SEISMIC PROTECTIVE SYSTEMS: PASSIVE ENERGY DISSIPATION

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These visuals were initially developed by Professor Finley Charney who presented them at the 2001 MBDSI. Modifications were made by Professor Michael Symans who presented the visuals at the 2002 and 2003 MBDSI and updated them for the cancelled 2004 MBDSI.

Major Objectives

- Illustrate why use of passive energy dissipation systems may be beneficial
- Provide overview of types of energy dissipation systems available
- Describe behavior, modeling, and analysis of structures with energy dissipation systems
- Review developing building code requirements

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Outline: Part I

- Objectives of Advanced Technology Systems and Effects on Seismic Response
- Distinction Between Natural and Added Damping
- Energy Distribution and Damage Reduction
- Classification of Passive Energy Dissipation Systems

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Outline: Part II

- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
- Models for Velocity-Dependent Dampers
- Effects of Linkage Flexibility
- Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
- Concept of Equivalent Viscous Damping
- Modeling Considerations for Structures with Passive Energy Dissipation Systems

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Outline: Part III

- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
- Representations of Damping
- Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
- Summary of MDOF Analysis Procedures

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Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems

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Objectives of Energy Dissipation and Seismic Isolation Systems

- Enhance performance of structures at all hazard levels by:
  - Minimizing interruption of use of facility *(e.g., Immediate Occupancy Performance Level)*
  - Reducing damaging deformations in structural and nonstructural components
  - Reducing acceleration response to minimize contents-related damage

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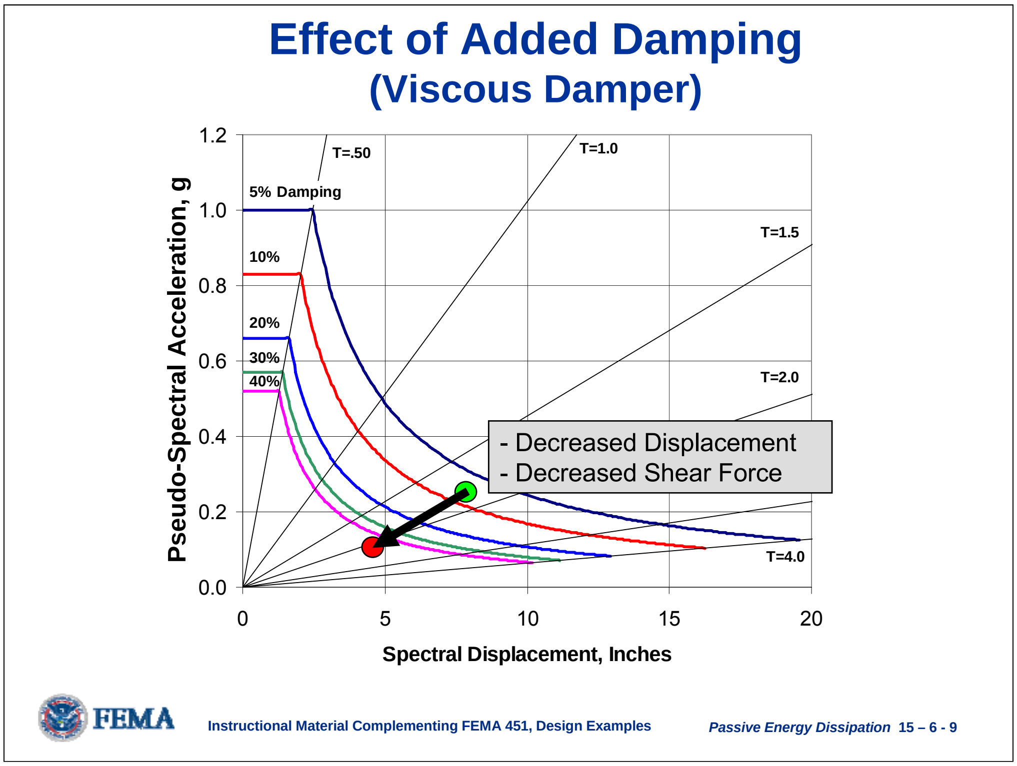
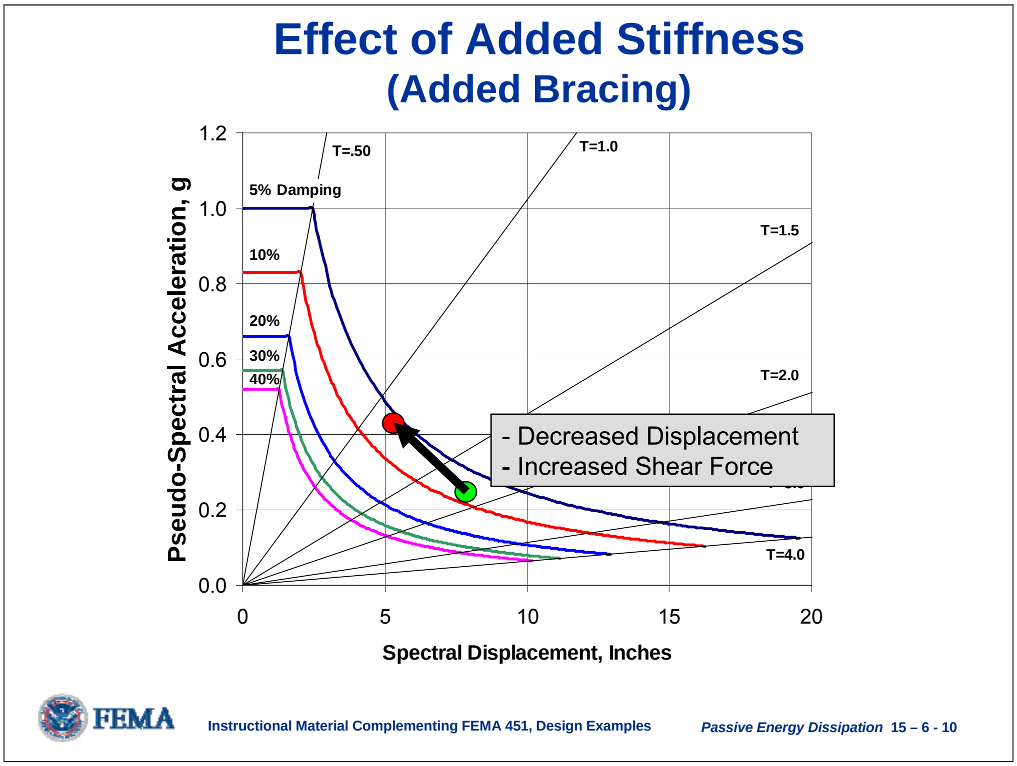
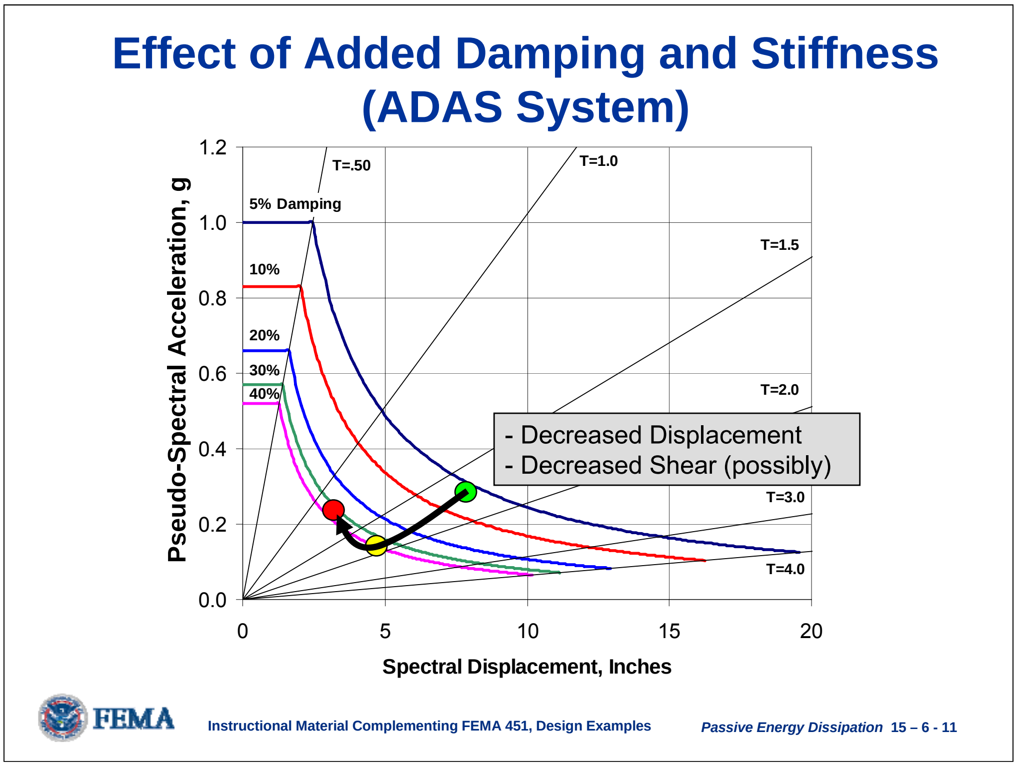


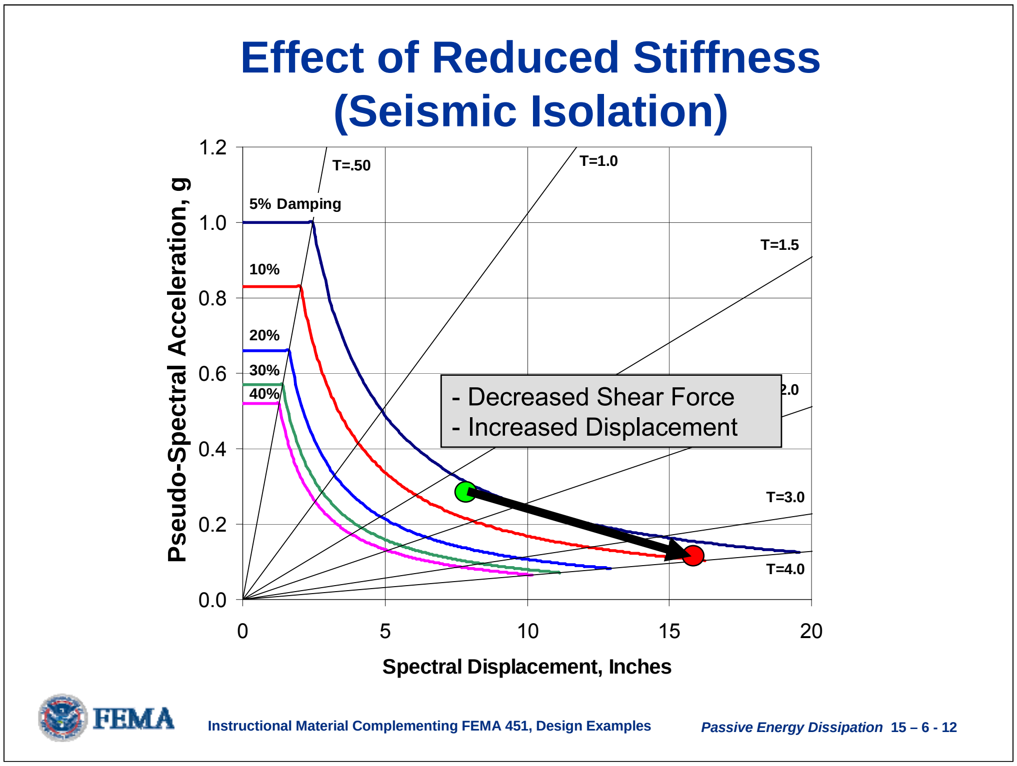
This slide shows a series of elastic design response spectra in the form of ADRS curves. In an ADRS spectrum, lines of constant period radiate out from the origin. A SDOF elastic structure having a natural period of 1.5 seconds and a damping ratio of 5% has a peak pseudo-acceleration and displacement response as indicated by the green circle. A viscous damper is added such that the damping ratio is increased from 5% to 30%, resulting in a reduction in both the peak pseudo-acceleration (and thus shear force) and peak displacement demands as indicated by the red circle. As the arrow indicates, the response moves along the constant period line since adding viscous dampers does not affect the natural period of the structure. However, in reality, the presence of the damper does affect the natural period since the damper is connected to a framing system that has flexibility. This issue is explored later in this set of slides.



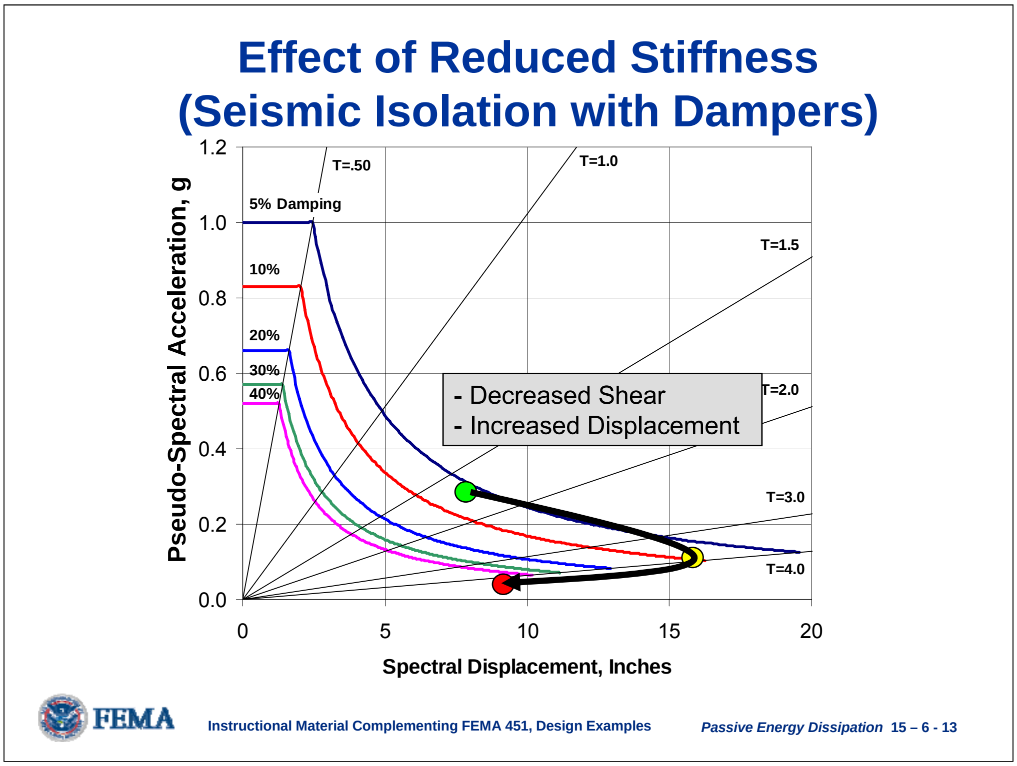
A SDOF elastic structure having a natural period of 1.5 seconds and a damping ratio of 5% has a peak pseudo-acceleration and displacement response as indicated by the green circle. A bracing system is added such that the natural period decreases to 1.0 seconds (approximately a 125% increase in stiffness), resulting in a reduction in peak displacement and an increase in peak pseudo-acceleration (and thus an increase in shear force) as indicated by the red circle. As the arrow indicates, the response moves along the 5%-damped design response spectrum.



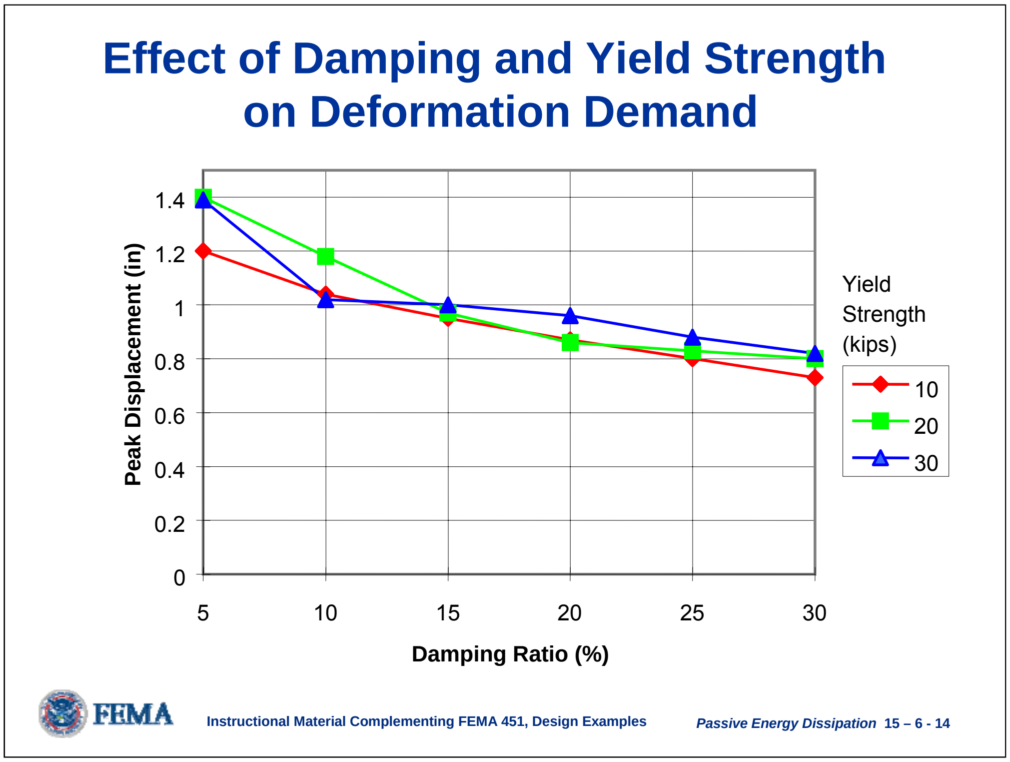
A SDOF elastic structure having a natural period of 1.5 seconds and a damping ratio of 5% has a peak pseudo-acceleration and displacement response as indicated by the green circle. An ADAS system is added such that the natural period decreases to 1.0 seconds (approximately a 125% increase in stiffness) and the equivalent viscous damping ratio increases to 30%, resulting in a reduction in peak displacement and a possible reduction in peak pseudo-acceleration (and thus shear force) as indicated by the red circle. As the arrow indicates, the response may be considered to first move along the constant natural period line due to the added equivalent viscous damping and then along the 30%-damped design spectrum due to the added stiffness. Note that the use of equivalent viscous damping to account for energy dissipation by metallic yielding devices (i.e., ADAS devices) is a major approximation and may lead to erroneous predictions of seismic response.



A SDOF elastic structure having a natural period of 1.5 seconds and a damping ratio of 5% has a peak pseudo-acceleration and displacement response as indicated by the green circle. An isolation system is installed such that the natural period increases to 3.0 seconds (approximately 75% reduction in stiffness), resulting in an increase in peak displacement and reduction in peak pseudo-acceleration (and thus a reduction in shear force) as indicated by the red circle. The increased displacement occurs across the isolation system rather than within the structure. As the arrow indicates, the response moves along the 5%-damped design response spectrum. Note that it is implicitly assumed here that the isolated structure behaves as a SDOF system wherein all of the flexibility is at the isolation level.



A SDOF elastic structure having a natural period of 1.5 seconds and a damping ratio of 5% has a peak pseudo-acceleration and displacement response as indicated by the green circle. An isolation system is installed such that the natural period increases to 3.0 seconds (approximately 75% reduction in stiffness) and the damping ratio increases to 30%, resulting in a slightly increased peak displacement and a reduction in peak pseudo-acceleration (and thus a reduction in shear force) as indicated by the red circle. The increased displacement occurs across the isolation system rather than within the structure. As the arrow indicates, the response first moves along the 5%-damped design response spectrum due to the reduced stiffness and then along the constant natural period line due to the increased damping. Note that it is implicitly assumed here that the isolated structure behaves as a SDOF system wherein all of the flexibility is at the isolation level.



This slide shows the peak displacement of a SDOF inelastic system with a bilinear force-deformation relation and various levels of yield strength and damping ratio. As the yield strength is modified, the effect on the peak displacement is minimal. This is consistent with the so-called "equal displacement" rule which says that, for systems having a medium-to-high elastic natural period and varying yield strengths, the elastic and inelastic peak displacements will be approximately equal. On the other hand, as the damping ratio is increased from 5% to 30%, the peak displacement of all of the inelastic systems reduces by about 30 to 40%. Thus, an increase in damping can significantly influence the response of both elastic and inelastic systems.

