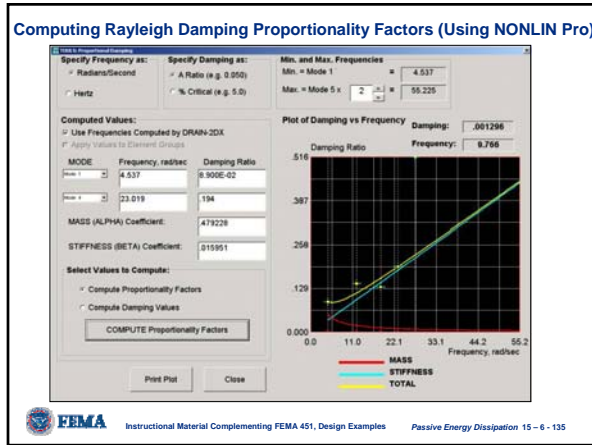
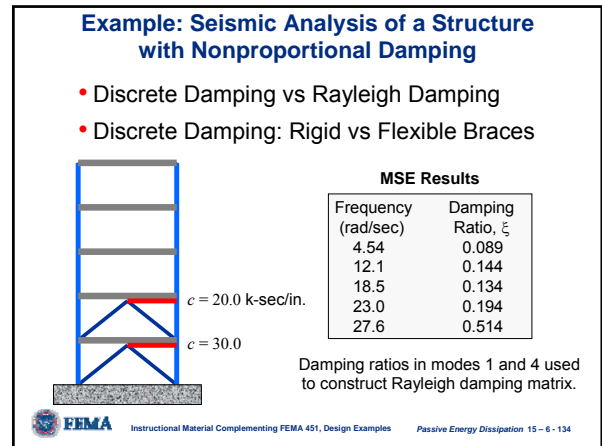
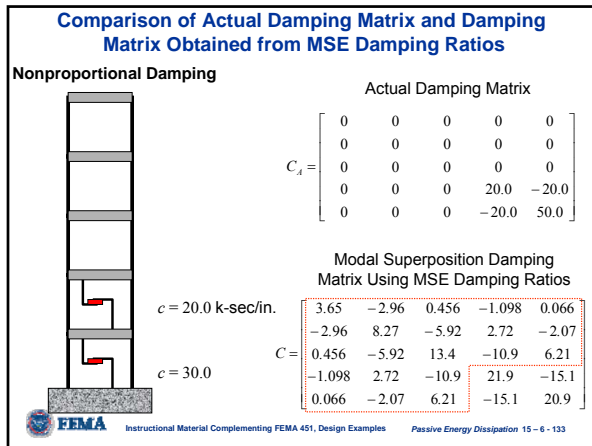


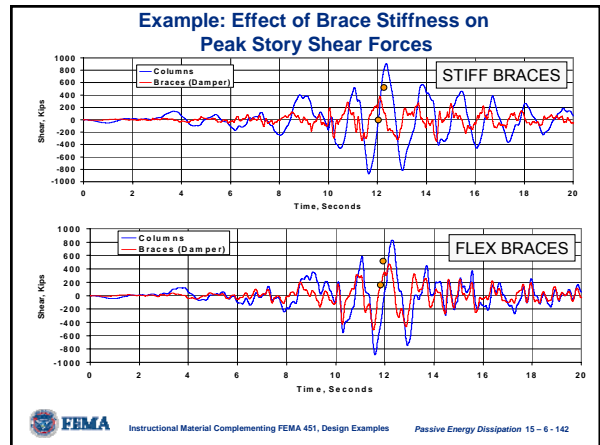
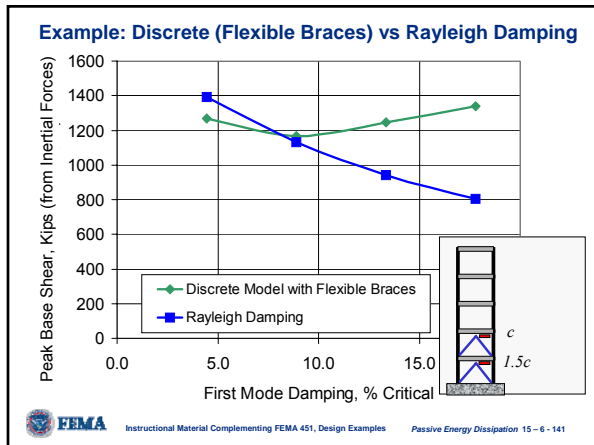
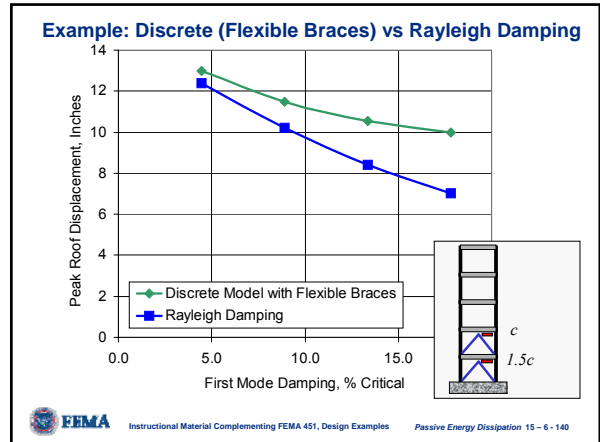
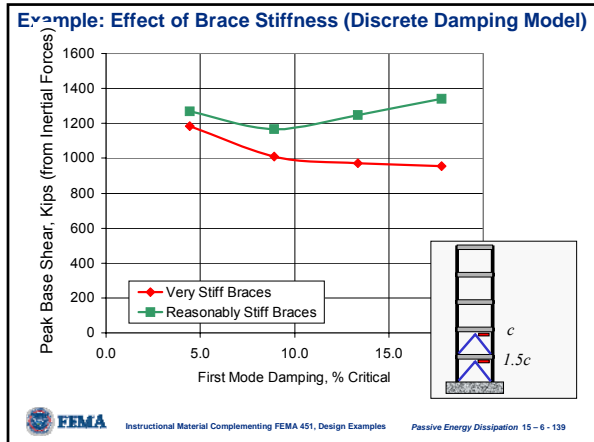
Actual Damping Matrix