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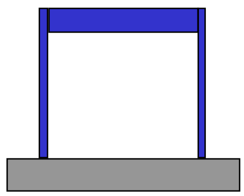
Distinction Between Natural and Added Damping

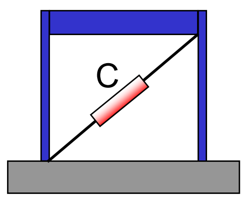
Natural (Inherent) Damping



ξ is a structural property, dependent on system mass, stiffness, and inherent energy dissipation mechanisms

$$\xi_{NATURAL} = 0.5 \text{ to } 7.0\%$$

Added Damping

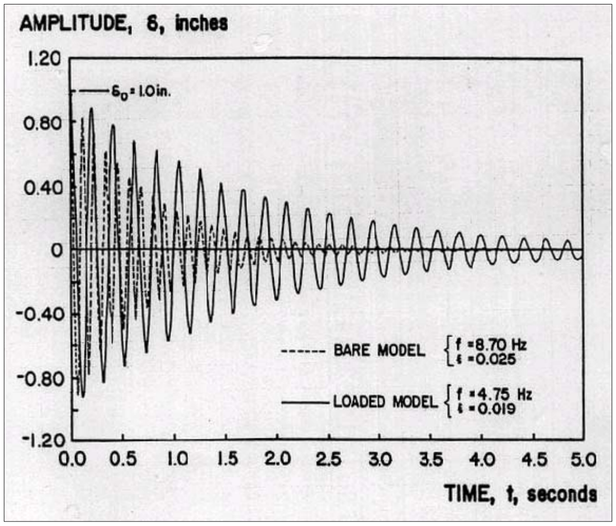


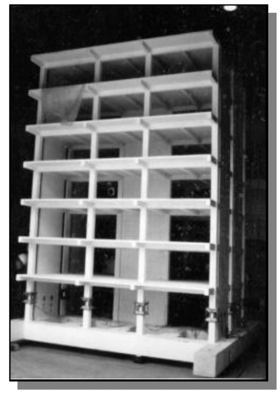
ξ is a structural property, dependent on system mass, stiffness, and the added damping coefficient C

$$\xi_{ADDED} = 10 \text{ to } 30\%$$

The inherent damping is associated with a variety of mechanisms that dissipate energy from the structure. All of these mechanisms are typically combined and modeled via a linear viscous dashpot. Of course, the dashpot does not physically exist, but rather provides for a mathematically convenient model of damping and, furthermore, experimental test results show that the model is reasonable for capturing the global response of structures, particularly when the inherent equivalent viscous damping ratio is below about 10%. For higher levels of damping, the equivalent viscous damping model may not work as well. The addition of damping to a structure involves physically inserting discrete dampers within the structure. The primary purpose of the added dampers is to provide a source of energy dissipation.

1981/1982 US-JAPAN PROJECT
Response of Bare Frame Before and After Adding Ballast





Model Weight

Bare Model	**18 kips**
Loaded Model	**105 kips**

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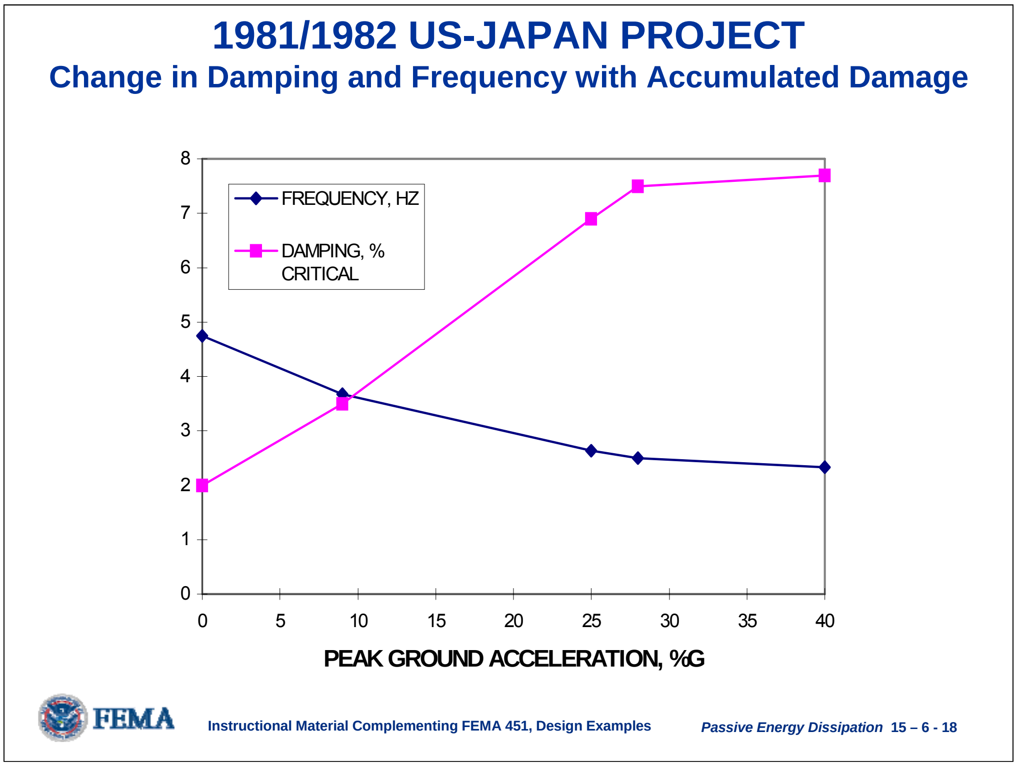
This slide shows a 1/5-scale concrete building model tested at UC Berkeley. For dynamic similitude, the bare model weight was increased by a factor of about 5.8. The plot shows free vibration displacement response in the fundamental mode wherein the displacement is measured at the roof. Using the measured period of free vibration and the log-decrement method, the following results were obtained:

Bare model: f = 8.7 Hz, damping ratio = 2.5%

Loaded model: f = 4.8 Hz (reduction of 45%), damping ratio = 1.9% (reduction of 24%)

As expected, an increase in the mass of the model results in a reduction in both the natural frequency and damping ratio. The theoretical reduction for the natural frequency and damping ratio would be 59% since both quantities are inversely proportional to the square root of the mass. For the tested structure, the reduction in damping ratio was less than was expected theoretically (damping ratio = 1.04%) and was actually attributed to the closing of micro-cracks in the concrete. This is consistent with the observation that the natural frequency did not reduce as much as theoretically expected (3.6 Hz) (i.e., the closing of the cracks resulted in a stiffer structure). In summary, the behavior of real structures is generally more complicated than simple theory suggests.



The natural frequencies and damping ratios were measured using free vibration tests performed subsequent to seismic loading tests. With a peak ground acceleration near zero, the natural frequency and damping ratio corresponds to elastic response. Under increasing seismic loading, the concrete frame is damaged and the dynamic properties change. The natural frequency decreases due to a loss in lateral stiffness while the effective damping ratio increases due to accumulated damage in the form of concrete cracking and loss of bond to reinforcement. To avoid damage, an alternate source of energy dissipation should be provided.

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Reduction in Seismic Damage

Energy Balance:

Hysteretic Energy

$$E_I = E_S + E_K + (E_{DI} + E_{DA}) + E_H$$

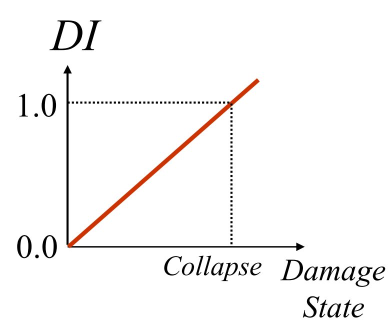
Inherent Damping

Added Damping

Damage Index:

$$DI(t) = \frac{u_{max}}{u_{ult}} + \rho \frac{E_H(t)}{F_y u_{ult}}$$

Source: Park and Ang (1985)



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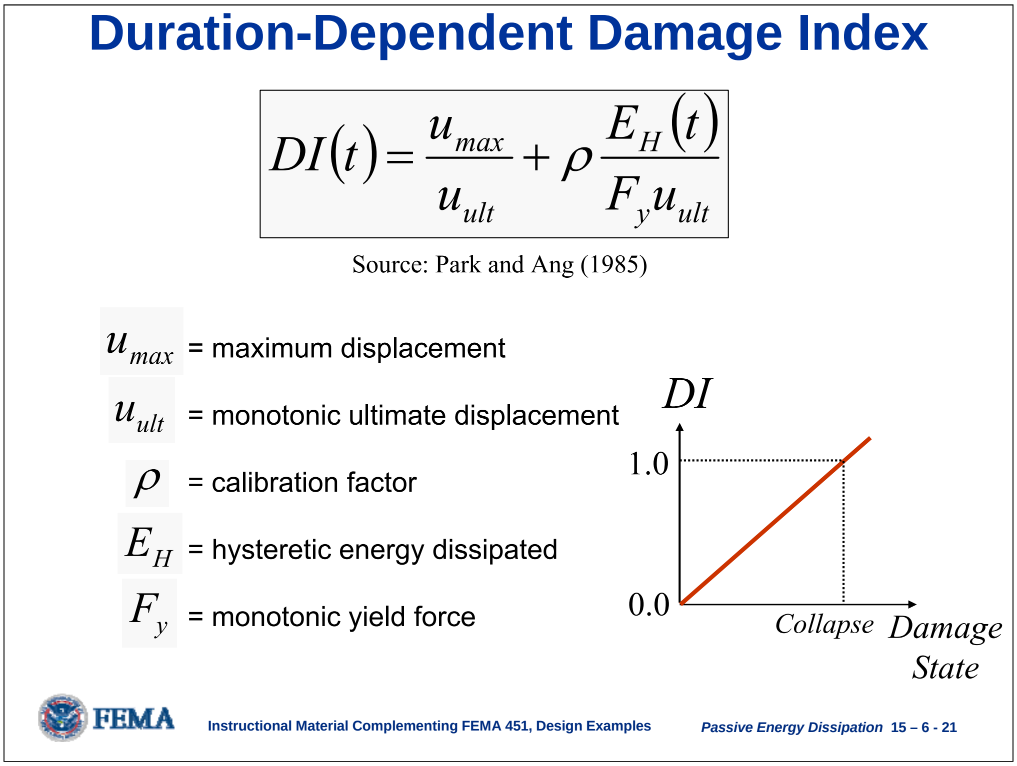
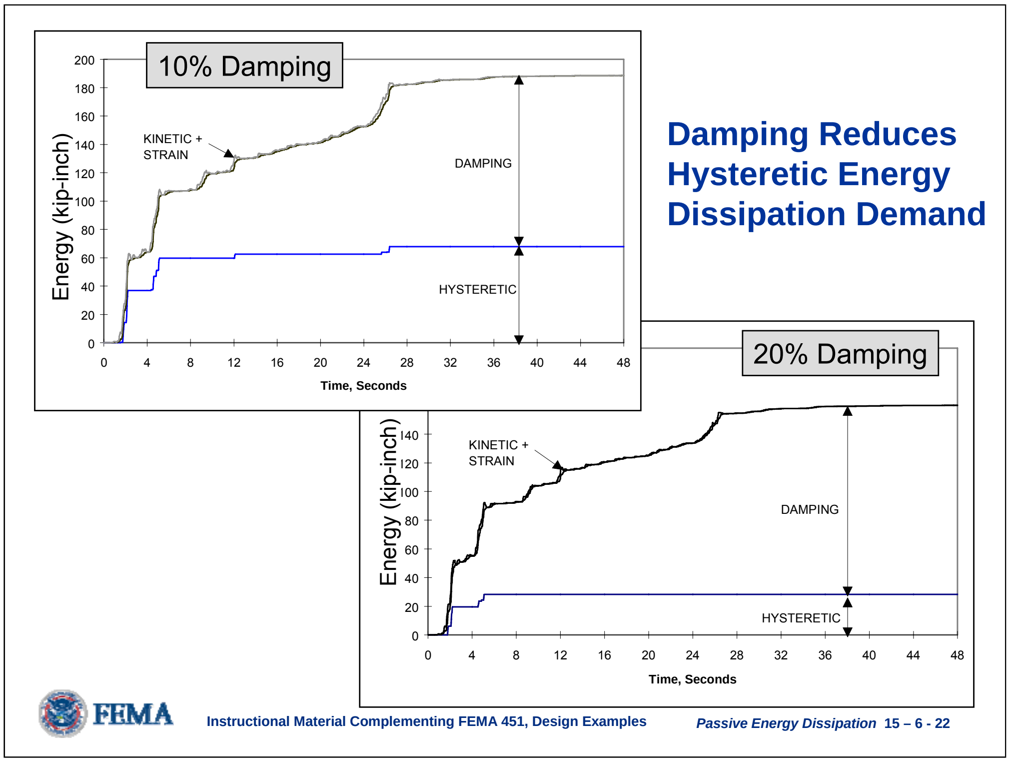
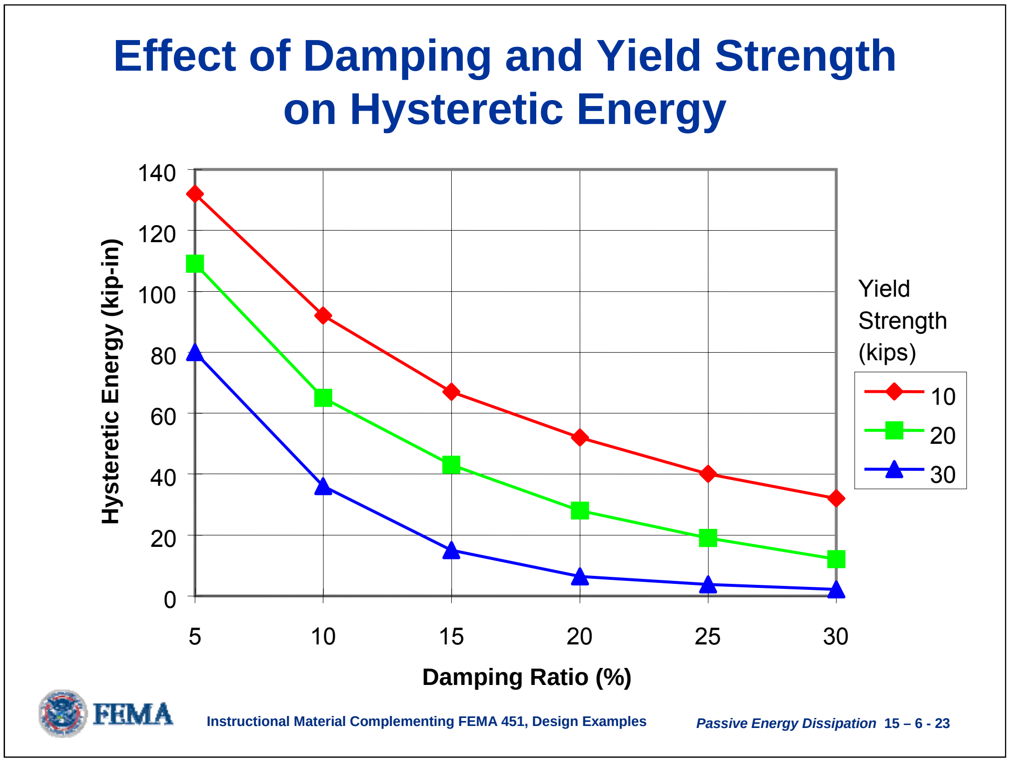
Seismic damage can be reduced by providing an alternate source of energy dissipation. The energy balance must be satisfied at each instant in time. For a given amount of input energy, the hysteretic energy dissipation demand can be reduced if a supplemental (or added) damping system is utilized. A damage index can be used to characterize the time-dependent damage to a structure. For the definition given, the time-dependence is in accordance with the time-dependence of the hysteretic energy dissipation. The calibration factor ρ accounts for the type of structural system and is calibrated such that a damage index of unity corresponds to incipient collapse. Damage index values less than about 0.2 indicate little or no damage. Note that this particular damage index is one of the first duration-dependent damage indices to be proposed.



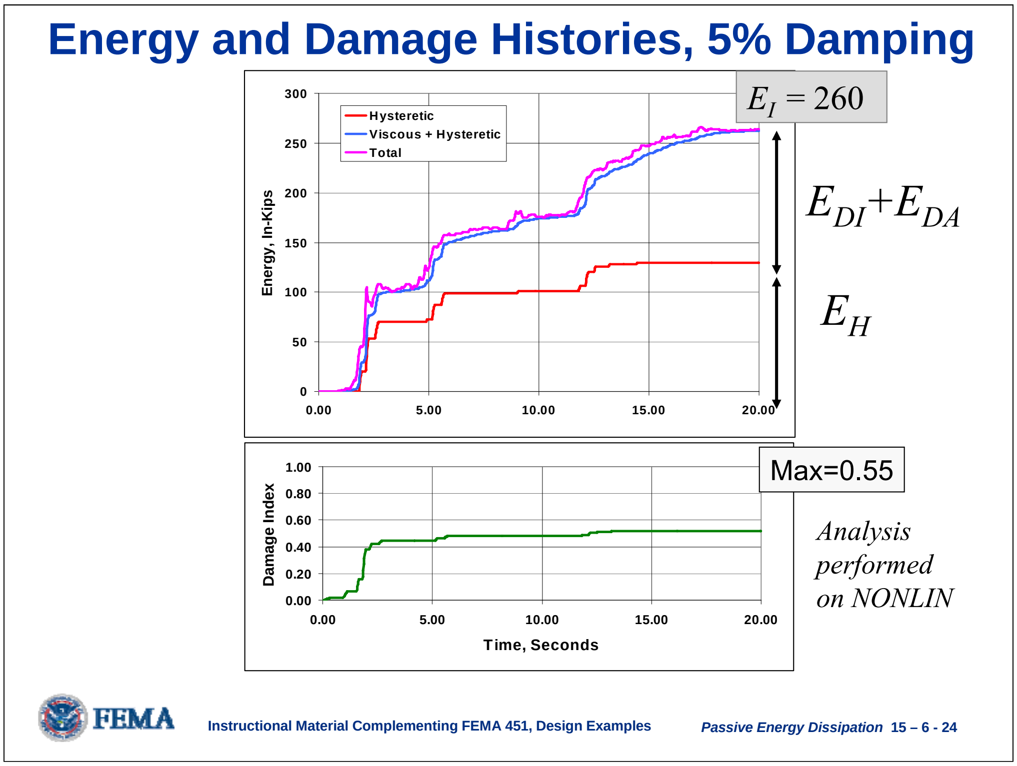
This slide defines the variables in the damage index.



Energy response histories are shown for a SDOF elasto-plastic system subjected to seismic loading. An increase in added damping reduces the hysteretic energy dissipation demand by about 57%. Note that the input energy also changes since the input energy is dependent on the deformations of the structure. In this case, the input energy has reduced, although this is not always the case.

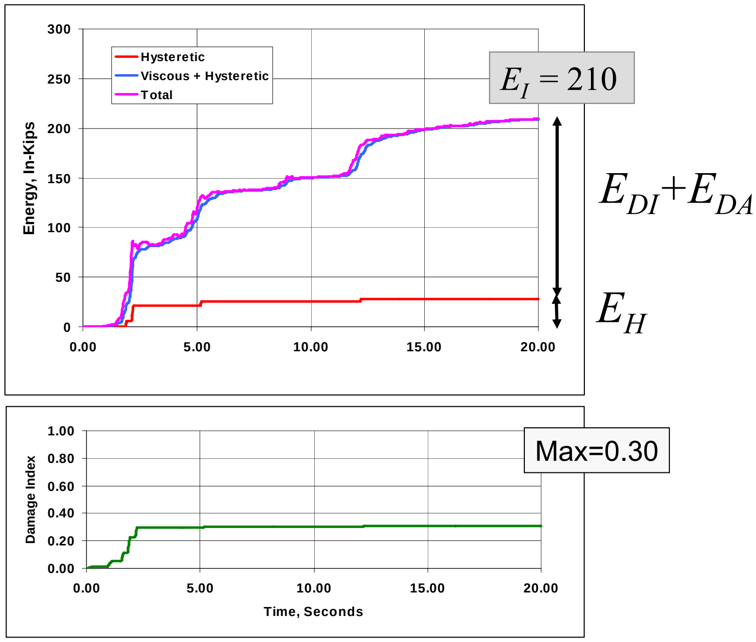


SDOF elasto-plastic system subjected to seismic loading. For a given damping ratio, increased yield strengths result in reduced hysteretic energy dissipation demand. Of course if the yield strength is high enough, the structure will remain elastic and hysteretic energy dissipation demand becomes zero. For a given yield strength, as the damping ratio is increased, the hysteretic energy dissipation demand reduces significantly.



SDOF elasto-plastic system subjected to 1940 Imperial Valley Earthquake. With 5% viscous damping, the cumulative input energy of 260 kip-in is approximately equally divided among hysteretic energy dissipation and viscous damping. Note that the damage index follows the time-dependence of the hysteretic energy and that both quantities remain constant after about 15 seconds, indicating that damage is not accumulated beyond this point. The maximum damage index is 0.55 which indicates significant damage.