$$C_A = \begin{bmatrix} 10.0 & -10.0 & 0 & 0 & 0 \\ -10.0 & 20.0 & -10.0 & 0 & 0 \\ 0 & -10.0 & 20.0 & -10.0 & 0 \\ 0 & 0 & -10.0 & 30.0 & -20.0 \\ 0 & 0 & 0 & -20.0 & 50.0 \end{bmatrix}$$

Modal Superposition Damping Matrix Using MSE Damping Ratios

$$C = \begin{bmatrix} 10.0 & -9.66 & -0.166 & -0.228 & -0.010 \\ -9.66 & 22.0 & -12.2 & -0.169 & -0.422 \\ -0.166 & -12.2 & 27.3 & -15.1 & -0.731 \\ -0.228 & -0.169 & -15.1 & 33.1 & -17.8 \\ -0.010 & -0.422 & -0.731 & -17.8 & 37.5 \end{bmatrix}$$





- ### Outline: Part III
- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
  - Representations of Damping
  - Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
  - Summary of MDOF Analysis Procedures
- Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 143

- ### Summary: MDOF Analysis Procedures (Linear Systems and Linear Dampers)
- Use discrete damper elements and explicitly include these dampers in the system damping matrix. Perform response history analysis of full system. **Preferred.**
  - Use discrete damper elements to estimate modal damping ratios and use these damping ratios in modal response history or modal response spectrum analysis. **Dangerous.**
  - Use discrete damper elements to estimate modal damping ratios and use these damping ratios in a response history analysis which incorporates Rayleigh proportional damping. **Dangerous.**
- Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 144



### Summary: MDOF Analysis Procedures (Linear Systems with Nonlinear Dampers)

- Use discrete damper elements and explicitly include these dampers in the system damping matrix. Perform response history analysis of full system. **Preferred.**
- Replace nonlinear dampers with “equivalent energy” based linear dampers, and then use these equivalent dampers in the system damping matrix. Perform response history analysis of full system. **Very Dangerous.**
- Replace nonlinear dampers with “equivalent energy” based linear dampers, use modal strain energy approach to estimate modal damping ratios, and then perform response spectrum or modal response history analysis. **Very Dangerous.**

### Summary: MDOF Analysis Procedures (Nonlinear Systems with Nonlinear Dampers)

- Use discrete damper elements and explicitly include these dampers in the system damping matrix. Explicitly model inelastic behavior in superstructure. Perform response history analysis of full system. **Preferred.**
- Replace nonlinear dampers with “equivalent energy” based linear dampers and use modal strain energy approach to estimate modal damping ratios. Use pushover analysis to represent inelastic behavior in superstructure. Use capacity-demand spectrum approach to estimate system deformations. **Do This at Your Own Risk!**

### Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems

### MDOF Solution for Non-Classically Damped Structures Using Complex Modal Analysis

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t)$$

#### Modal Analysis using Damped Mode Shapes:

- Treat  $C_I$  as modal damping and model  $C_A$  explicitly.
- Solve by modal superposition using damped (complex) mode shapes and frequencies.

*System (dampers and structure) must be linear.*

### Damped Eigenproblem

$$M\ddot{v}(t) + C_A\dot{v}(t) + Kv(t) = 0 \quad \leftarrow \text{EOM for Damped Free Vibration}$$

Assume  $C_I$  is negligible

Linear Structure

$$\text{State Vector: } Z = \begin{Bmatrix} \dot{v} \\ v \end{Bmatrix}$$

$$\text{State-Space Transformation: } \dot{Z} = HZ$$

$$\text{State Matrix: } H = \begin{bmatrix} -M^{-1}C_A & -M^{-1}K \\ I & 0 \end{bmatrix}$$

### Solution of Damped Eigenproblem

Assume Harmonic Response for n-th mode:  $Z_n = P_n e^{\lambda_n t}$

Substitute Response into State Space Equation:  $P_n \lambda_n = H P_n \quad \leftarrow \text{Damped Eigenproblem for n-th Mode}$

$$P \Lambda = H P \quad \leftarrow \text{Damped Eigenproblem for All Modes}$$

$$\text{Eigenvalue Matrix: } \Lambda = \begin{bmatrix} \lambda & \\ & \lambda^* \end{bmatrix} \quad \lambda = \text{diag}[\lambda_n]$$

(\* = complex conjugate)

$$\text{Eigenvector Matrix: } P = \begin{bmatrix} \Phi \lambda & \Phi^* \lambda^* \\ \Phi & \Phi^* \end{bmatrix}$$

## Extracting Modal Damping and Frequency from Complex Eigenvalues

Complex Eigenvalue for Mode n:  $\lambda_n = -\xi_n \omega_n \pm i \omega_n \sqrt{1 - \xi_n^2}$  ← Analogous to Roots of Characteristic Equation for SDOF Damped Free Vibration Problem