Energy and Damage Histories, 20% Damping



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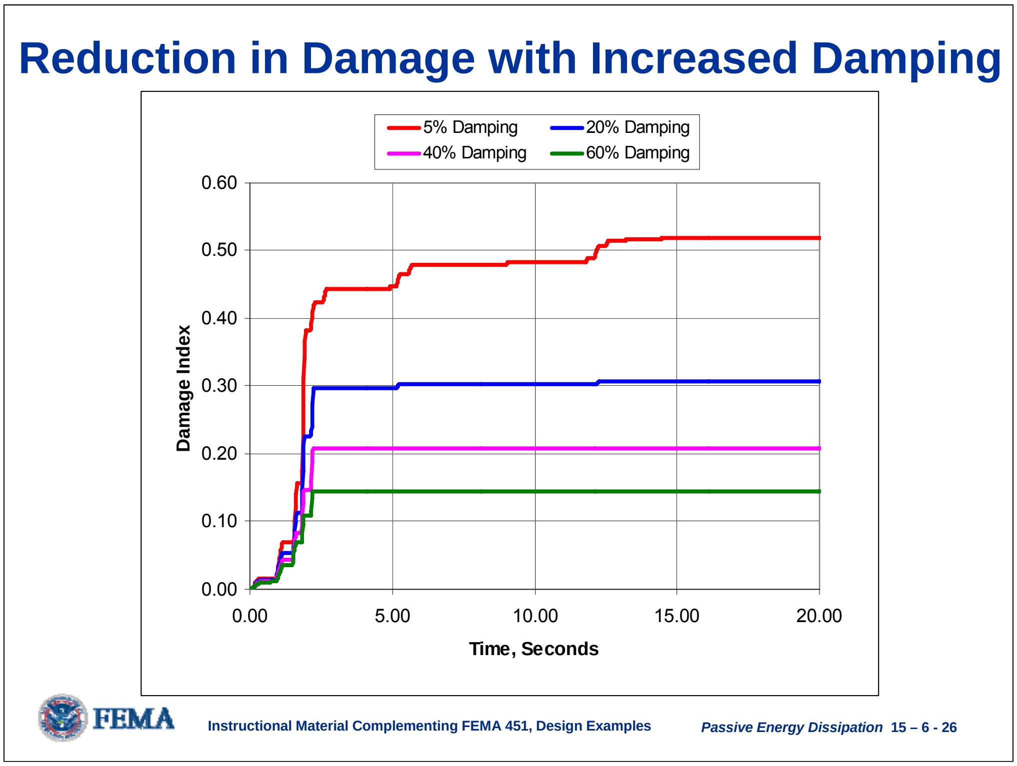
The results shown here are for the same SDOF elasto-plastic system subjected to the 1940 Imperial Valley Earthquake but with 20% viscous damping rather than 5%. The cumulative input energy in this case is 210 kip-in and is primarily dissipated by viscous damping with the hysteretic energy dissipation demand being quite small (reduced by about 77% compared to the 5%-damped case). Note that the damage index follows the time-dependence of the hysteretic energy and has been reduced to a maximum value of 0.3, about a 45% reduction as compared to the value of 0.55 for the 5%-damped case. A maximum damage index of 0.3 suggests that the structure would still be damaged but much less than for a damage index of 0.55. Both the damage index and hysteretic energy dissipation remain constant after about 12 seconds, indicating that damage is not accumulated beyond this point.



The results shown here are for the same SDOF elasto-plastic system subjected to the 1940 Imperial Valley Earthquake but with various levels of viscous damping. The effect of increased viscous damping on the damage index indicates that the benefit of adding damping can be significant in terms of reducing damage, although there are diminishing returns as higher levels of damping are approached. In other words, for a given incremental change in the damping ratio, the incremental change in the damage index is larger when the damping ratios are small. For example, as the damping ratio increases from 5% to 20%, the damage index reduces by about 45%. As the damping ratio increases from 40% to 60%, the damage index reduction is about 33%. The low value of the damage index (about 0.15) for the 60%-damped case is indicative of nearly elastic response.

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Classification of Passive Energy Dissipation Systems

Velocity-Dependent Systems
- Viscous fluid or viscoelastic solid dampers
- May or may not add stiffness to structure

Displacement-Dependent Systems
- Metallic yielding or friction dampers
- Always adds stiffness to structure

Other
- Re-centering devices (shape-memory alloys, etc.)
- Vibration absorbers (tuned mass dampers)

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Velocity-dependent systems consist of dampers whose force output is dependent on the rate of change of displacement (thus, we often call these systems rate-dependent). Viscous fluid dampers, the most commonly utilized energy dissipation system, are generally exclusively velocity-dependent and thus add no additional stiffness to a structure (assuming no flexibility in the damper framing system). Viscoelastic solid dampers exhibit both velocity and displacement-dependence. Displacement-dependent systems consist of dampers whose force output is dependent on the displacement and NOT the rate of change of the displacement (thus, we often call these systems rate-independent). More accurately, the force output of displacement-dependent dampers generally depends on both the displacement and the sign of the velocity. Other energy absorbing devices are available but are not commonly used and will not be presented herein.

Outline: Part II

- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
- Models for Velocity-Dependent Dampers
- Effects of Linkage Flexibility
- Displacement-Dependent Damping Systems: Steel Plate Dampers,Unbonded Brace Dampers, and Friction Dampers
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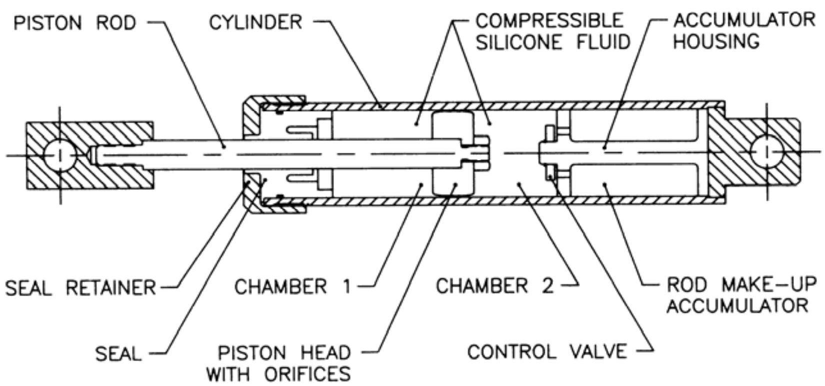
Cross-Section of Viscous Fluid Damper



Source: Taylor Devices, Inc.

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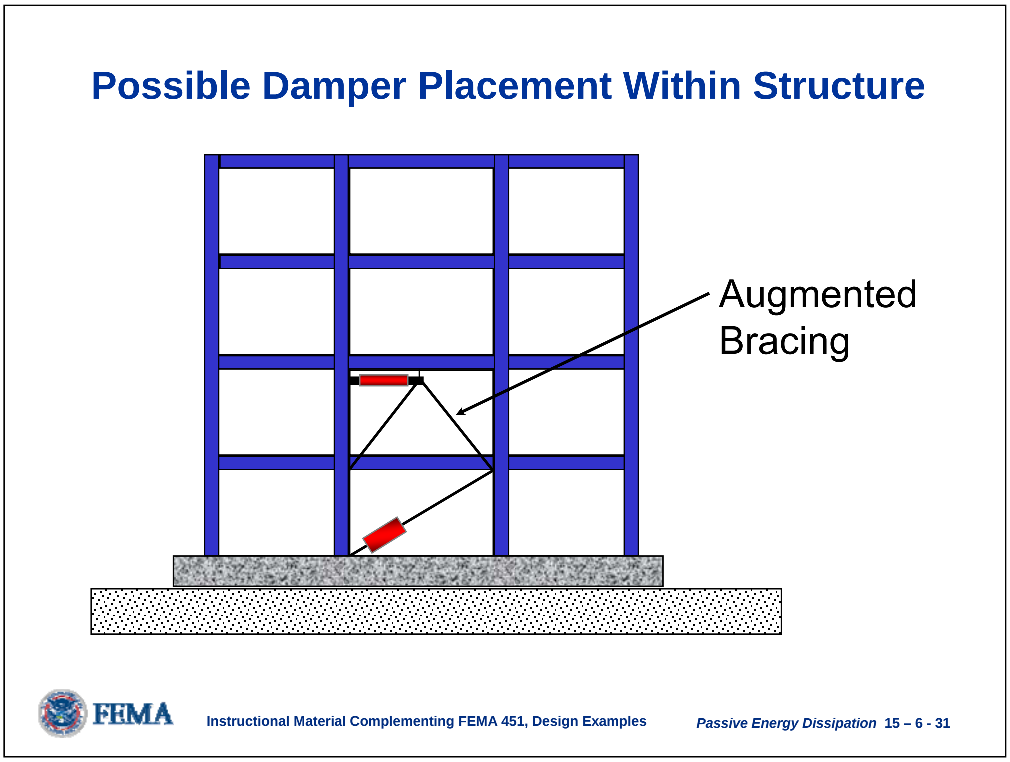
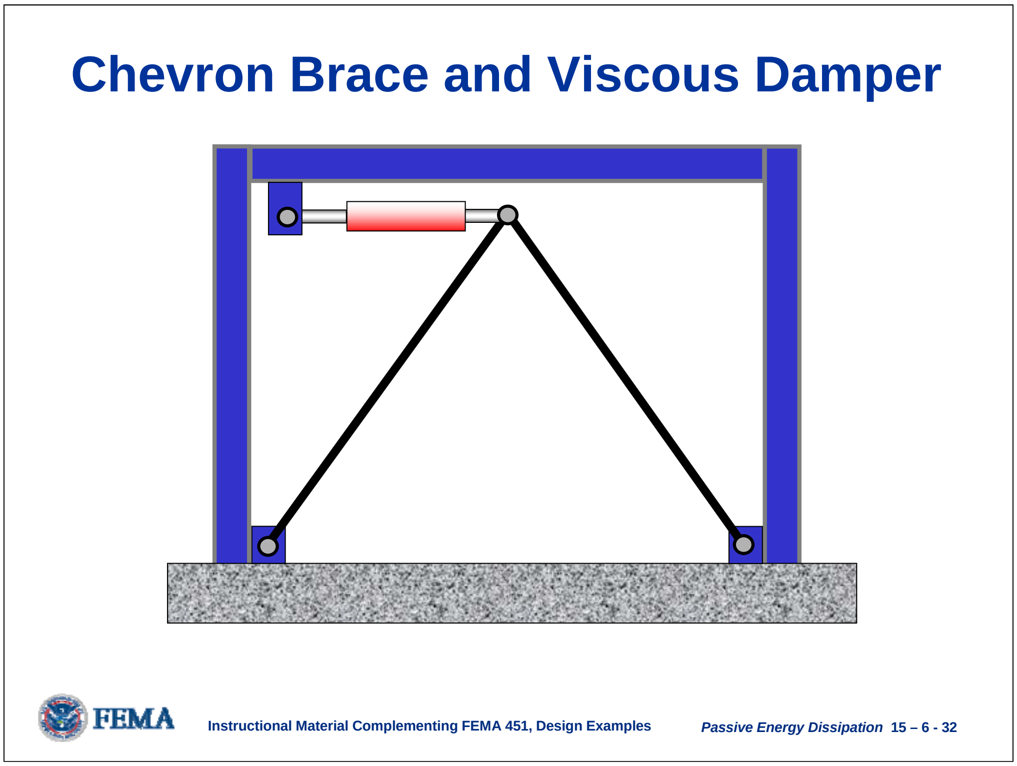
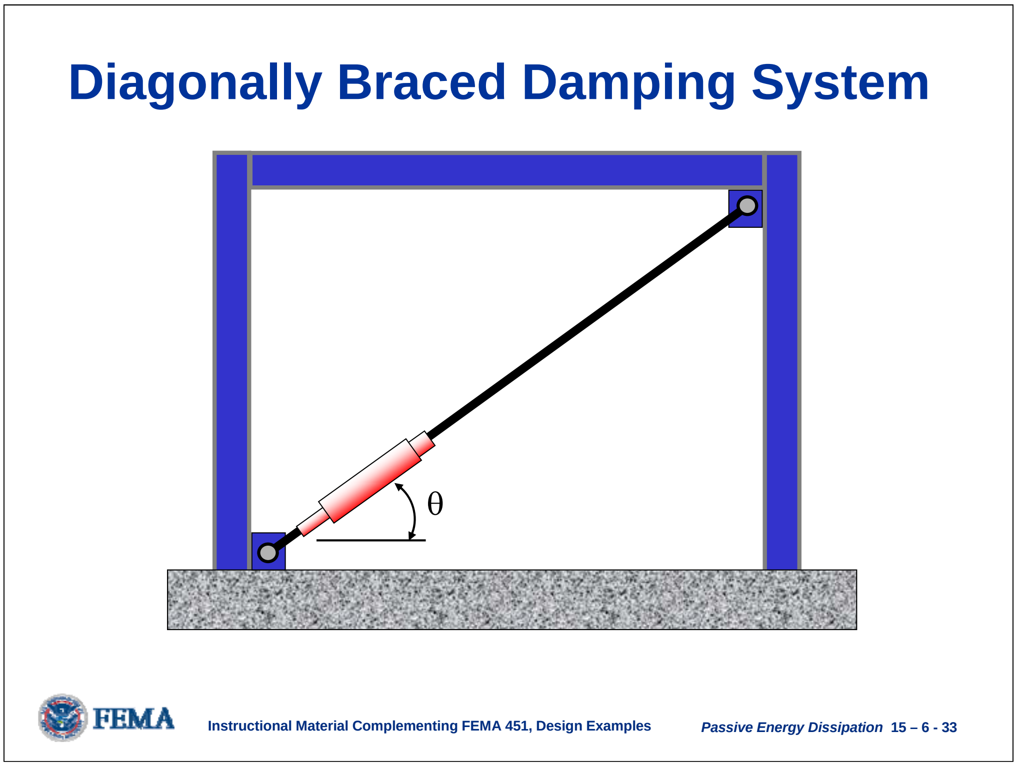
A viscous fluid damper consists of a hollow cylinder filled with a fluid. As the damper piston rod and piston head are stroked, fluid is forced to flow through orifices either around or through the piston head. The fluid flows at high velocities, resulting in the development of friction and thus heat. The heat is dissipated harmlessly to the environment. Interestingly, although the damper is called a viscous fluid damper, the fluid typically has a relatively low viscosity. The term viscous fluid damper comes from the macroscopic behavior of the damper which is essentially the same as an ideal viscous dashpot (i.e., the force output is directly related to the velocity). Note that the damper shown above is “single-ended” in that the piston rod is only on one side of the piston head. This type of design is relatively compact but can lead to the development of stiffness at relatively low frequencies of motion (to be explained later). A different design which has been used in all applications to date within the United States, makes use of a “double-ended” or “run-through” piston rod. In this case, the piston rod projects outward from both sides of the piston head and exits the damper at both ends of the main cylinder. Although larger in size and more prone to seal leakage (since there is a second seal), the advantage of this configuration is that there is virtually no stiffness over a wide range of frequencies.



Dampers are commonly installed either within chevron bracing or diagonal bracing.



The chevron bracing arrangement is attractive since the full capacity of the damper is utilized to resist lateral motion. However, the bracing is subjected primarily to axial forces and thus, to be effective, the bracing must have high axial stiffness. Excessive flexibility in the brace reduces the effectiveness of the damper. Note that, in many installations, the chevron bracing arrangement is inverted such that damper is located near the floor rather than near the ceiling. This facilitates installation and future inspection of the dampers.



The diagonal bracing arrangement may be less effective since only a component of the damper force (the damper axial force multiplied by the square of the cosine of the angle of inclination) resists lateral motion. However, the bracing is subjected only to axial forces and thus is inherently stiff. As an example, for a damper inclined at 45 degrees, the damper effectiveness is reduced by 50% due to the inclination. For convenient access, the damper is commonly installed near the lower corner.

Fluid Dampers within Inverted Chevron Brace

Pacific Bell North Area Operation Center (911 Emergency Center)
Sacramento, California
(3-Story Steel-Framed Building Constructed in 1995)

62 Dampers: 30 Kip Capacity, +/-2 in. Stroke

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For aesthetic reasons, the dampers in structures are often left exposed.

Fluid Damper within Diagonal Brace

Huntington Tower
Boston, MA

San Francisco State
Office Building
San Francisco, CA

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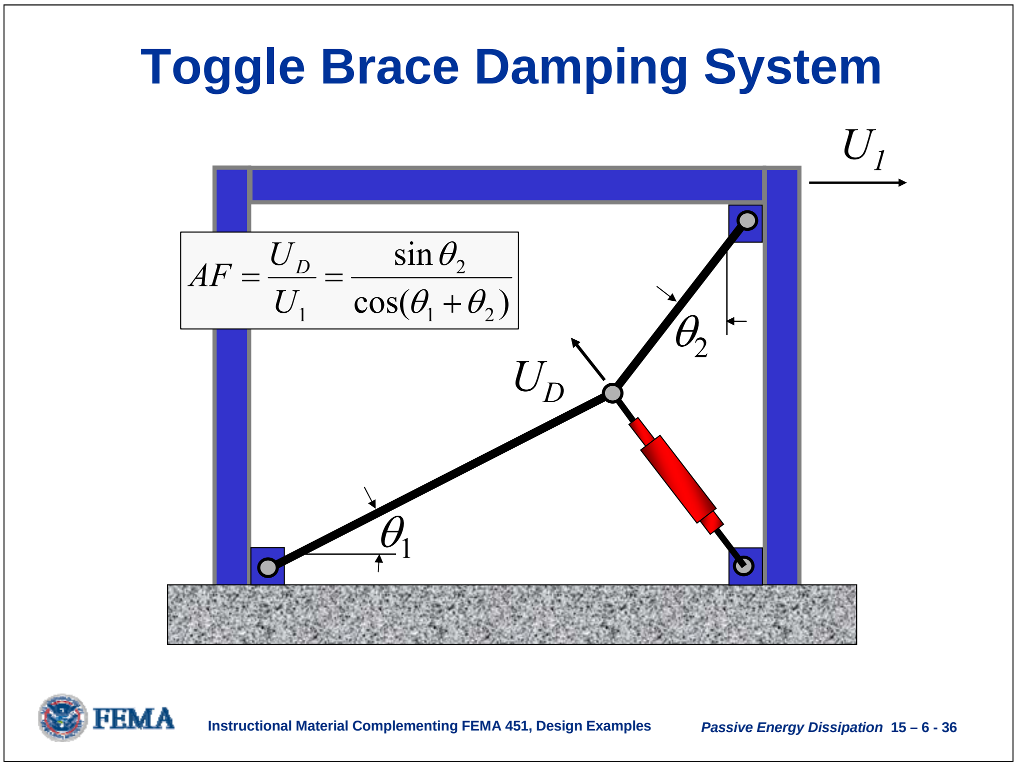
As per common practice, the damper is positioned near the bottom corner of the structural framing and is pin-connected to the framing.



For stiff structures, the motion of the damper can be amplified via a mechanical linkage known as a toggle brace system. The axial displacement of the damper is amplified with respect to the lateral displacement by the Amplification Factor, AF.

Toggle Brace Deployment

Huntington Tower, Boston, MA

- New 38-story steel-framed building
- 100 direct-acting and toggle-brace dampers
- 1300 kN (292 kips), +/- 101 mm (+/- 4 in.)
- Dampers suppress wind-induced vibration

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Passive Energy Dissipation 15 – 6 - 37

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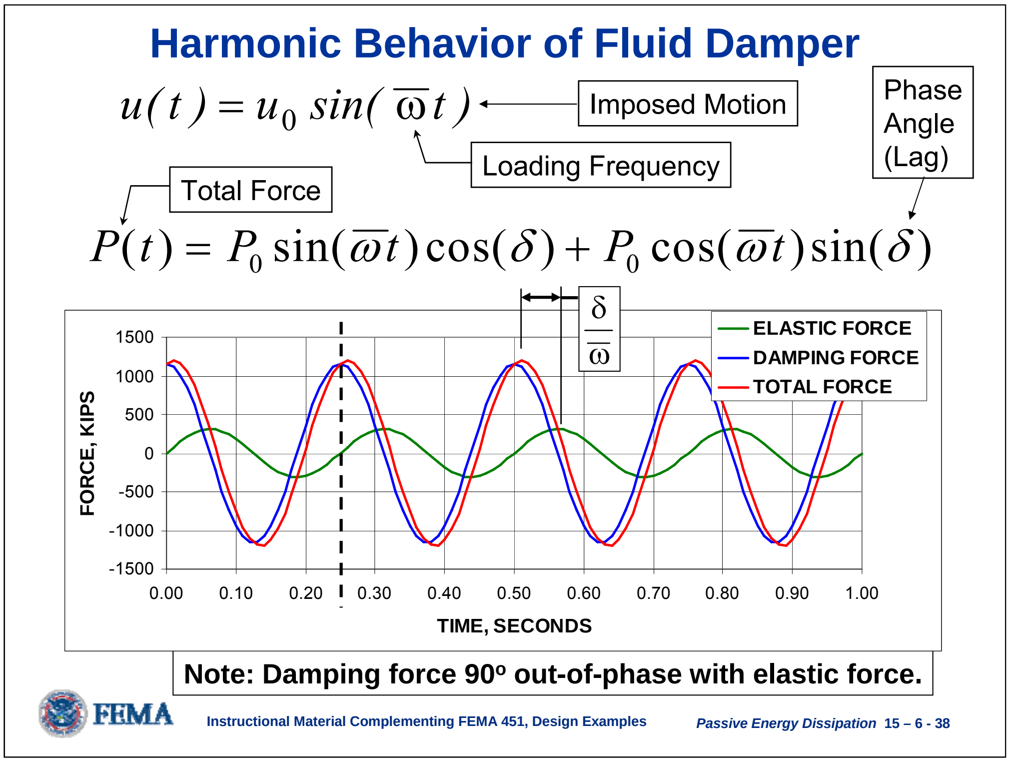
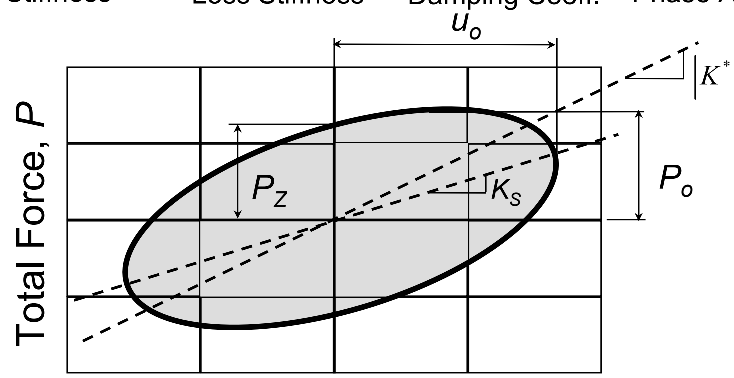


The frequency-dependent behavior of velocity-dependent dampers is typically obtained via harmonic testing. In this test, a harmonic displacement at a given frequency is imposed on the damper and the force required to impose the motion is measured. Due to the velocity-dependence of the damper, the measured force is out-of-phase with respect to the imposed displacement. This phase difference is characterized by the phase angle. The elastic (or restoring) force is proportional to displacement, the damping force is proportional to velocity, and the measured (or total) force is related to both displacement and velocity. The dashed vertical line is used to show that, at any instant in time, the damping force is 90 degrees out-of-phase with respect to the elastic force.

$$P(t) = K_S u(t) + C\dot{u}(t)$$

$K_S = \dfrac{P_0}{u_0}cos(\delta)$	$K_L = \dfrac{P_0}{u_0}sin(\delta)$	$C = \dfrac{K_L}{\overline{\omega}}$	$\delta = \sin^{-1}\left(\dfrac{P_Z}{P_0}\right)$
Storage Stiffness	Loss Stiffness	Damping Coeff.	Phase Angle



Damper Displacement, u

$$P_Z = K_L u_o = P_o\, sin(\delta)$$

$$E_D = \pi P_Z u_o = \pi P_o u_o\, sin(\delta)$$

The total damper force is related to both the displacement and velocity. The storage stiffness characterizes the ability of the damper to store energy. The loss stiffness and damping coefficient characterize the ability of the damper to dissipate energy. The phase angle indicates the degree to which the damper stores and dissipates energy. For example, if the phase angle is 90 degrees, the storage stiffness is zero and thus the damper acts as a pure viscous dashpot. Conversely, if the phase angle is 0 degrees, the loss stiffness is zero and the damper acts as a pure elastic spring. In terms of the damper hysteresis loop, the storage stiffness is the slope of the loop at the maximum displacement. The slope associated with the maximum displacement and maximum force is equal to the magnitude of the complex stiffness (discussed further on next slide). The width of the loop at zero displacement is proportional to the loss stiffness. The area within the loop, which is also proportional to the loss stiffness, is equal to the energy dissipated per cycle.

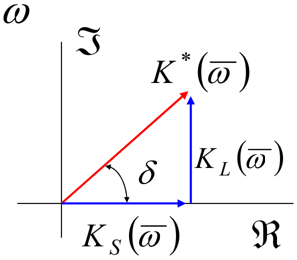
Frequency-Domain Force-Displacement Relation

$$P(t) = K_S u(t) + C\dot{u}(t)$$

Apply Fourier Transform:

$$P(\overline{\omega}) = K_S u(\overline{\omega}) + K_L i\overline{\omega} u(\overline{\omega}) / \overline{\omega}$$

$$P(\overline{\omega}) = [K_S + iK_L] u(\overline{\omega})$$



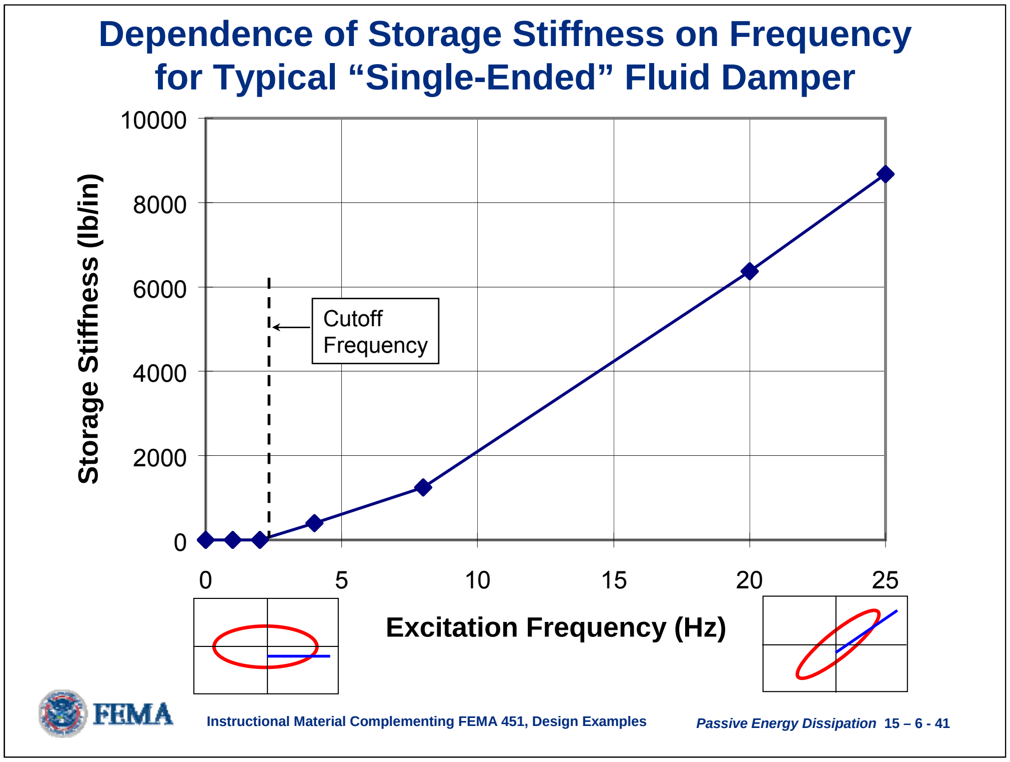
Complex Stiffness:

$$K^*(\overline{\omega}) = \frac{P(\overline{\omega})}{u(\overline{\omega})}$$

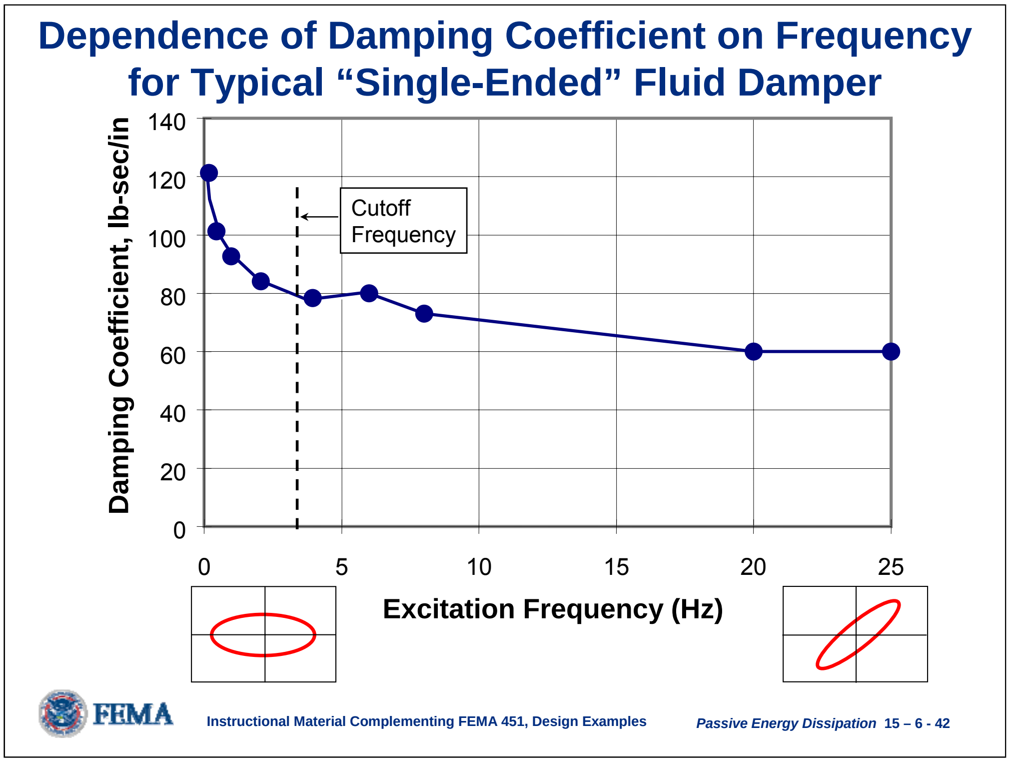
$$P(\overline{\omega}) = K^*(\overline{\omega}) u(\overline{\omega}) \longleftarrow \boxed{\text{Compact Force-Displ. Relation for Viscoelastic Dampers}}$$

Note: $\Re(K^*) = K_S$ and $\Im(K^*) = K_L$

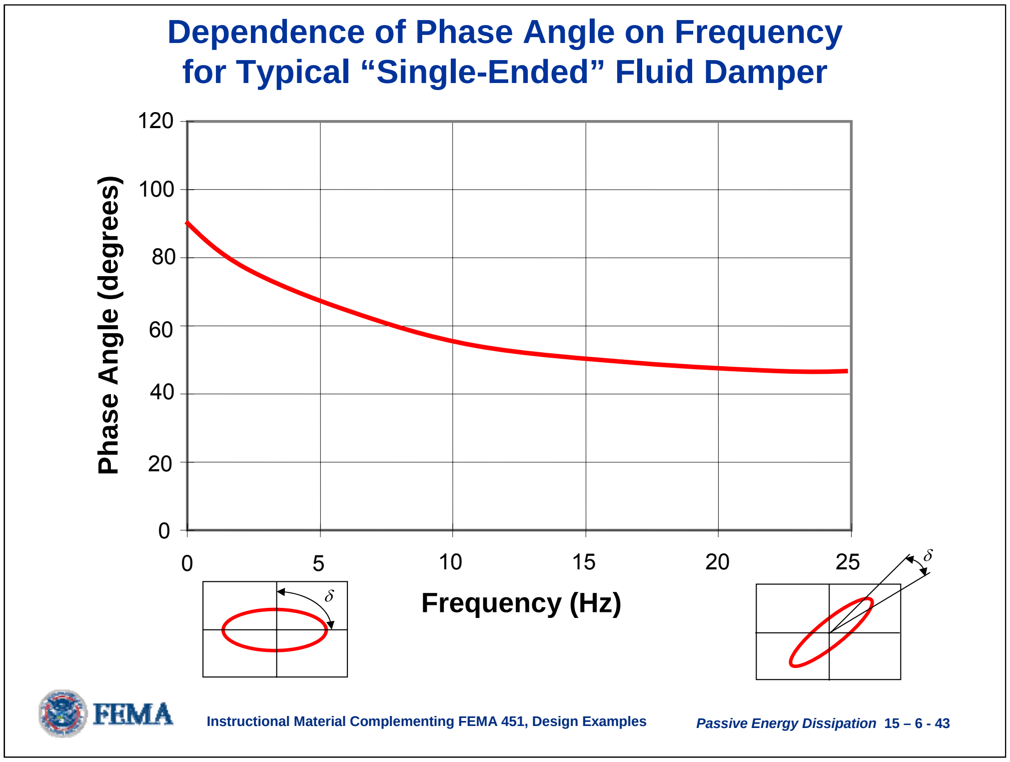
The time-dependent force-displacement relation for viscoelastic dampers can be rewritten as a frequency-dependent force-displacement relation in the frequency domain. The resulting expression is a compact form in which the complex stiffness characterizes both the ability to store and dissipate energy. The transformation to the frequency-domain can be performed either by replacing the imposed sinusoidal displacement and resulting force with equivalent complex exponential functions (not shown here) or by applying the Fourier transform to the time-dependent force-displacement relation (shown here). Note that the magnitude of the complex stiffness is equal to the peak force divided by the peak displacement (see previous slide).



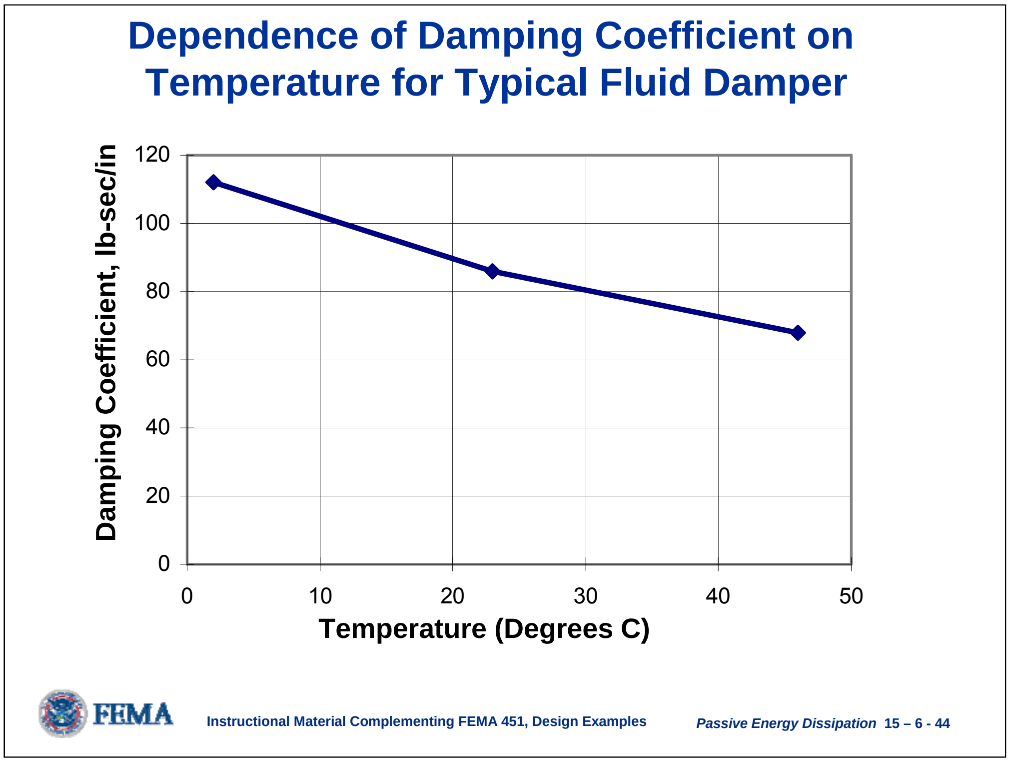
The data shown is for a 2-kip capacity "single-ended" damper that was tested by Constantinou and Symans (1992). The damper is a "single-ended" design and contains a piston rod accumulator that is used to account for the fluid volume displaced by the piston rod as it enters the damper. The accumulator control valves open and close to allow fluid to enter and leave the accumulator as needed so as to minimize the development of stiffness due to compression of the damper fluid. Below the cut-off frequency, the valves operate such that the storage stiffness is negligible. Beyond the cut-off frequency, the valves cannot operate fast enough and the damper develops stiffness. The hysteresis loops demonstrate the effect of frequency on the storage stiffness.



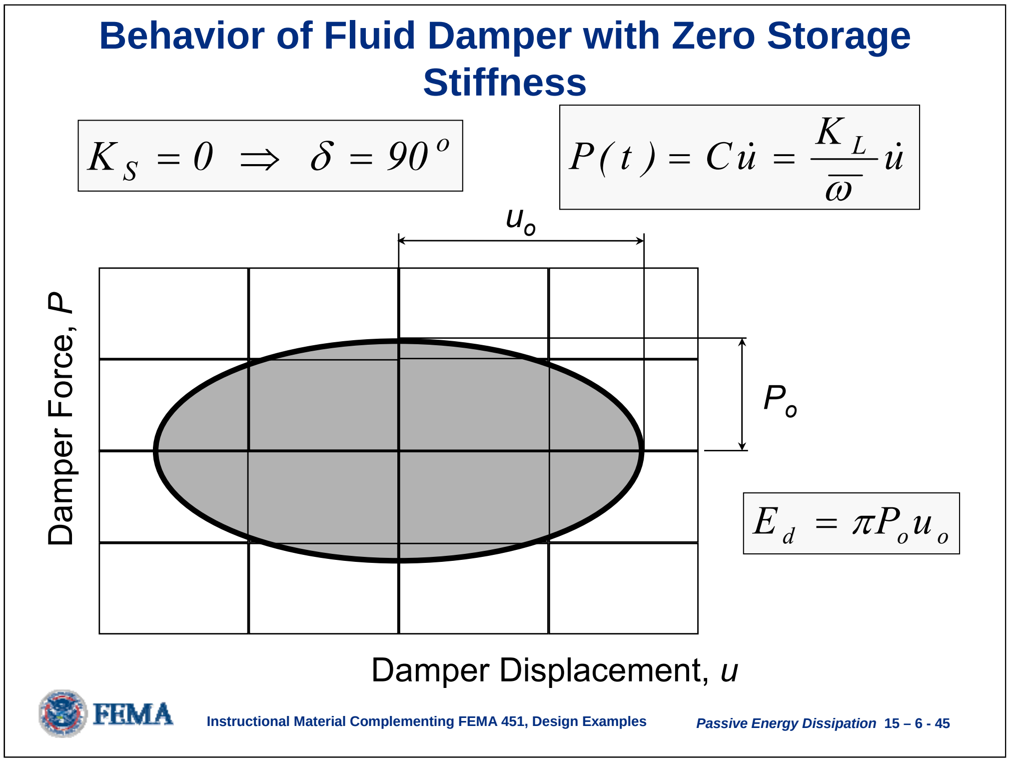
The data shown is for a 2-kip capacity “single-ended” damper that was tested by Constantinou and Symans (1992). Again, the damper contains a piston rod accumulator that is used to account for the fluid volume displaced by the piston rod as it enters the damper. At very low frequencies, the damping coefficient is difficult to quantify and artificially tends toward high values due to the mathematical expression for C (i.e., small values are in both the numerator and denominator). As the frequency is increased, the damping coefficient reduces at a relatively slow rate. The hysteresis loops demonstrate the effect of frequency on the damping coefficient.



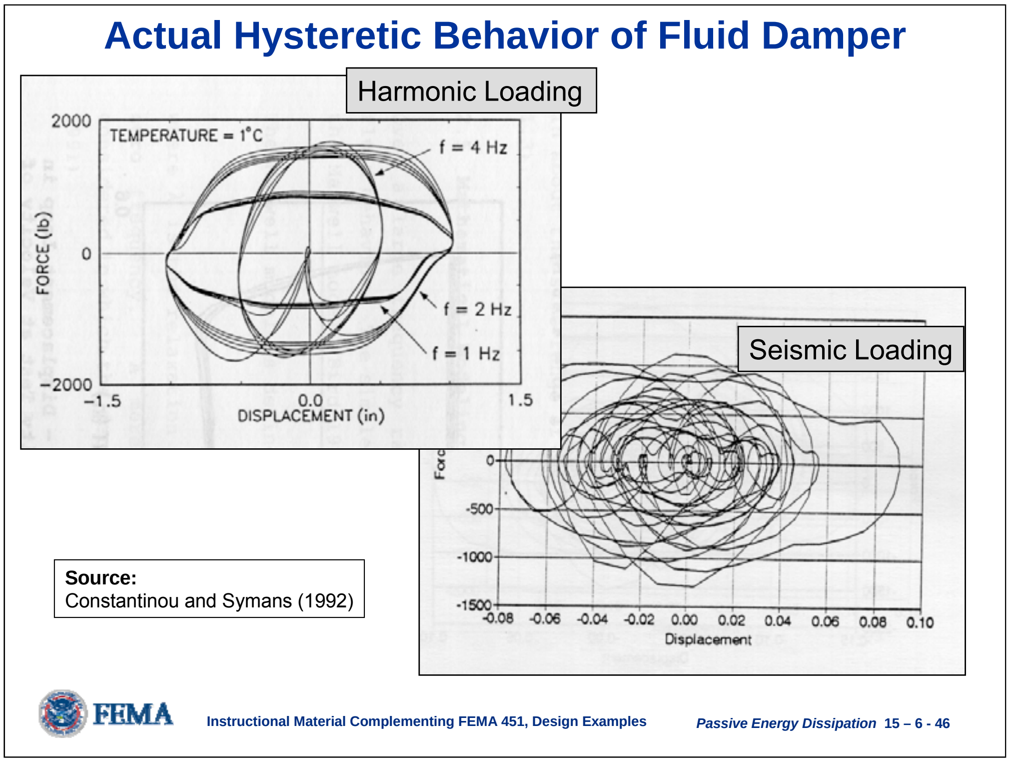
The phase angle between the peak displacement and peak force approaches 90 degrees as the frequency approaches zero. In this case, the storage stiffness is zero and the damper behaves as a pure viscous dashpot. As the frequency of motion increases, the phase angle reduces and the storage stiffness has a non-zero value.