Modal Frequency:  $\omega_n = |\lambda_n|$

Modal Damping Ratio:  $\xi_n = -\frac{\Re(\lambda_n)}{|\lambda_n|}$

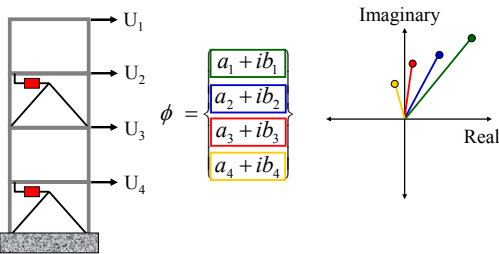
Note:  $i = \sqrt{-1}$

## Extracting Damped Mode Shapes

$$P = \begin{bmatrix} \Phi \Lambda & \Phi^* \Lambda^* \\ \Phi & \Phi^* \end{bmatrix}$$

↑  
Damped Mode Shapes

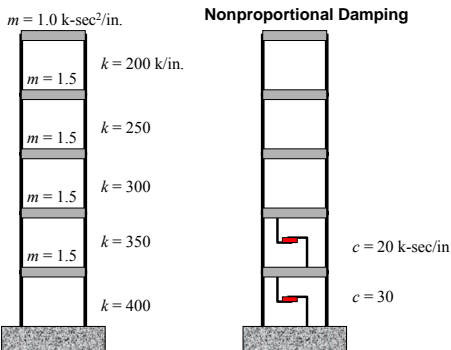
## Damped Mode Shapes



## Outline: Part IV

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## Example: Damped Mode Shapes and Frequencies



## Example: Damped Mode Shapes and Frequencies System with Non-Classical Damping

### Using UNDAMPED MODE SHAPES

	Frequency (rad/sec)	Damping Ratio
1	4.54	0.089
2	12.1	0.144
3	18.4	0.134
4	23.0	0.194
5	27.6	0.516

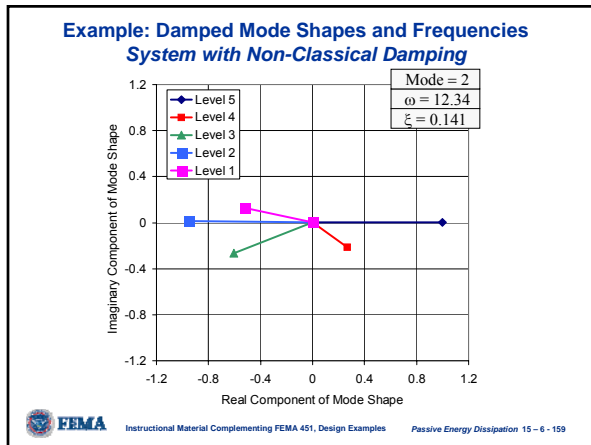
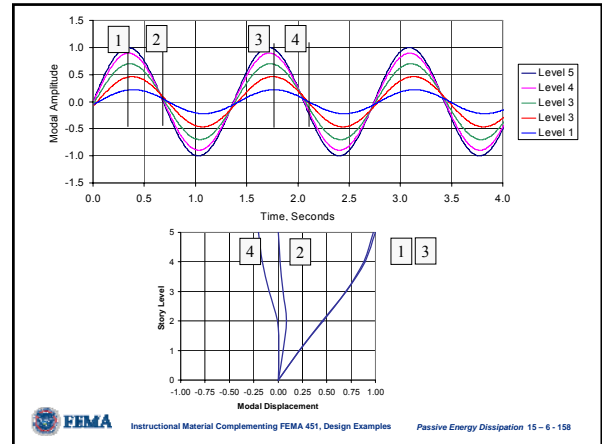
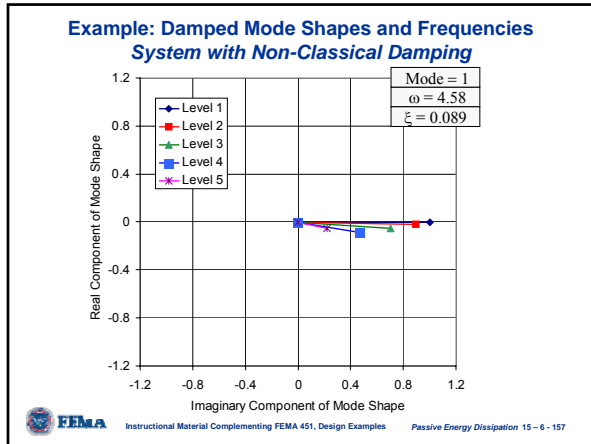
### Using DAMPED MODE SHAPES\*

	Frequency (rad/sec)	Damping Ratio
1	4.58	0.089
2	12.3	0.141
3	18.9	0.064
4	24.0	0.027
5	25.1	0.770

\*Table is for model with VERY STIFF braces.

↑  
Obtained from MSE Method

**Significant Differences in Higher Mode Damping Ratios**



- ### Outline: Part IV
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- FEMA Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 160

### An Unexpected Effect of Passive Damping

The larger the damping coefficient  $C$ , the *smaller* the damping ratio  $\xi$ .

**Why?**

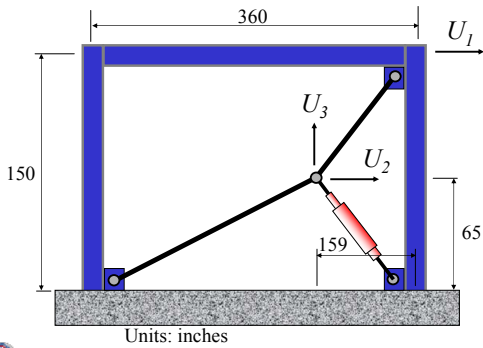
Note:  
Occurs for toggle-braced systems only.

**Huntington Tower**  
 - 111 Huntington Ave, Boston, MA  
 - New 38-story steel-framed building  
 - 100 Direct-acting and toggle-brace dampers  
 - 1300 kN (292 kips), +/- 101 mm (+/- 4 in.)  
 - Dampers suppress wind vibration

FEMA Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 161



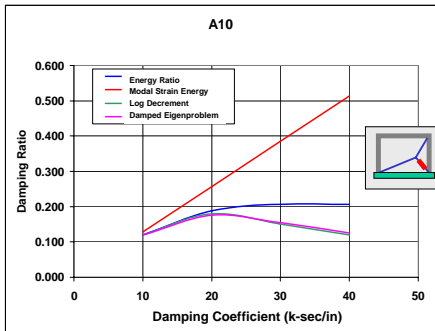
### Example: Toggle Brace Damping System



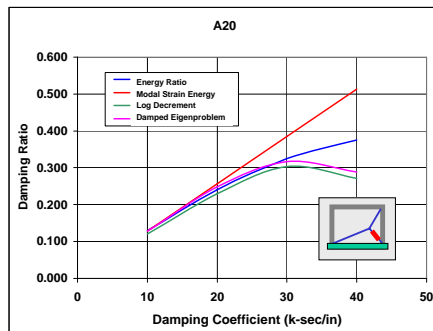
### Methods of Analysis Used to Determine Damping Ratio

- Energy Ratios for Steady-State Harmonic Loading:  $\xi = E_D/4\pi E_S$
  - Modal Strain Energy
  - Free Vibration Log Decrement
  - Damped Eigenproblem
- $C = 10$  to  $40$  k-sec/in (increments of 10)  
 $A = 10$  to  $100$  in<sup>2</sup> (increments of 10)

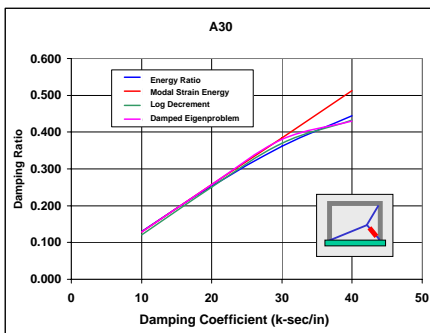
### Computed Damping Ratios for System With A = 10



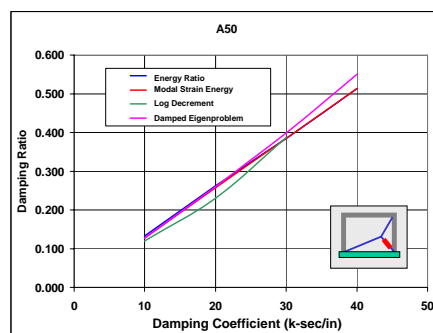
### Computed Damping Ratios for System With A = 20



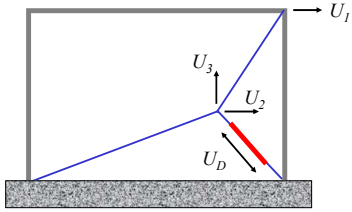
### Computed Damping Ratios for System With A = 30



### Computed Damping Ratios for System With A = 50

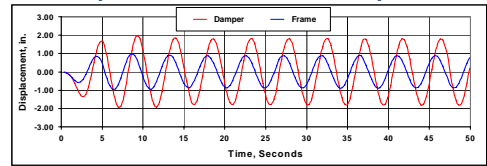


## Why Does Damping Ratio Reduce for Low Brace Area/Damping Coefficient Ratios?

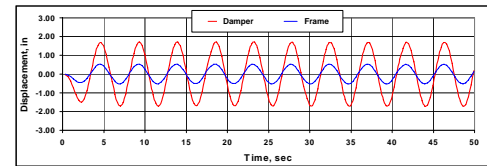


Displacement in Damper is *Out-of-Phase* with Displacement at DOF 1

## Phase Difference Between Damper Displacement and Frame Displacement

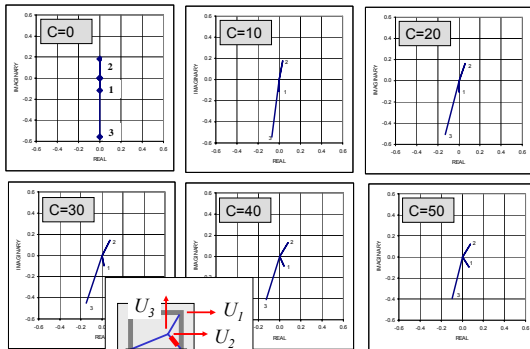


A=10  
C=40  
A/C=0.25



A=50  
C=40  
A/C=1.25

## Damped Mode Shapes for System With A=20 in<sup>2</sup>



## Interim Summary Related to Modeling and Analysis (1)

- Viscously damped systems are very effective in reducing damaging deformations in structures.
- With minor exceptions, viscously damped systems are non-classical, and *must* be modeled explicitly using dynamic time history analysis.
- Avoid the use of the Modal Strain Energy method (it may provide unconservative results)

## Interim Summary Related to Modeling and Analysis (2)

- Damped mode shapes provide phase angle information that is essential in assessing and improving the efficiency of viscously damped systems. This is particularly true for linkage systems (e.g. toggle-braced systems).
- If damped eigenproblem analysis procedures are not available, use overlaid response history plots of damper displacement and interstory displacement to assess damper efficiency. (This would be required for nonlinear viscously damped systems.)