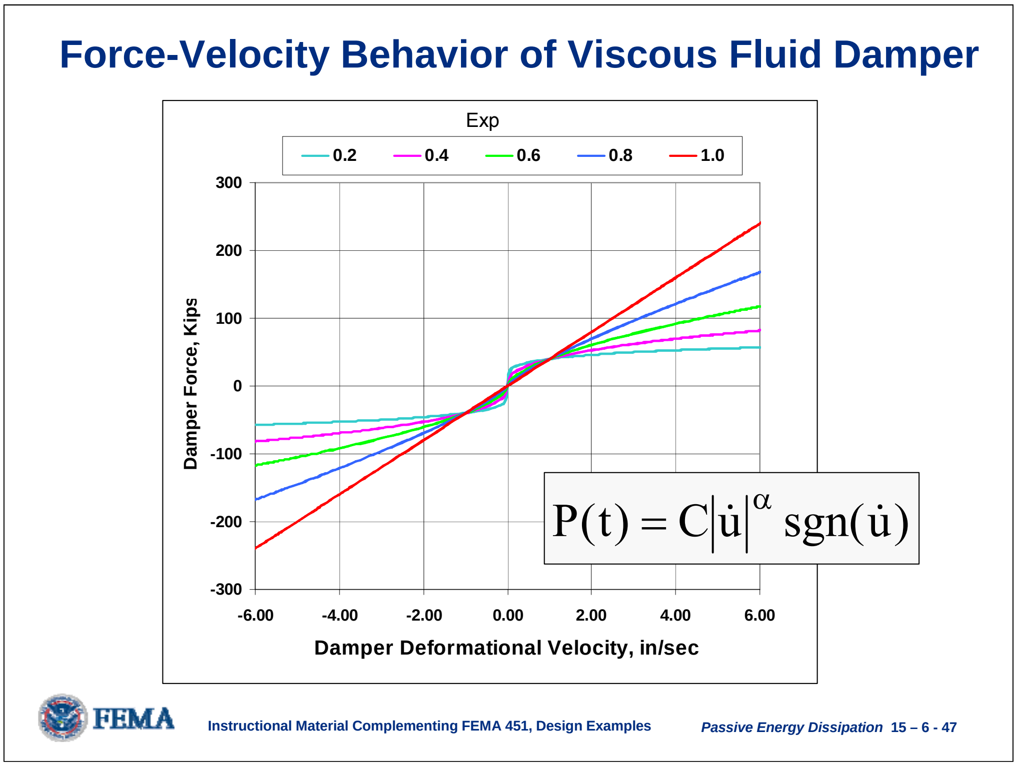
The data shown is for a 2-kip capacity “single-ended” damper that was tested by Constantinou and Symans (1992). Due to a special design of the orifices within the piston head that allows for temperature compensation, the damping coefficient has a relatively minor dependence on the ambient temperature. For viscoelastic solid dampers, the temperature-dependence is much stronger.



If a fluid damper has zero storage stiffness, it behaves as a pure viscous dashpot with a classical elliptical hysteresis loop with zero slope at the peak displacement (assuming harmonic excitation). The phase angle is 90 degrees indicating that the damper force is 90 degrees out-of-phase with respect to the damper displacement (i.e., the force is dependent only on velocity). As can be seen in the hysteresis loop, when the force is maximum, the displacement is zero, and vice-versa. The energy dissipated per cycle is equal to the area within the ellipse.



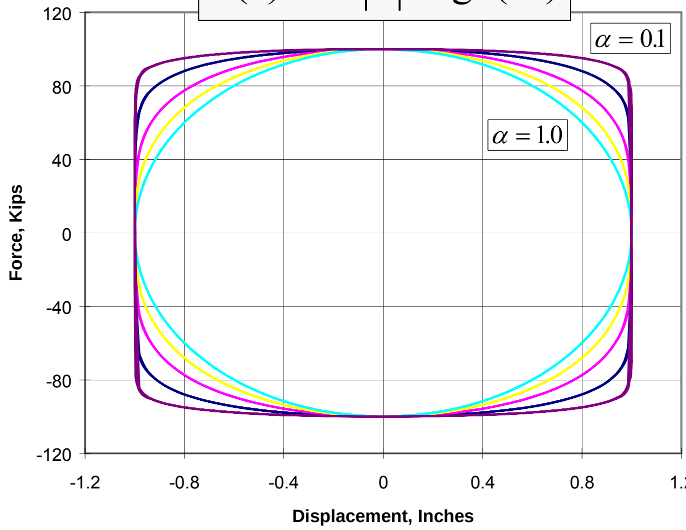
The hysteresis loops shown are for harmonic loading and seismic loading. The results from three different harmonic tests are shown. For each test, the loops are approximately elliptical in shape. The hysteresis loop for the seismic loading consists of a number of hysteresis loops, each of which is approximately elliptical in shape.



Thus far, we have considered dampers having linear viscoelastic behavior. Fluid dampers are often designed to behave as nonlinear dampers in which the force output is nonlinearly related to the velocity. A typical force-velocity relation and plots of the relation for 5 different velocity exponents is shown above. Physically realizable values for the velocity exponent range from about 0.2 to 2.0. For seismic applications, values of 1.0 and below are commonly used.

Nonlinear Fluid Dampers

$$P(t) = C|\dot{u}|^{\alpha}\,\mathrm{sgn}(\dot{u})$$



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The hysteresis loops for nonlinear fluid dampers move from an ellipse for the linear case (velocity exponent = 1.0) to a rectangle (velocity exponent = 0.0). Thus, for a given force and displacement amplitude, the energy dissipated per cycle for a nonlinear fluid damper is larger than for a linear fluid damper.

Energy Dissipated Per Cycle for Linear and Nonlinear Viscous Fluid Dampers

Linear Damper: $E_D = \pi P_o u_o$

Hysteretic Energy Factor

Nonlinear Damper: $E_D = \lambda P_o u_o$

$$\lambda = 4 \times 2^{\alpha} \frac{\Gamma^2\left(1+\frac{\alpha}{2}\right)}{\Gamma(2+\alpha)}$$

Γ = Gamma Function

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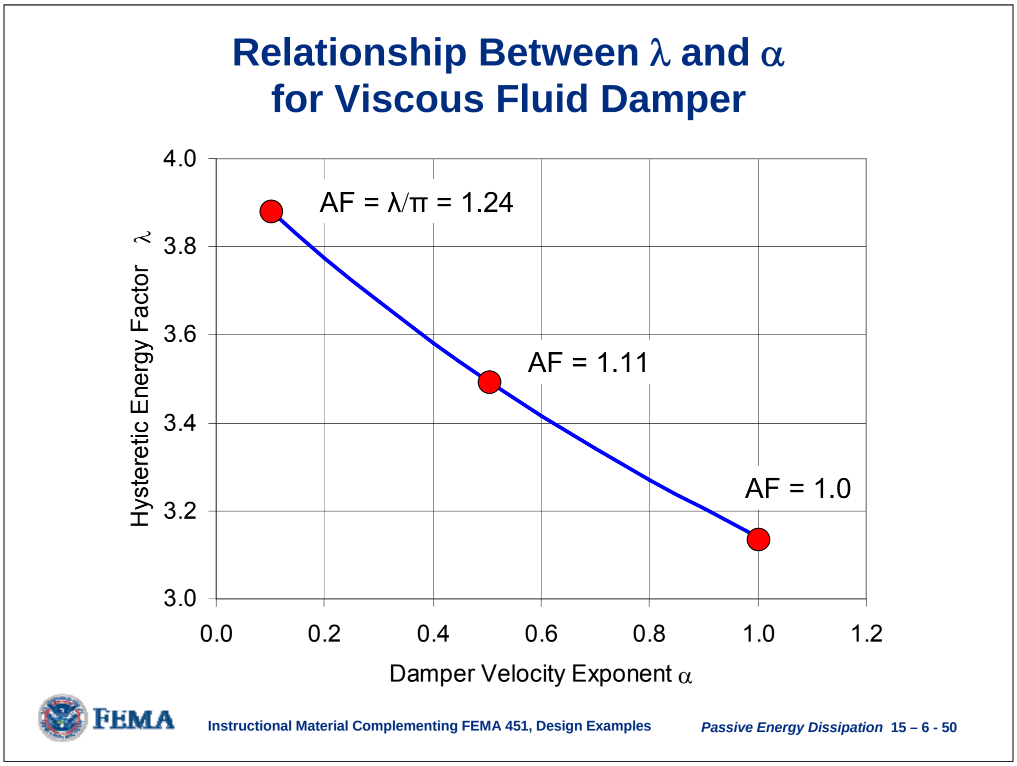
For a linear and nonlinear damper subjected to harmonic motion, the energy dissipated per cycle is obtained by integrating the damper force over the damper displacement for one cycle of motion. For the linear case, the energy dissipated per cycle is given by the equation for an ellipse.



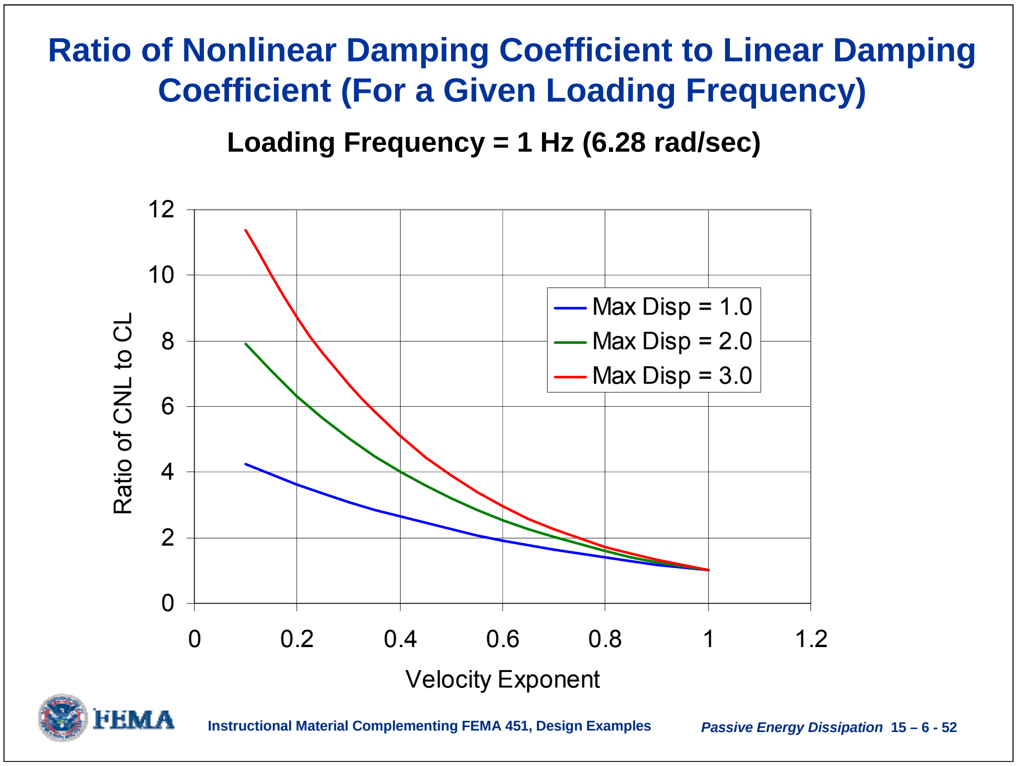
As indicated by the hysteretic energy factor, for a given force and displacement amplitude, the energy dissipated per cycle for a nonlinear fluid damper is larger than for a linear fluid damper and increases as the velocity exponent decreases. The amplification factor (AF) represents the factor by which the energy dissipated per cycle is increased for the nonlinear damper as compared to the linear damper (i.e., AF = hysteretic energy factor divided by pi).

Relationship Between Nonlinear and Linear Damping Coefficient for Equal Energy Dissipation Per Cycle

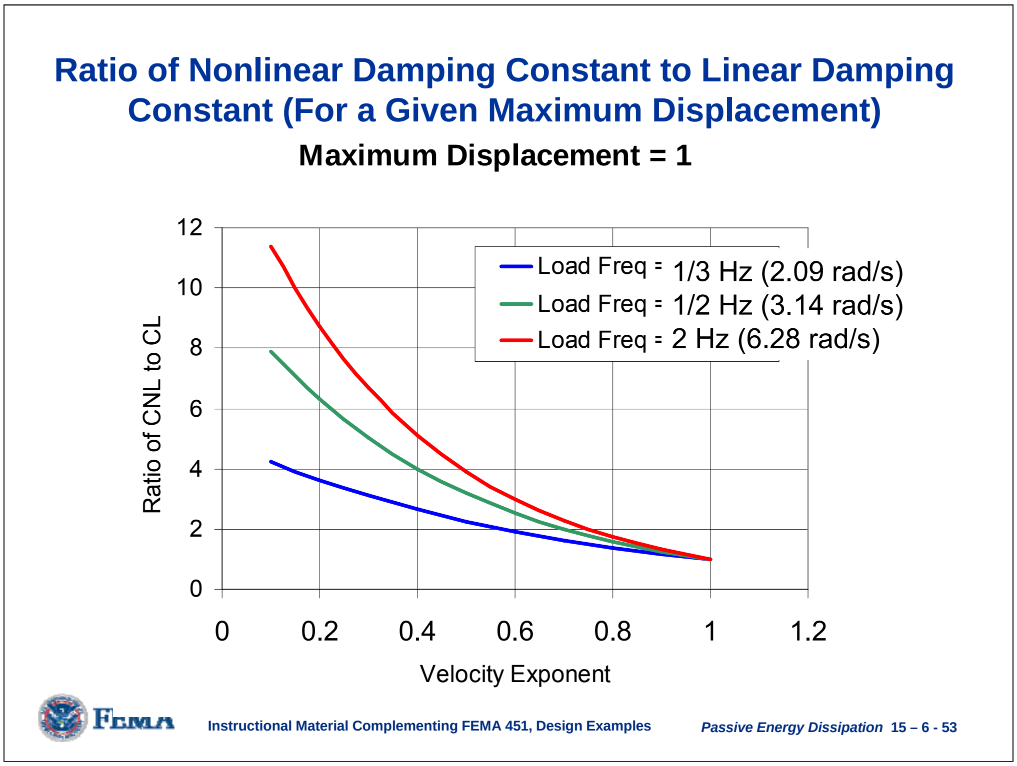
$$\frac{C_{NL}}{C_L} = \frac{\pi}{\lambda}(u_o\overline{\omega})^{1-\alpha}$$

Note: Ratio is frequency- and displacement-dependent and is therefore meaningful only for steady-state harmonic response.

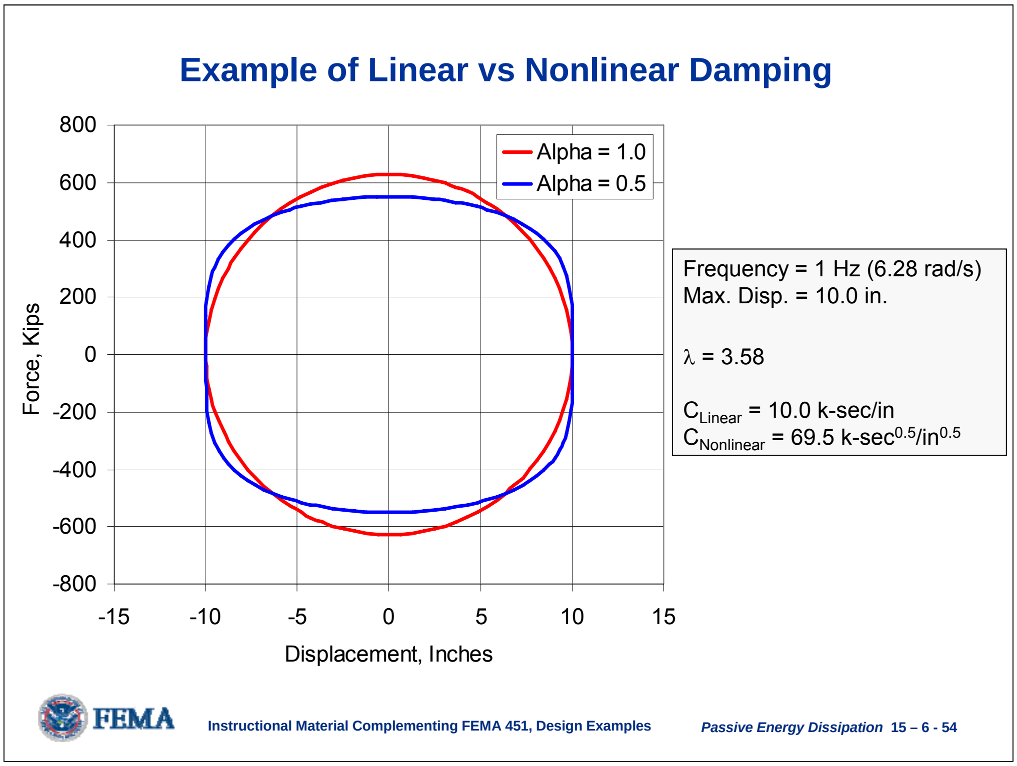
For a given harmonic motion (i.e., amplitude and frequency), the energy dissipated per cycle in a linear and nonlinear damper will generally not be equal. The values of the respective damping coefficients that result in equal energy dissipation per cycle is given by the above expression. Note that when the velocity exponent of the nonlinear damper approaches unity (i.e., the damper approaches a linear damper), the above ratio approaches unity as would be expected.



To achieve the same energy dissipation per cycle at a given frequency of harmonic motion, the damping coefficient of a nonlinear damper must be increased as the displacement amplitude increases.



To achieve the same energy dissipation per cycle at a given harmonic displacement amplitude, the damping coefficient of a nonlinear damper must be increased as the frequency increases.



In this example, the C value for the nonlinear damper must be 6.95 times as large as for the linear damper to produce the same amount of energy dissipation per cycle for the identical harmonic loading conditions (i.e., same frequency and amplitude).

Recommendations Related to Nonlinear Viscous Dampers

- Do NOT attempt to linearize the problem when nonlinear viscous dampers are used. Perform the analysis with discrete nonlinear viscous dampers.
- Do NOT attempt to calculate effective damping in terms of a damping ratio (ξ) when using nonlinear viscous dampers.
- DO NOT attempt to use a free vibration analysis to determine equivalent viscous damping when nonlinear viscous dampers are used.

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The behavior of nonlinear viscous dampers is very different from that of linear viscous dampers and thus attempts to transform the nonlinear problem to an equivalent linear problem should generally be avoided.

Advantages of Fluid Dampers

- High reliability
- High force and displacement capacity
- Force Limited when velocity exponent < 1.0
- Available through several manufacturers
- No added stiffness at lower frequencies
- Damping force (possibly) out of phase with structure elastic forces
- Moderate temperature dependency
- *May* be able to use linear analysis

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If a fluid damper is linear or nearly linear, the damping force will be out-of-phase with respect to the peak elastic forces in the structural framing.

If the fluid damper is linear or nearly linear, linear analysis may be used if the structure remains elastic.