## Outline: Part IV

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## Computer Software Analysis Capabilities

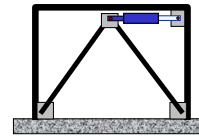
	SAP2000; ETABS	DRAIN	RAM Perform
Linear Viscous Fluid Dampers	Yes	Yes	Yes
Nonlinear Viscous Fluid Dampers	Yes	NO	Yes*
Viscoelastic Dampers	Yes	Yes	Yes
ADAS Type Systems	Yes	Yes	Yes
Unbonded Brace Systems	Yes	Yes	Yes
Friction Systems	Yes	Yes	Yes
General System Yielding	Pending	Yes	Yes

\*Piecewise Linear



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## Modeling Linear Viscous Dampers in DRAIN



Use a Type-1 truss bar element with stiffness proportional damping:

$$K = \frac{AE}{L} \quad C = \beta K$$

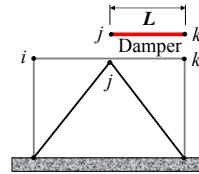
For dampers with low stiffness:

$$\text{Set } A = L, E = 0.01 \text{ and } \beta = C_{\text{Damper}}/0.01$$

Result:

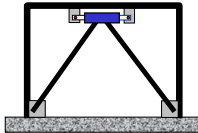
$$K = 0.01 \quad C = C_{\text{Damper}}$$

$$F = C\dot{u} = \beta K\dot{u} = C_{\text{Damper}}\dot{u}$$

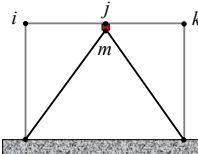


Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 176

## Modeling Linear Viscous Dampers in DRAIN



Dampers may be similarly modeled using the zero-length "Type-4" connection element.

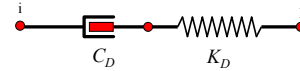


Nodes *j* and *m* have the same coordinates



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## Modeling Viscous/Viscoelastic Dampers Using the SAP2000 NLLINK Element



The damper is modeled as a Maxwell Element consisting of a linear or nonlinear dashpot in series with a linear spring.

To model a linear viscous dashpot,  $K_D$  must be set to a large value, but not too large or convergence will not be achieved. To achieve this, it is recommended that the relaxation time ( $\lambda = C_D/K_D$ ) be an order of magnitude less than the loading time step  $\Delta t$ . For example, let  $K_D = 100C_D/\Delta t$ . Sensitivity to  $K_D$  should be checked.

SAP2000 often has difficulty converging when nonlinear dampers are used and the velocity exponent is less than 0.4.

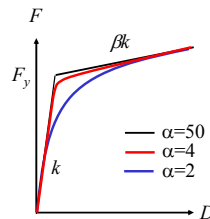


Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 178

## Modeling ADAS, Unbonded Brace, and Friction Dampers using the SAP2000 NLLINK Element

$$F = \beta kD + (1 - \beta)F_y Z$$

$$\dot{Z} = \frac{k}{F_y} \begin{cases} \dot{D} (1 - |Z|^\alpha) & \text{if } \dot{D} > 0 \\ \dot{D} & \text{otherwise} \end{cases}$$



Note: *Z* is an internal hysteretic variable with magnitude less than or equal to unity. The yield surface is associated with a magnitude of unity.

For bilinear behavior, use  $\alpha$  of approximately 50. Larger values can produce strange results.



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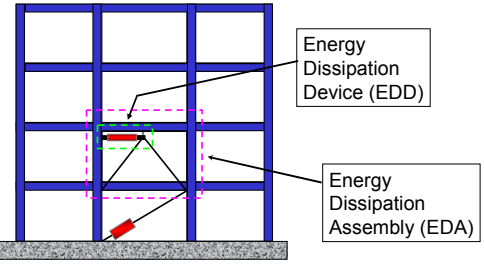


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### 1993 - Tentative General Requirements for the Design and Construction of Structures Incorporating Discrete Passive Energy Dissipation Devices (1 of 3)

- Draft version developed by Energy Dissipation Working Group (EDWG) of Base Isolation Subcommittee of Seismology Committee of SEAONC (Not reviewed/approved by SEAOC; used as basis for 1994 NEHRP Provisions)
- Philosophy: For Design Basis Earthquake (10/50), confine inelastic behavior to energy dissipation devices (EDD); gravity load resisting system to remain elastic
- Established terminology and nomenclature for energy dissipation systems (EDS)
- Classified systems as rate-independent or rate-dependent (included metallic, friction, viscoelastic, and viscous dampers)
- Required at least two vertical lines of dampers in each principal direction of building; dampers to be continuous from the base of the building
- Prescribed analysis and testing procedures

### 1993 - Tentative General Requirements for the Design and Construction of Structures Incorporating Discrete Passive Energy Dissipation Devices (2 of 3)



### Energy Dissipation Nomenclature

### 1993 - Tentative General Requirements for the Design and Construction of Structures Incorporating Discrete Passive Energy Dissipation Devices (3 of 3)

- Elastic structures with rate-dependent devices: Linear dynamic procedures (response spectrum or response history analysis)
- Inelastic structures or structures with rate-independent devices: Nonlinear dynamic response history analysis
- Prototype tests on full-size specimens (not required if previous tests performed and documented by ICBO)
- General acceptability criteria for energy dissipation systems:
  - Remain stable at design displacements
  - Provide non-decreasing resistance with increasing displacement (for rate-independent systems)
  - Exhibit no degradation under repeated cyclic load at design displ.
  - Have quantifiable engineering parameters
- Independent engineering review panel required to oversee design and testing

### 1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (1 of 4)

Part 1 - Provisions & Part 2 - Commentary (FEMA 222A & 223A)

- Includes Appendix to Chapter 2 entitled: Passive Energy Dissipation Systems
- Material is based on:
  - 1993 draft SEAONC EDWG document
  - Proceedings of ATC 17-1 Seminar on Seismic Isolation, Passive Energy Dissipation, and Active Control (March 1993)
  - Special issue of Earthquake Spectra (August 1993)
- Applicable to wide range of EDD's; therefore requires EDD performance verification via prototype testing
- Performance objective identical to conventional structural system (i.e., life-safety for design EQ)
- At least two EDD per story in each principal direction, distributed continuously from base to top of building unless adequate performance (drift limits satisfied and member curvature capacities not exceeded) with incomplete vertical distribution can be demonstrated
- Members that transmit damper forces to foundation designed to remain elastic

### 1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (2 of 4)

Part 1 - Provisions & Part 2 - Commentary (FEMA 222A & 223A)

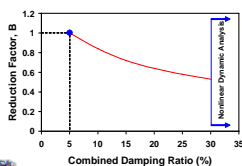
#### Analysis/Design Procedure for Linear Viscous Energy Dissipation Systems

$$V_{min} = BV = BC_S W$$

$V_{min}$  = Minimum base shear for design of structure with EDS  
(Use for linear static (ELF) or linear dynamic (Modal) analysis)

$V$  = Minimum base shear for design of structure without EDS

$B$  = Reduction factor to account for energy dissipation provided by EDS (based on combined, inherent plus added damping, damping ratio)



**Note:** After publication, it was recognized that this procedure may not be appropriate since it allows reduction in forces due to both inelastic structural response (R-factor) and added damping (B-factor). For yielding structures, added damping will not reduce forces.

### 1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (3 of 4)

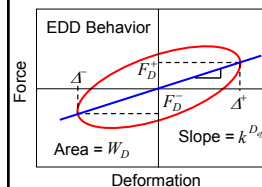
Part 1 - Provisions & Part 2 - Commentary (FEMA 222A & 223A)

#### Analysis/Design Procedure for EDD's other than Linear Viscous Dampers

1) Preliminary Design: Linear dynamic modal analysis using effective stiffness and damping coefficient of energy dissipation devices. Use B-factor to reduce modal base shears.

$$k^{D_{eff}} = \frac{F_D^+ + F_D^-}{\Delta^+ + \Delta^-} \quad \text{Eq. (C2A.3.2.1a)}$$

Effective Device Stiffness at Design Displacement



$$c_{eq} = 2m\omega_n \xi_{eq} = \frac{2m\omega_n W_D}{4\pi W_S} = \frac{W_D T}{2\pi^2 \Delta^+}$$

Eq. (2A.3.2.1)  
Equivalent Device Damping Coefficient

$$\xi_{combined} = \xi_{str} + \frac{\sum W_D}{4\pi SE}$$

Eq. (C2A.3.2.1c)  
Combined Equivalent Damping Ratio

2) Performance Verification: Nonlinear response history analysis

**1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (4 of 4)**  
 Part 1 – Provisions & Part 2 – Commentary (FEMA 222A & 223A)

- For nonlinear response-history analysis, mathematical modeling should account for:
  - Plan and vertical spatial distribution of EDD's
  - Dependence of EDD's on loading frequency, temperature, sustained loads, nonlinearities, and bilateral loads
- Prototype Tests on at least two full-size EDD's (unless prior testing has been documented)
  - 200 fully reversed cycles corresponding to wind forces
  - 50 fully reversed cycles corresponding to design earthquake
  - 10 fully reversed cycles corresponding to maximum capable earthquake
- Acceptability criteria from prototype testing of EDD's:
  - Hysteresis loops have non-negative incremental force-carrying capacities (for rate-independent systems only)
  - Exhibit limited effective stiffness degradation under repeated cyclic load
  - Exhibit limited degradation in energy loss per cycle under repeated cyclic load
  - Have quantifiable engineering parameters
  - Remain stable at design displacements

**1997 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures**  
 Part 1 – Provisions & Part 2 – Commentary (FEMA 302 & 303)

- Includes an appendix to Chapter 13 entitled: **Passive Energy Dissipation**
- The appendix in the 1994 NEHRP Provisions was deleted since it was deemed to be insufficient for design and regulation. It was replaced with 3 paragraphs that provide very general guidance on passive energy dissipation systems.