Disadvantages of Fluid Dampers

- Somewhat higher cost
- Not force limited (particularly when exponent = 1.0)
- Necessity for nonlinear analysis in most practical cases (as it has been shown that it is generally not possible to add enough damping to eliminate all inelastic response)

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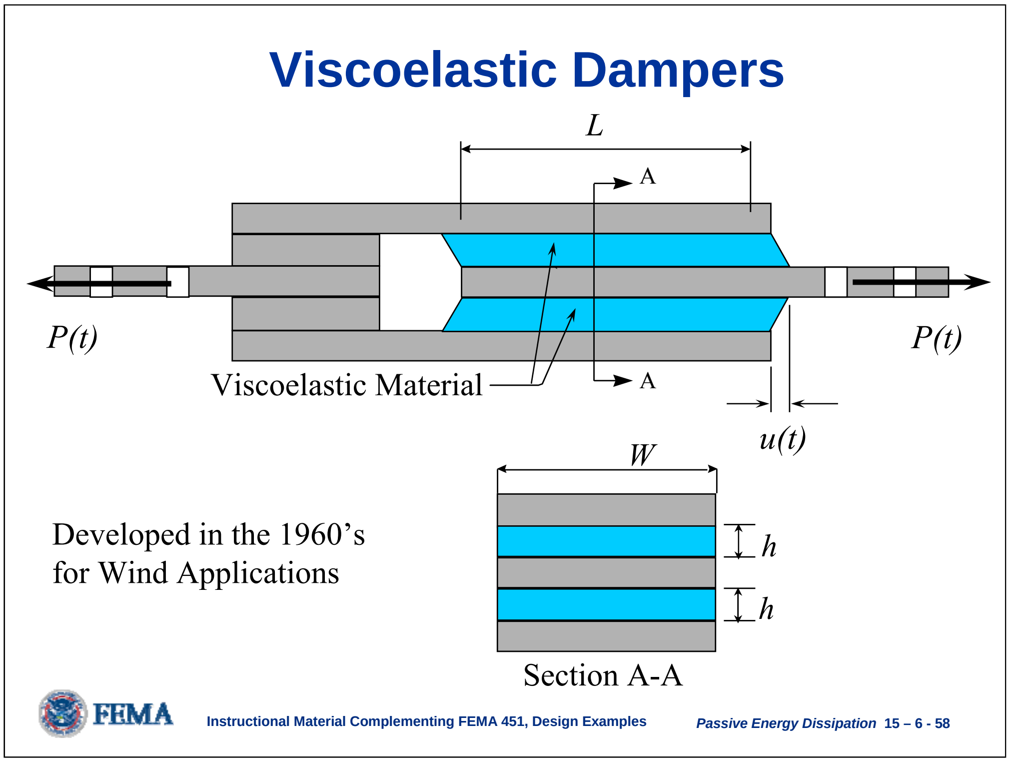
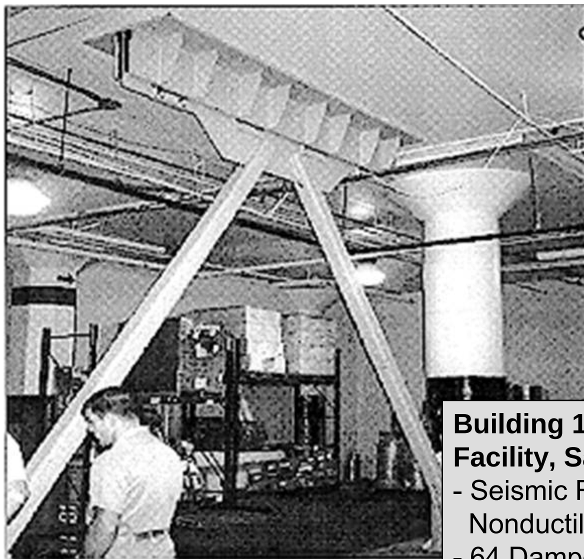
Nonlinear analysis is generally applicable for seismic analysis of structures with fluid dampers since, under strong earthquakes, it may be difficult to completely eliminate inelastic response. Also, the velocity exponent is less than unity in most applications, resulting in the need for nonlinear analysis.



Viscoelastic dampers consist of solid elastomeric pads (viscoelastic material) bonded to steel plates. The steel plates are attached to the structure within chevron or diagonal bracing. As one end of the damper displaces with respect to the other, the viscoelastic material is sheared. The shearing action results in the development of heat which is dissipated to the environment. By its very nature, viscoelastic dampers exhibit both elasticity and viscosity (i.e., they are displacement- and velocity-dependent). To date, the number of applications of viscoelastic dampers for wind and seismic vibration control is quite small relative to the number of applications of fluid viscous dampers.

Implementation of Viscoelastic Dampers



Building 116, US Naval Supply Facility, San Diego, CA
- Seismic Retrofit of 3-Story Nonductile RC Building
- 64 Dampers Within Chevron Bracing Installed in 1996

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No annotation is provided for this slide.

Harmonic Behavior of Viscoelastic Damper

$$u(t) = u_0 \sin(\overline{\omega} t)$$

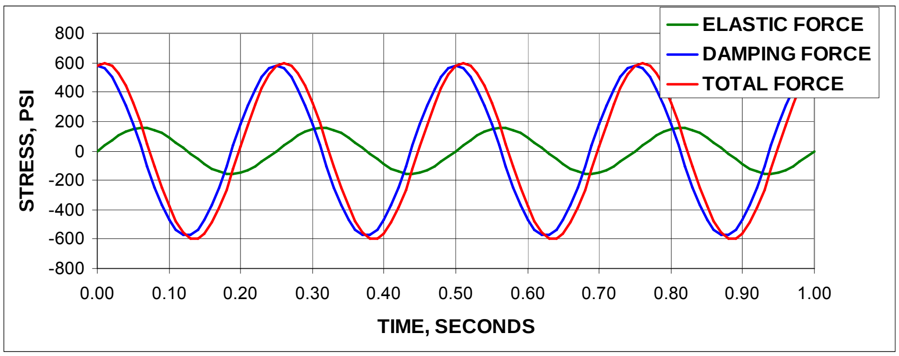
Imposed Motion

Loading Frequency

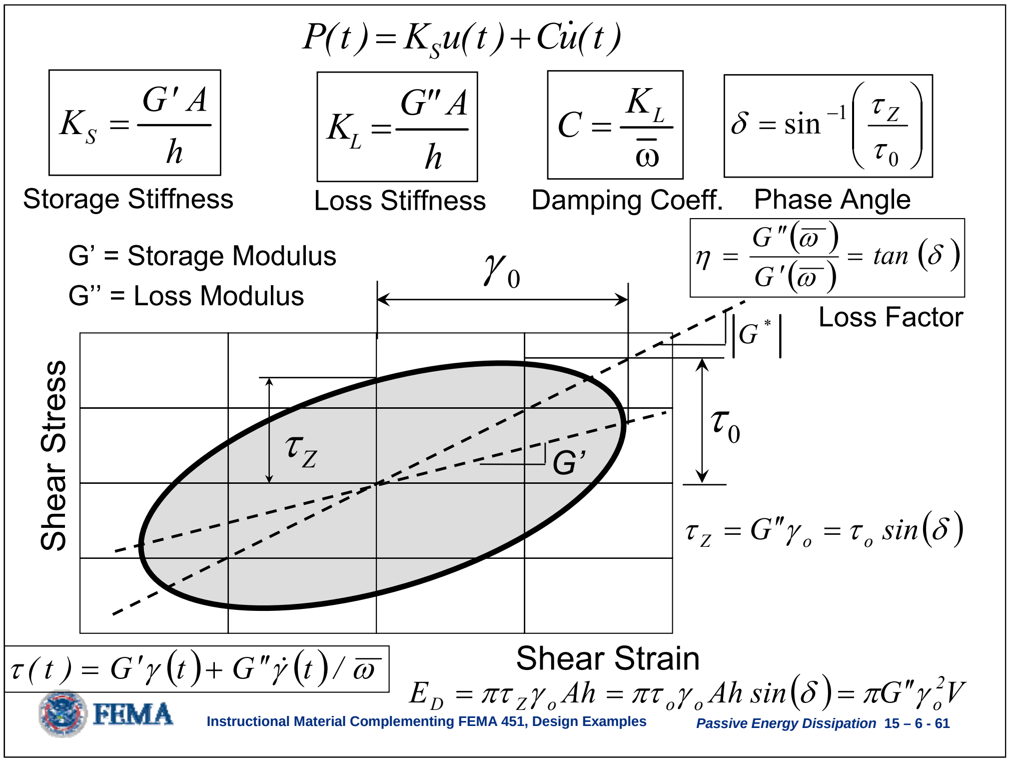
Phase Angle (Lag)

Total Force

$$P(t) = P_0 \sin(\overline{\omega} t)\cos(\delta) + P_0 \cos(\overline{\omega} t)\sin(\delta)$$



The frequency-dependent behavior of viscoelastic dampers is typically obtained via harmonic testing. In this test, a harmonic displacement at a given frequency is imposed on the damper and the force required to impose the motion is measured. Due to the velocity-dependence of the damper, the measured force is out-of-phase with respect to the displacement. The elastic force is proportional to displacement, the damping force is proportional to velocity, and the measured (or total) force is related to both displacement and velocity. For viscoelastic materials, the behavior is typically presented in terms of stresses and strains rather than forces and displacements.



The total damper force is related to both the displacement and velocity. The storage stiffness characterizes the ability of the damper to store energy. The loss stiffness and damping coefficient characterize the ability of the damper to dissipate energy. The phase angle indicates the degree to which the damper stores and/or dissipates energy. For example, if the phase angle is 90 degrees, the storage stiffness is zero and thus the damper acts as a pure viscous dashpot. Conversely, if the phase angle is 0 degrees, the loss stiffness is zero and the damper acts as a pure spring. Viscoelastic materials have phase angles that are in between 0 and 90 degrees. In terms of the damper stress-strain hysteresis loop, the storage modulus (which is proportional to the storage stiffness) is the slope of the loop at the maximum strain. The slope associated with the maximum strain and maximum stress is equal to the magnitude of the complex shear modulus (discussed further on next slide). The width of the loop at zero strain is proportional to the loss modulus (which, in turn, is proportional to the loss stiffness). The area within the loop, which is also proportional to the loss modulus, is equal to the energy dissipated per cycle. Note that the shear and loss moduli are material properties whereas the storage and loss stiffness are damper properties.

Frequency-Domain Stress-Strain Relation

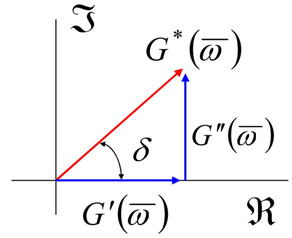
$$\tau(t) = G'\gamma(t) + G''\dot{\gamma}(t)/\overline{\omega}$$

Apply Fourier Transform:

$$\tau(\overline{\omega}) = G'\gamma(\overline{\omega}) + G''i\overline{\omega}\gamma(\overline{\omega})/\overline{\omega}$$

$$\tau(\overline{\omega}) = [G' + iG'']\gamma(\overline{\omega})$$

$$\tau(\overline{\omega}) = G'[1 + i\eta]\gamma(\overline{\omega})$$



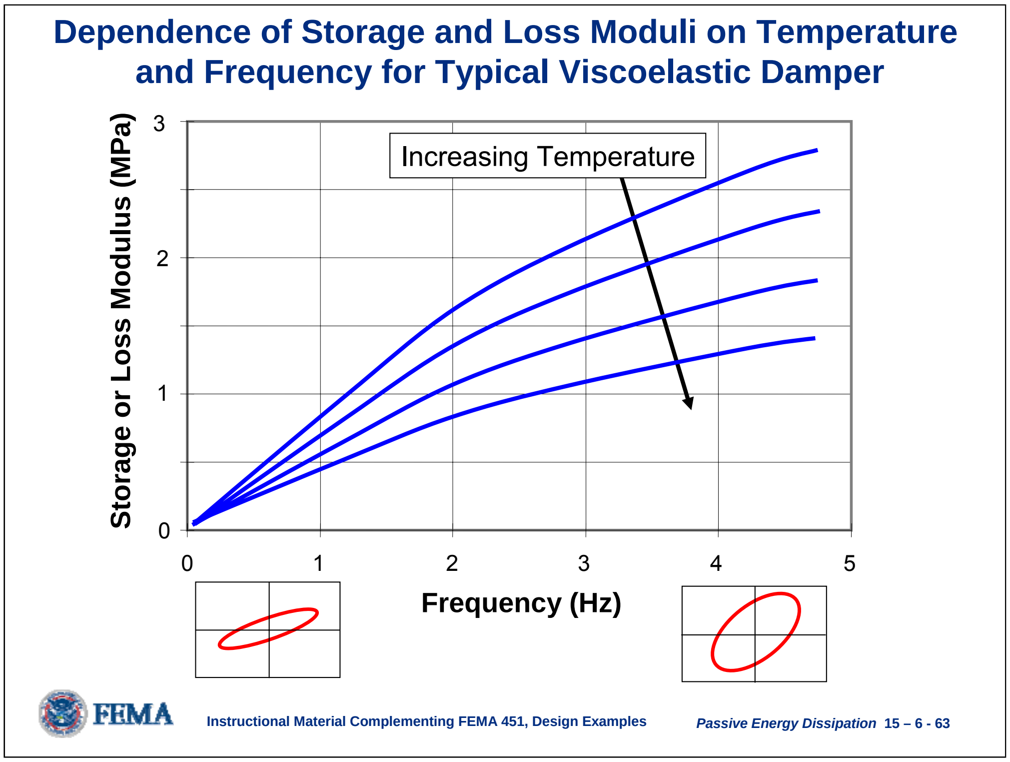
Complex Shear Modulus:

$$G^*(\overline{\omega}) = \frac{\tau(\overline{\omega})}{\gamma(\overline{\omega})} = G'[1 + i\eta]$$

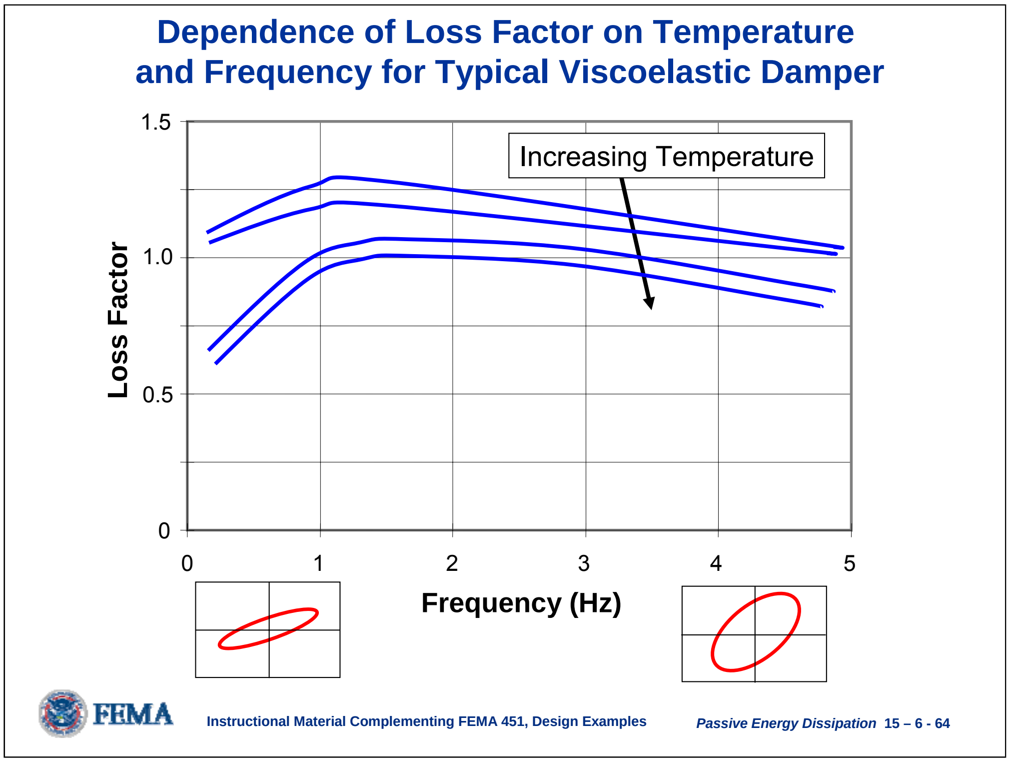
$$\tau(\overline{\omega}) = G^*(\overline{\omega})\gamma(\overline{\omega}) \longleftarrow$$

Compact Stress-Strain Relation for Viscoelastic Materials

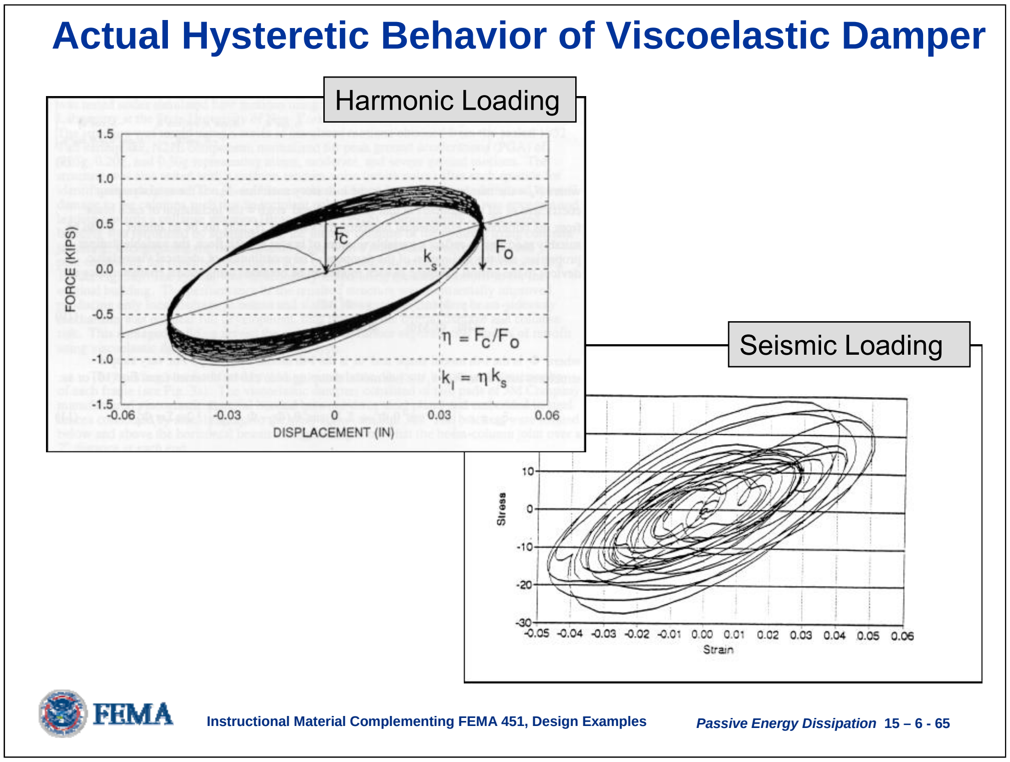
The time-dependent stress-strain relation for viscoelastic dampers can be rewritten as a frequency-dependent stress-strain relation in the frequency domain. The resulting expression is a compact form in which the complex shear modulus characterizes both the ability to store and dissipate energy. The transformation to the frequency-domain can be performed either by replacing the imposed sinusoidal strain and resulting stress with equivalent complex exponential functions (not shown here) or by applying the Fourier transform to the time-dependent stress-strain relation (shown here). Note that the magnitude of the complex shear modulus is equal to the peak stress divided by the peak strain (see previous slide).



At a given frequency and strain amplitude, the storage and loss moduli have similar values. Increases in temperature can significantly reduce the moduli. The effect of frequency on the moduli is indicated by the two hysteresis loops. At low frequencies, the hysteresis loop has a small slope and is narrow, thus the storage and loss moduli are small. At higher frequencies, the hysteresis loop has a larger slope and is wide, thus the storage and loss moduli are larger.



At a given frequency and strain amplitude, the loss factor is approximately unity. Over a range of frequencies, the loss factor has minimal dependence on the frequency of excitation.



The hysteresis loops shown are for harmonic loading and seismic loading. For the harmonic loading test, the temperature of the viscoelastic material increases during the test. The increased temperature reduces both the storage stiffness and the loss stiffness. As expected, for the harmonic loading the shape of the hysteresis loops is elliptical. The hysteresis loop for the seismic loading consists of a number of hysteresis loops, each of which is approximately elliptical in shape.

Advantages of Viscoelastic Dampers

- High reliability
- *May* be able to use linear analysis
- Somewhat lower cost

If the viscoelastic damper material properties are not strongly frequency- or temperature-dependent over the expected range of frequencies of motion, linear analysis may be used if the structure remains elastic.

Disadvantages of Viscoelastic Dampers

- Strong Temperature Dependence
- Lower Force and Displacement Capacity
- Not Force Limited
- Necessity for nonlinear analysis in most practical cases (as it has been shown that it is generally not possible to add enough damping to eliminate all inelastic response)

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Nonlinear analysis is generally applicable for seismic analysis of structures with viscoelastic dampers since, under strong earthquakes, it may be difficult to completely eliminate inelastic response.