**1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)**  
 1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (1 of 9)

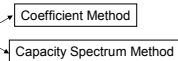
- Chapter 9 entitled: Seismic Isolation and Energy Dissipation (Developed by New Technologies Team under ATC Project 33)
- Performance-based document
  - Rehabilitation objectives based on desired performance levels for selected hazard levels
- Global Structural Performance Levels
  - Operational (OP)
  - Immediate Occupancy (IO)
  - Life-Safety (LS)
  - Collapse Prevention (CP)

} Most Applicable Performance Levels
- Hazard levels
  - Basic Safety Earthquake 1 (BSE-1): 10/50 event
  - Basic Safety Earthquake 2 (BSE-2): 2/50 event (Maximum Considered EQ - MCE)
- Rehabilitation Objectives
  - Limited Objectives (less than BSO)
  - Basic Safety Objective (BSO): LS for BSE-1 and CP for BSE-2
  - Enhanced Objectives (more than BSO)

} Applicable Rehabilitation Objectives

**1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)**  
 1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (2 of 9)

- Simplified vs. Systematic Rehabilitation
  - Simplified: For simple structures in areas of low to moderate seismicity
  - Systematic: Considers all elements needed to attain rehabilitation objective
- Systematic Rehabilitation methods of analysis:
  - Linear static procedure (LSP)
  - Linear dynamic procedure (LDP)
  - Nonlinear static procedure (NSP)
  - Nonlinear dynamic procedure (NDP)



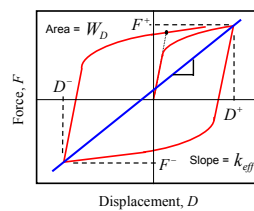
**1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)**  
 1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (3 of 9)

- **Basic Principles:**
  - Dampers should be spatially distributed (at each story and on each side of building)
  - Redundancy (at least two dampers along the same line of action; design forces for dampers and damper framing system are reduced as damper redundancy is increased)
  - For BSE-2, dampers and their connections designed to avoid failure (i.e., not weak link)
  - Members that transmit damper forces to foundation designed to remain elastic
- **Classification of EDD's**
  - Displacement-dependent
  - Velocity-dependent
  - Other (e.g., shape memory alloys and fluid restoring force/damping dampers)

Manufacturing quality control program should be established along with prototype testing programs and independent panel review of system design and testing program

**1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)**  
 1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (4 of 9)

**Mathematical Modeling of Displacement-Dependent Devices**



$$F = k_{eff} D \quad \text{Eq. (9-20) Force in Device}$$

$$k_{eff} = \frac{|F^+| + |F^-|}{|D^+| + |D^-|} \quad \text{Eq. (9-21) Effective Stiffness of Device}$$

$$\beta_{eff} = \frac{1}{2\pi} \frac{W_D}{k_{eff} D_{ave}^2} \quad \text{Eq. (9-39) Equivalent Viscous Damping Ratio of Device}$$



1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)  
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (5 of 9)

**Mathematical Modeling of Solid Viscoelastic Devices**

Force

EDD Behavior

Area =  $W_D$

Slope =  $k_{eff}$

Deformation

Loss Stiffness

Storage Stiffness

$$F = k_{eff}D + C\dot{D}$$

Eq. (9-22)  
Force in Device

$$k_{eff} = \frac{|F^+| + |F^-|}{|D^+| + |D^-|} = K^*$$

Eq. (9-23)  
Effective Stiffness of Device

$$C = \frac{W_D}{\pi\omega_1 D_{ave}^2} = \frac{K^*}{\omega_1}$$

Eq. (9-24)  
Damping Coefficient of Device

Average Peak Displ.

Circular frequency of mode 1

FEMA Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 193

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)  
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (6 of 9)

**Mathematical Modeling of Fluid Viscoelastic and Fluid Viscous Devices**

Fluid Viscoelastic Devices:

$$F + \lambda\dot{F} = C\dot{D}$$

Maxwell Model

Fluid Viscous Devices:

$$F = C_0|\dot{D}|^\alpha \text{sgn}(\dot{D})$$

Eq. (9-25)  
Linear or Nonlinear Dashpot Model

**Caution:** Only use fluid viscous device model if  $\alpha \neq 0$  for frequencies between  $0.5 f_i$  and  $2.0 f_i$ ; Otherwise, use fluid viscoelastic device model.

FEMA Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 194

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)  
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (7 of 9)

**Pushover Analysis for Structures with EDD's (Part of NSP)**

Roof Displ.

Base Shear

Performance point without dampers

Performance point with dampers

With Viscous Dampers

No dampers

With Friction Dampers

No dampers

With ADAS Dampers

No dampers

With Friction Dampers

No dampers

With Viscoelastic Dampers

No dampers

Reduced Displacement

Reduced Damage

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1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)  
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) 8 of 9)

**Design Process for Velocity-Dependent Dampers using NSP**

Steps

- 1) Estimate Target Displacement (performance point)
- 2) Calculate Effective Damping Ratio and Secant Stiffness of building at Target Displacement
- 3) Use Effective Damping and Secant Stiffness to calculate revised Target Displacement
- 4) Compare Target Displacement from Steps 1 and 4. If within tolerance, stop. Otherwise, return to Step 1.

$$\beta_{eff} = \beta + \frac{\sum W_j}{4\pi W_k}$$

Effective damping ratio of building with dampers at Target Displ.;  
j = index over devices

$$W_k = \frac{1}{2} \sum_i F_i \delta_i$$

Maximum strain energy in building with dampers at Target Displ.;  
i = index over floor levels

FEMA Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 196

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)  
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (9 of 9)

**Design Process for Velocity-Dependent Dampers using NSP (2)**

$$W_j = \frac{2\pi^2}{T_s} C_j \delta_{Tj}^2$$

Work done by j-th damper with building subjected to Target Displacement (assumes harmonic motion with amplitude equal to Target Displacement and frequency corresponding to Secant Stiffness at Target Displacement)

$$\beta_{eff} = \beta + \frac{T_s \sum_j C_j \cos^2 \theta_j \phi_{Tj}^2}{4\pi \sum_i m_i \phi_i^2}$$