Outline: Part II

- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
- Models for Velocity-Dependent Dampers
- Effects of Linkage Flexibility
- Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
- Concept of Equivalent Viscous Damping
- Modeling Considerations for Structures with Passive Damping Systems

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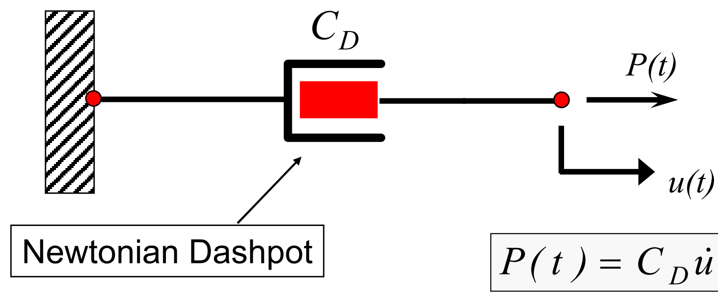
Modeling Viscous Dampers:
Simple Dashpot

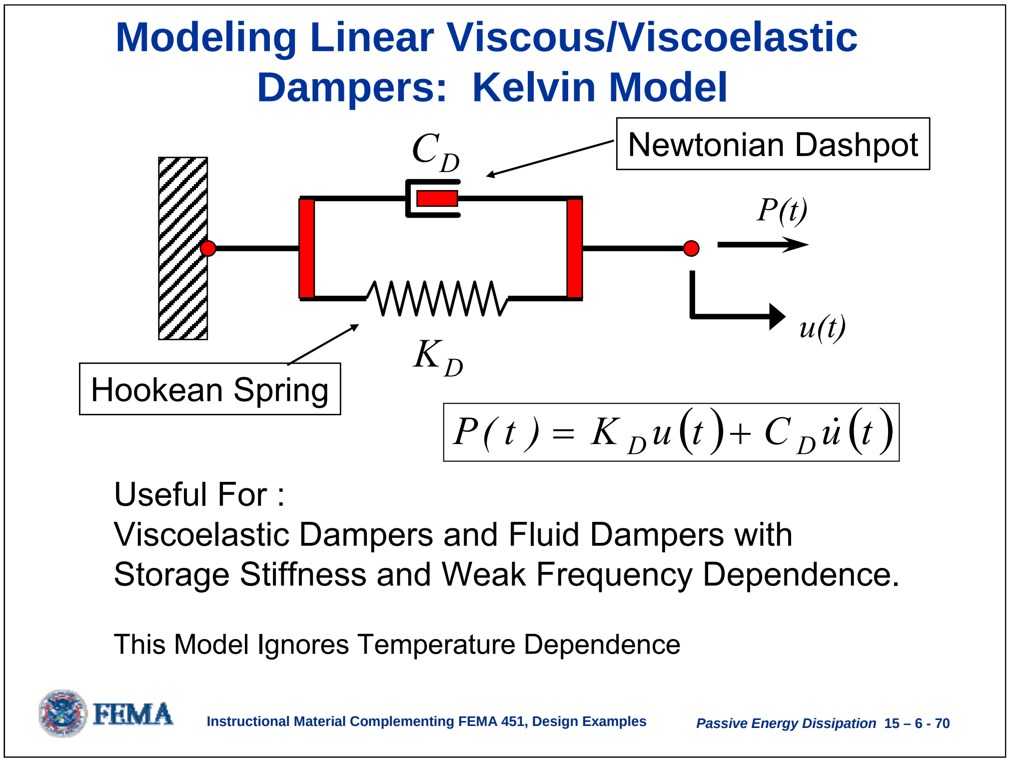


$$P(t) = C_D \dot{u}(t)$$

Useful For :
Fluid Dampers with Zero Storage Stiffness

This Model Ignores Temperature Dependence

A simple dashpot can be used to model dampers that exhibit viscosity and little or no elasticity. This model employs a Newtonian dashpot in which the force is proportional to the velocity. The proportionality constant is the damping coefficient. Note that, in many applications, a nonlinear dashpot model would be appropriate (e.g., nonlinear fluid viscous dampers).



A Kelvin model can be used to model dampers that exhibit both elasticity and viscosity. The model consists of a Hookean (linear) spring and Newtonian (linear) dashpot in parallel. The force required to displace the model is equal to the sum of the forces in the spring and dashpot. If a constant force is applied to the model, over a long duration of time, the resisting force becomes fully elastic (i.e., the force is proportional to the displacement). Thus, the Kelvin model is said to represent the behavior of "solids."

Kelvin Model (Continued)

$$\boxed{P(t) = K_D u(t) + C_d \dot{u}(t)}$$

Apply Fourier Transform:

$$P(\overline{\omega}) = \left[K_D + i\overline{\omega} C_d\right] u(\overline{\omega})$$

Complex Stiffness:

$$K^*(\overline{\omega}) = K_D + i\overline{\omega} C_d$$

Storage Stiffness:

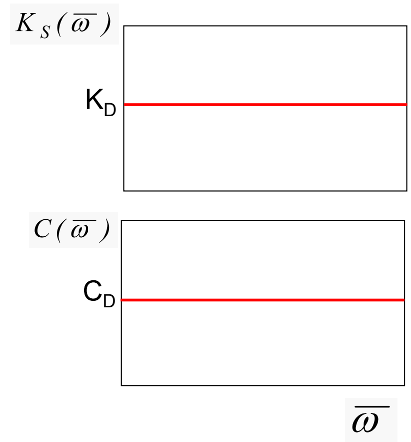
$$K_S(\overline{\omega}) = \Re\left[K^*(\overline{\omega})\right] = K_D$$

Loss Stiffness:

$$K_L(\overline{\omega}) = \Im\left[K^*(\overline{\omega})\right] = C_D \overline{\omega}$$

Damping Coefficient:

$$C(\overline{\omega}) = \frac{K_L(\overline{\omega})}{\overline{\omega}} = C_D$$



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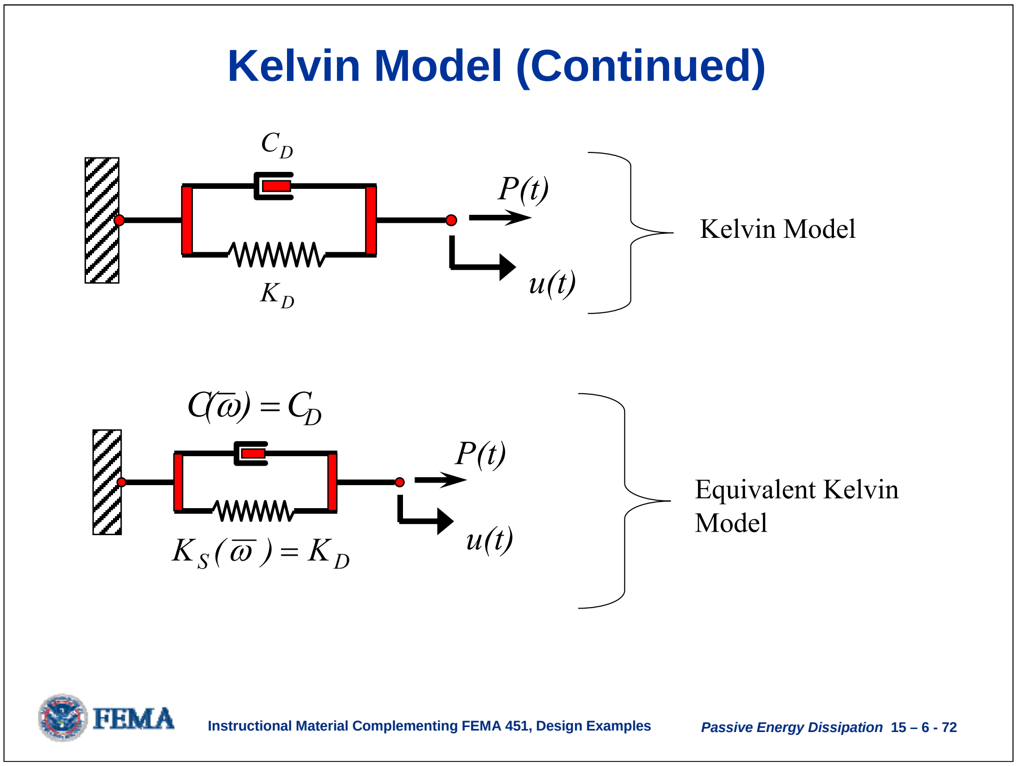
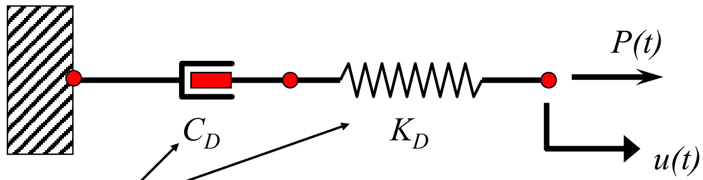
The frequency-dependency of the storage stiffness and damping coefficient of a linear model of viscoelasticity can be identified by expressing the force output in the frequency-domain via application of the Fourier transform. The resulting complex stiffness contains the storage stiffness (real part) and the loss stiffness (imaginary part). For the physical damper, the storage stiffness is the proportionality factor for the displacement and the loss stiffness divided by the frequency is the damping coefficient which is the proportionality factor for the velocity. In general, the storage stiffness and damping coefficient will be frequency-dependent.

For the Kelvin model, it is evident by inspection of the mathematical model, that the model parameters, K_D and C_D, are the storage stiffness and damping coefficient and are frequency-independent. The formal process for showing that this is true is provided on this slide. Although the storage stiffness and damping coefficient of the Kelvin model have no frequency-dependence, the model does exhibit frequency dependence since the force is related to the velocity. The frequency dependence is weak since the storage stiffness and damping coefficient are independent of frequency.



The force output of the equivalent Kelvin model is equal to the sum of the spring and dashpot force. The spring force is proportional to the displacement with the frequency-independent storage stiffness being the proportionality factor. The dashpot force is proportional to the velocity with the frequency-independent damping coefficient being the proportionality factor. Although it may not appear that this analysis of the Kelvin model has been very productive, it provides a simple illustration of the process by which the storage stiffness and damping coefficient of a linear model of viscoelasticity are determined. We will use the same procedure next for the Maxwell model which is slightly more complicated than the Kelvin model.

Modeling Linear Viscous/Viscoelastic Dampers: Maxwell Model



Newtonian Dashpot; Hookean Spring

$$P(t) + \frac{C_D}{K_D}\dot{P}(t) = C_D\dot{u}(t)$$

Useful For :
Viscoelastic Dampers and Fluid Dampers with Strong Frequency Dependence.
This Model Ignores Temperature Dependence

A Maxwell model can be used to model dampers that exhibit both elasticity and viscosity. The model consists of a Hookean (linear) spring and Newtonian (linear) dashpot in series. The force in the spring is equal to the force in the dashpot. If a constant force is applied to the model, over a long duration of time, the resisting force becomes viscous (i.e., the force is proportional to the velocity). Thus, the Maxwell model is said to represent the behavior of “fluids.” Note that, within the SAP2000 structural analysis program, the Maxwell model is available to model linear viscous dampers, viscoelastic dampers, and nonlinear viscous dampers (wherein the linear dashpot is replaced with a nonlinear dashpot).

Maxwell Model (Continued)

$$\boxed{P(t)+\frac{C_D}{K_D}\dot{P}(t)=C_d\dot{u}(t)}$$

Apply Fourier Transform:

$$P(\overline{\omega})+i\overline{\omega}\frac{C_D}{K_D}P(\overline{\omega})=i\overline{\omega}C_d u(\overline{\omega})$$

Complex Stiffness:

$$K^*(\overline{\omega})=\frac{C_D\lambda\overline{\omega}^2}{1+\lambda^2\overline{\omega}^2}+i\frac{C_D\overline{\omega}}{1+\lambda^2\overline{\omega}^2}$$

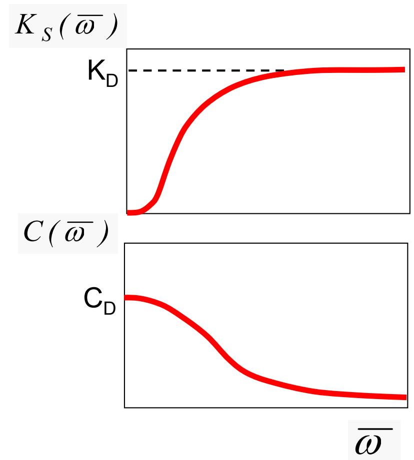
Relaxation Time: $\lambda=C_D/K_D$

Storage Stiffness:

$$K_S(\overline{\omega})=\Re\left[K^*(\overline{\omega})\right]=\frac{K_D\lambda^2\overline{\omega}^2}{1+\lambda^2\overline{\omega}^2}$$

Loss Stiffness:

$$K_L(\overline{\omega})=\Im\left[K^*(\overline{\omega})\right]=\frac{C_D\overline{\omega}}{1+\lambda^2\overline{\omega}^2}$$



Damping Coefficient:

$$C(\overline{\omega})=\frac{K_S(\overline{\omega})}{\overline{\omega}}=\frac{C_D}{1+\lambda^2\overline{\omega}^2}$$

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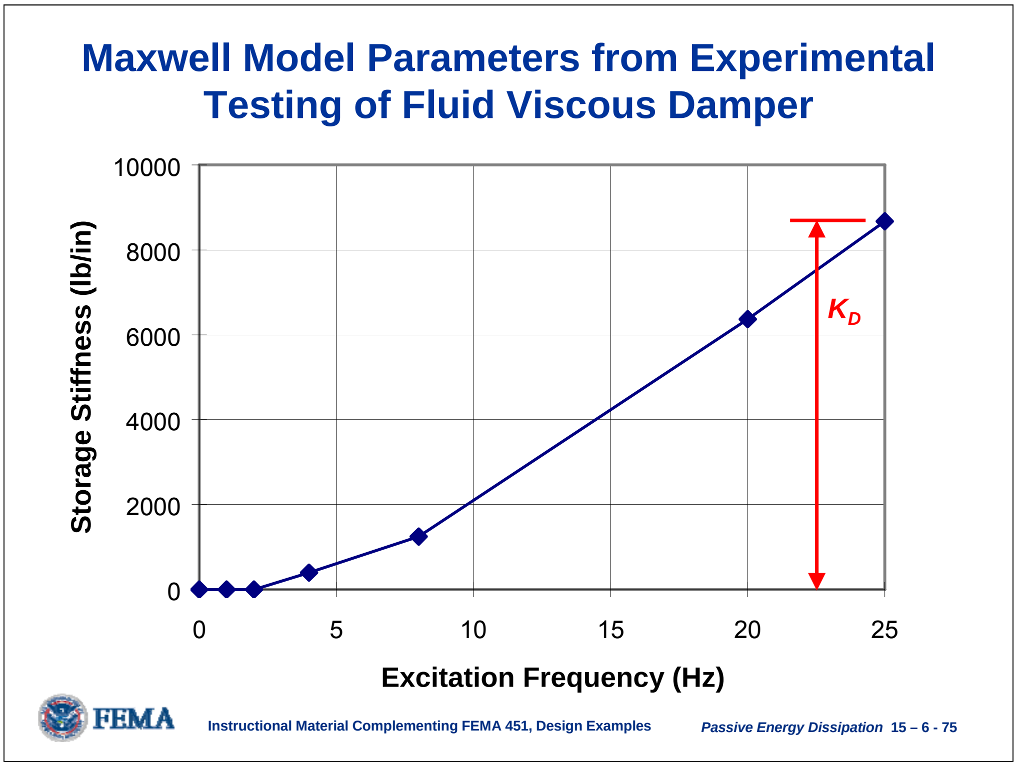
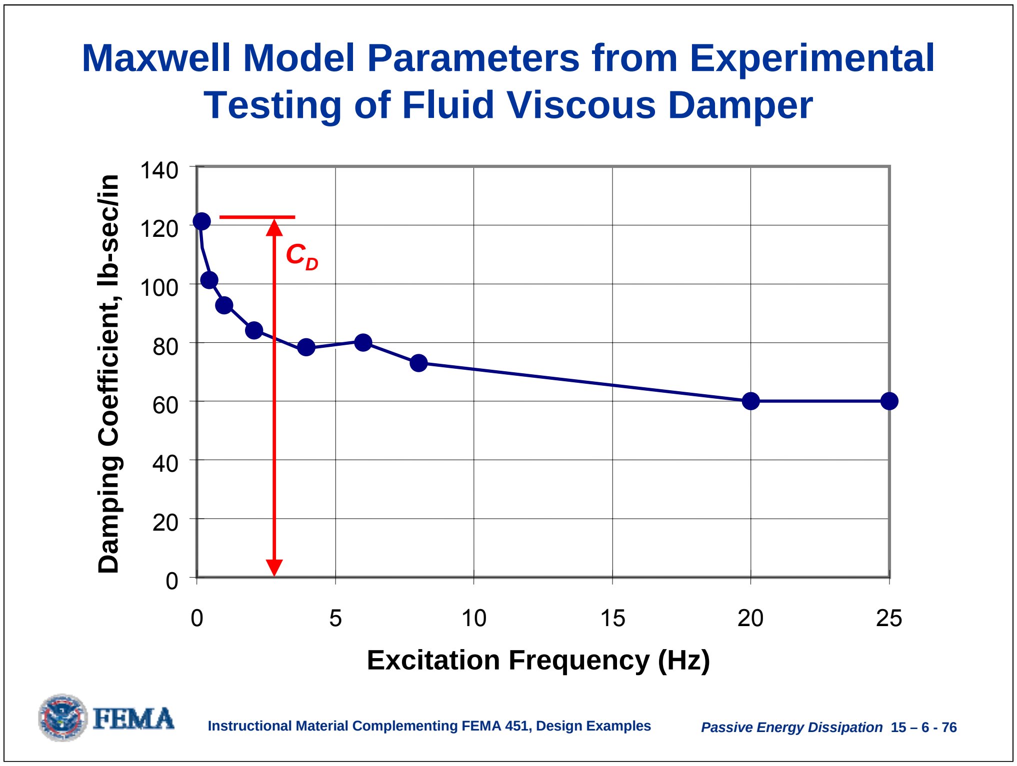
The frequency dependence of the Maxwell model is said to be strong since the storage stiffness (ability to store energy) and damping coefficient (ability to dissipate energy) are dependent on frequency. The stiffness of the spring in the Maxwell model, K_D, is equal to the storage stiffness at high frequencies. The damping coefficient of the dashpot in the Maxwell model, C_D, is equal to the damping coefficient at low frequencies.

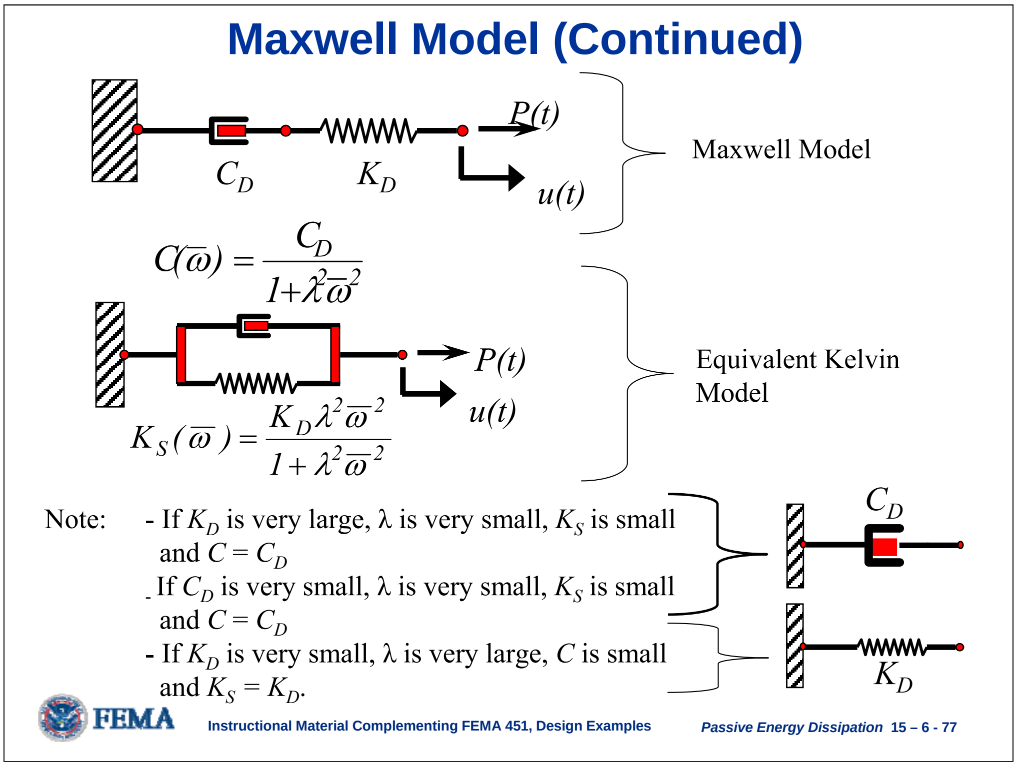
From a materials point-of-view, the relaxation time is determined from a relaxation test in which a constant strain is suddenly applied to the damper material. The resulting stress decays (or relaxes) with time due to a transfer of stored energy in the spring to dissipated energy in the dashpot. The relaxation time is the time required for the stress to reach approximately 37% (1/e) of the initial stress. For a purely elastic damper, the relaxation time goes to infinity. For a purely viscous damper, the relaxation time goes to zero. For a viscoelastic damper, the relaxation time is greater than zero but finite.