Alternate expression for Effective Damping Ratio that uses modal amplitudes of first mode shape

**Checking Building Component Behavior (Forces and Deformations)**

For velocity-dependent dampers, must check component behavior at three stages:

- 1) Maximum Displacement
- 2) Maximum Velocity
- 3) Maximum Acceleration

FEMA Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 197

**2000 - Prestandard and Commentary for the Seismic Rehabilitation of Buildings (FEMA 356)**

- Prestandard version of 1997 NEHRP Guidelines and Commentary for the Seismic Rehabilitation of Buildings (FEMA 273 & 274)
- Prepared by ASCE for FEMA
- Prestandard = Document has been accepted for use as the start of the formal standard development process (i.e., it is an initial draft for a consensus standard)

FEMA Instructional Material Complementing FEMA 451, Design Examples Passive Energy Dissipation 15 - 6 - 198

**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (1 of 8)**  
Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

- Appendix to Chapter 13 entitled **Structures with Damping Systems** (completely revised/updated version of 1994 and 1997 Provisions; Brief commentary provided)

- **Intention:**

- Apply to all energy dissipation systems (EDS)
- Provide design criteria compatible with conventional and enhanced seismic performance
- Distinguish between design of members that are part of EDS and members that are independent of EDS.

-The seismic force resisting system must comply with the requirements for the system's Seismic Design Category, except that the damping system may be used to meet drift limits.

*No reduction in detailing is thereby allowed, even if analysis shows that the damping system is capable of producing significant reductions in ductility demand or damage.*



**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (2 of 8)**  
Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

- Members that transmit damper forces to foundation designed to remain elastic

- Prototype tests on at least two full-size EDD's (reduced-scale tests permitted for velocity-dependent dampers)

- Production testing of dampers prior to installation.

- Independent engineering panel for review of design and testing programs

- Residual mode concept introduced for linear static analysis. This mode, which is in addition to the fundamental mode, is used to account for the combined effects of higher modes. Higher mode interstory-velocities can be significant and thus are important for velocity-dependent dampers.



**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (3 of 8)**  
Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

Methods of Analysis:

- Linear Static (Equivalent Lateral Force\*)  
- OK for Preliminary Design
- Linear Dynamic (Modal Response Spectrum\*)  
- OK for Preliminary Design
- Nonlinear Static (Pushover\*)  
- May Produce Significant Errors
- Nonlinear Dynamic (Response History)  
- Required if  $S_1 > 0.6 g$  and may be used in all other cases

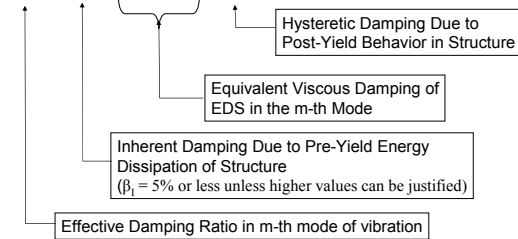
\*The Provisions allow final design using these procedures, but only under restricted circumstances.



**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (4 of 8)**  
Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

**Effective Damping Ratio**  
(used to determine factors, B, that reduce structure response)

$$\beta_m = \beta_I + \beta_{Vm}\sqrt{\mu} + \beta_H$$



**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (5 of 8)**  
Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

**Equivalent Viscous Damping from EDS**

$$\beta_m = \beta_I + \beta_{Vm}\sqrt{\mu} + \beta_H$$

$$\beta_{Vm} = \frac{\sum_j W_{mj}}{4\pi W_m}$$

Equivalent Viscous Damping in m-th mode (due to EDS)

$$W_m = \frac{1}{2} \sum_i F_{im} \delta_{im}$$

Maximum Elastic Strain Energy of structure in m-th mode

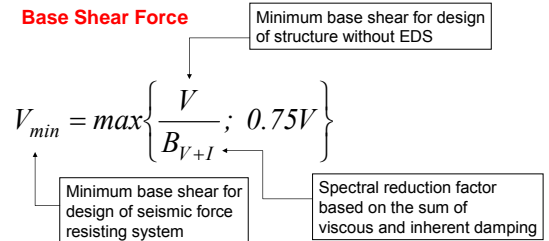
$\sqrt{\mu}$  Adjustment factor that accounts for dominance of post-yielding inelastic hysteretic energy dissipation



**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (6 of 8)**  
Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

**Base Shear Force**

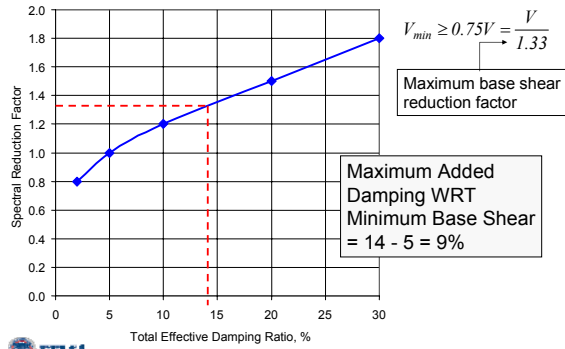
$$V_{min} = \max \left\{ \frac{V}{B_{V+I}} ; 0.75V \right\}$$



To protect against damper system malfunction, maximum reduction in base shear over a conventional structure is 25%



**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (7 of 8)**  
 Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)



**2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (8 of 8)**  
 Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