The data shown is for a 2-kip capacity "single-ended" fluid viscous damper that was tested by Constantinou and Symans (1992).



The data shown is for a 2-kip capacity "single-ended" fluid viscous damper that was tested by Constantinou and Symans (1992).



The force output of the equivalent Kelvin model has two components, a spring force and a dashpot force. The spring force is proportional to the displacement with the frequency-dependent storage stiffness being the proportionality factor. The dashpot force is proportional to the velocity with the frequency-dependent damping coefficient being the proportionality factor.

If the stiffness of the spring in the Maxwell model is very large or if the damping coefficient of the dashpot in the Maxwell model is very small, most of the deformation occurs in the dashpot and thus the Maxwell element behaves essentially as a viscous dashpot. If the stiffness of the spring in the Maxwell model is very small, most of the deformation occurs in the spring and thus the Maxwell element behaves essentially as a linear spring.

Outline: Part II

- Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
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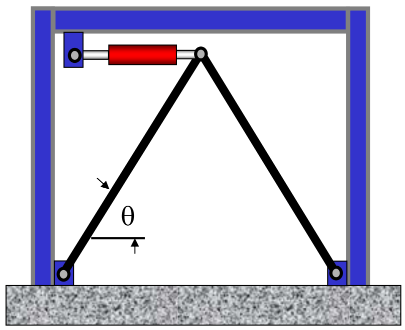
Effect of Linkage Flexibility on Damper Effectiveness



$$K_{Brace,Effective} = 2\frac{AE}{L}\cos^2\theta$$

Because the damper is always in series with the linkage, the damper-brace assembly acts like a Maxwell model.

Hence, the effectiveness of the damper is reduced. The degree of lost effectiveness is a function of the structural properties and the loading frequency.

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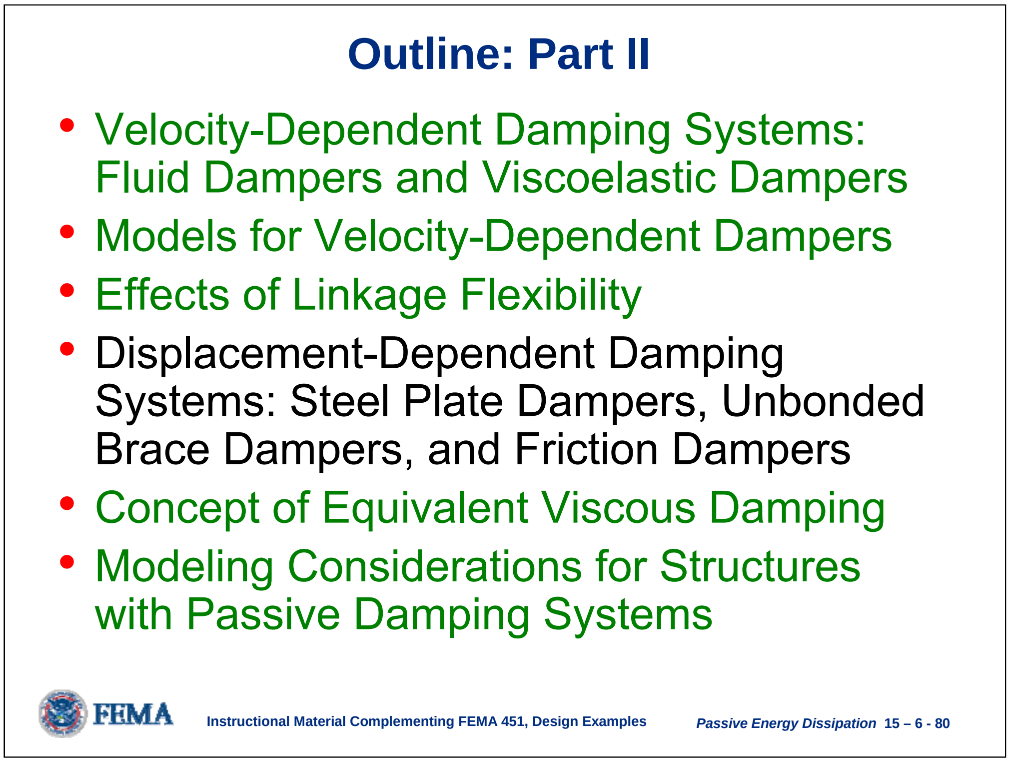
The lateral stiffness of the chevron bracing shown is proportional to the square of the cosine of the angle of inclination. As a result, the lateral stiffness is reduced significantly with respect to the axial stiffness of the braces. For example, if the angle is 60 degrees, the lateral stiffness of the chevron bracing (two braces) is 50% of the axial stiffness of a single brace.

Since the damper-brace assembly acts as a Maxwell model (spring and dashpot in series), the bracing flexibility can have a significant influence on the damper effectiveness. At the two extremes, if the bracing is very stiff, the damper will be essentially 100% effective and, if the bracing has zero stiffness, the damper will be completely ineffective. Also note that, since the damper-brace assembly acts as a Maxwell model, the storage stiffness and damping coefficient of the assembly are frequency-dependent. As the frequency of motion increases, the storage stiffness of the assembly increases while the damping coefficient reduces.



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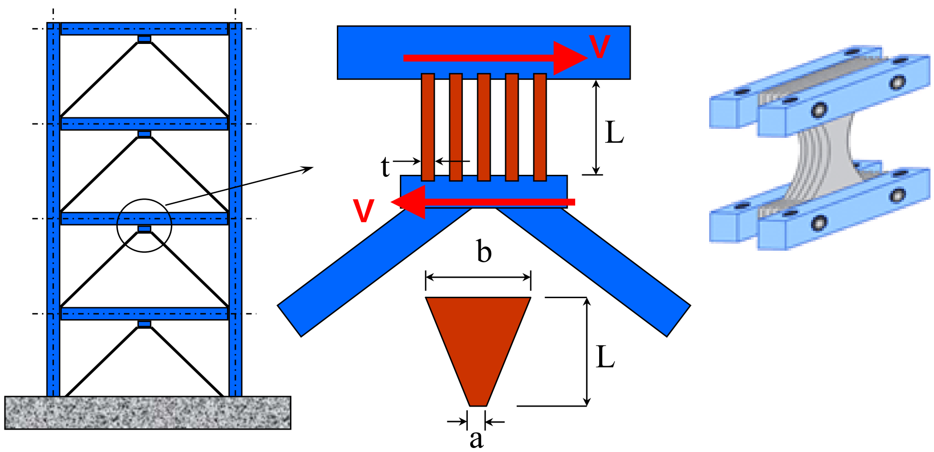
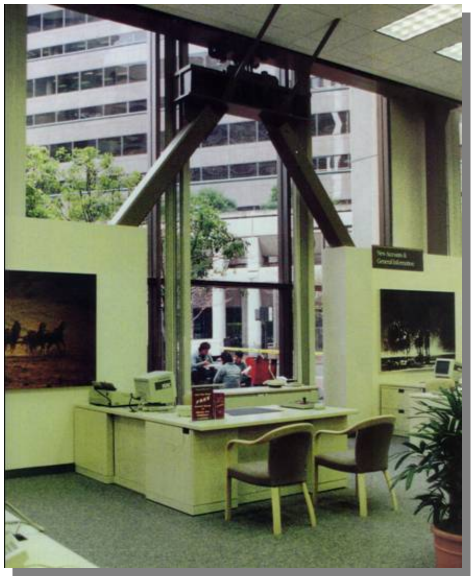
Steel Plate Dampers
(Added Damping and Stiffness System - ADAS)



ADAS dampers consist of a series of steel plates. The bottom of the plates are attached to the top of a chevron bracing arrangement and the top of the plates are attached to the floor level above the bracing. As the floor level above deforms laterally with respect to the chevron bracing, the steel plates are subjected to the shear force V. The shear forces induce bending moments over the height of the plates with bending occurring about the weak axis. The geometrical configuration of the plates is such that the bending moments produce a uniform flexural stress distribution over the height. Thus, inelastic action (energy dissipation) occurs uniformly over the full height of the plates. For example, in the case where the plates are fixed-pinned, the geometry is triangular (see center figure above). In the case where the plates are fixed-fixed, the geometry is an hourglass-shape (see right-most figure above). As the steel plates deform, they provide displacement-dependent stiffness and energy dissipation. The chevron bracing must be very stiff to ensure that the relative deformation of the ADAS device is approximately equal to that of the story. This may result in the chevron bracing being more costly than the ADAS device it supports. Note that the ADAS system remains patented.

Implementation of ADAS System



Wells Fargo Bank,
San Francisco, CA
- Seismic Retrofit of Two-Story Nonductile Concrete Frame; Constructed in 1967
- 7 Dampers Within Chevron Bracing Installed in 1992
- Yield Force Per Damper: 150 kips

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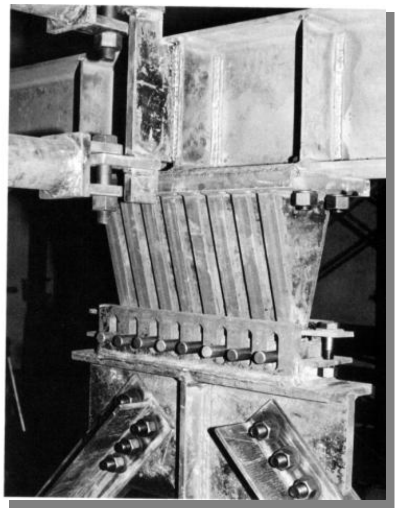
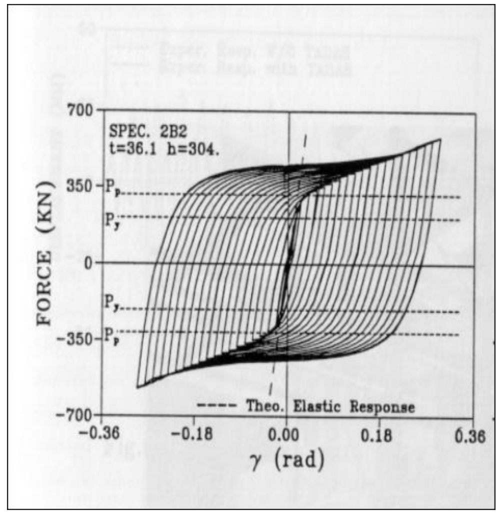
The ADAS system was developed in the 1980's. This is the only structure within the United States to incorporate the ADAS system. Three structures in Mexico City utilize the ADAS system.

Hysteretic Behavior of ADAS Device



ADAS Device
(Tsai et al. 1993)

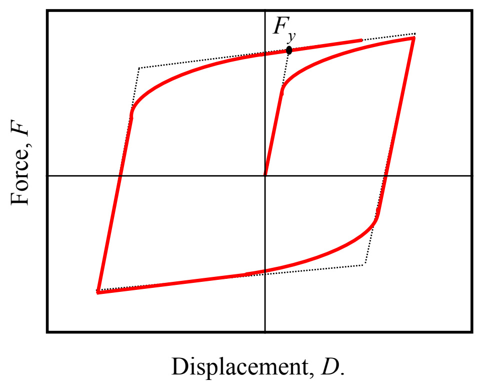


Experimental Response (Static)
(**Source:** Tsai et al. 1993)

The photograph shows a triangular-shaped ADAS device. The plates are fixed on top and pinned on the bottom. The hysteresis loops were obtained in a cyclic test wherein the displacement amplitude was monotonically increased. The horizontal axis represents the ratio of lateral displacement to the height of the plates. Note that the ADAS device exhibits increasing stiffness as the deformations increase. This is due to the effects of finite deformation, which becomes more significant as the deformations increase.

Ideal Hysteretic Behavior of ADAS Damper

(SAP2000 and ETABS Implementation)



Initial Stiffness

Secondary Stiffness Ratio

$$F = \beta k D + (1-\beta) F_y Z$$

$$\dot{Z} = \frac{k}{F_y} \begin{cases} \dot{D}\left(1 - |Z|^{\alpha}\right) & \textbf{\textit{if}}\ \dot{D} \cdot Z > 0 \\ \dot{D} & \textbf{\textit{otherwise}} \end{cases}$$

Yield Sharpness

Z is a Path Dependency Parameter

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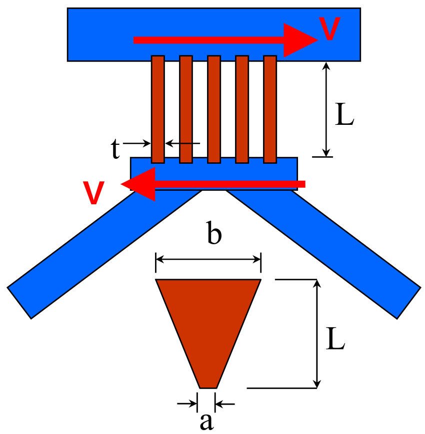
The hysteretic behavior of the ADAS device can be represented by various mathematical models. For example, SAP2000 and ETABS provides the analytical model shown here which was developed by Wen. This model can not capture the increasing stiffness at larger displacements.

Parameters of Mathematical Model of ADAS Damper



$$k = \frac{n(2 + a/b)EI_b}{L^3}$$

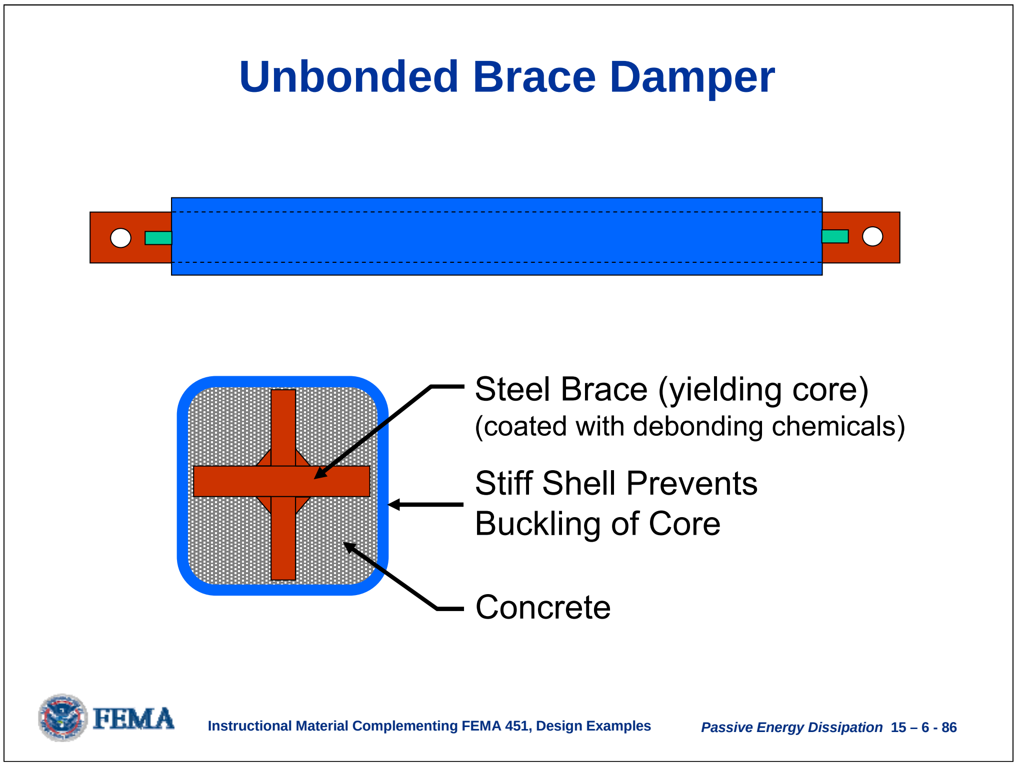
$$F_y = \frac{nf_y bt^3}{4L}$$

$n =$ Number of plates

$f_y =$ Yield force of each plate

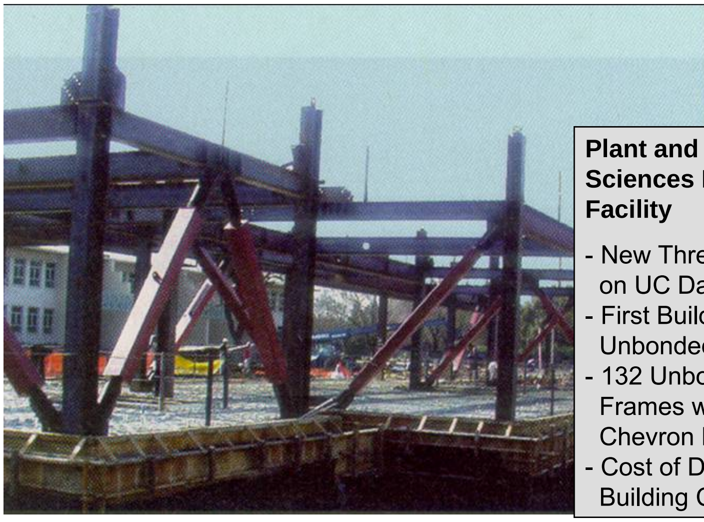
$I_b =$ Second moment of area of each plate at b (i.e, at top of plate)

Expressions for the initial elastic stiffness and yield force of an ADAS damper are provided here.



The Unbonded Brace Damper consists of a steel brace (usually having a low-yield strength) with a cruciform cross-section that is surrounded by a stiff steel tube. The region between the tube and brace is filled with a concrete-like material. A special coating is applied to the brace to prevent it from bonding to the concrete. Thus, the brace can slide with respect to the concrete-filled tube. The confinement provided by the concrete-filled tube allows the brace to be subjected to compressive loads without buckling (i.e., the Unbonded Brace Damper acts as a buckling-restrained brace that can yield in tension or compression). The axial compressive and tensile loads are carried entirely by the steel brace.

Implementation of Unbonded Brace Damper



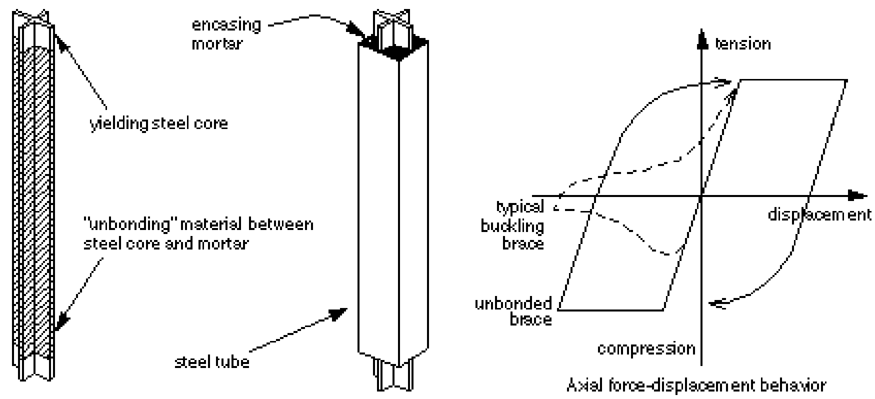
Plant and Environmental Sciences Replacement Facility

- New Three-Story Building on UC Davis Campus
- First Building in USA to Use Unbonded Brace Damper
- 132 Unbonded Braced Frames with Diagonal or Chevron Brace Installation
- Cost of Dampers = 0.5% of Building Cost

Source: ASCE Civil Engineering Magazine, March 2000.

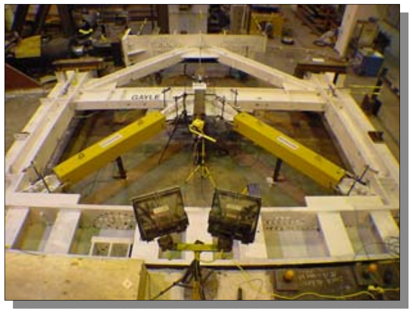
Unbonded Brace Dampers were developed in Japan in the 1980's and were patented by Nippon Steel. The first structure to use unbonded brace dampers in the United States is located on the UC Davis campus. Note that, under lateral load, the chevron bracing arrangement shown results in zero vertical load at the intersection point between the unbonded brace dampers and the beam above (since the dampers can resist both tension and compression). In this regard, the unbonded brace dampers may be regarded as superior to a conventional chevron bracing arrangement where the compression member is expected to buckle elastically, leaving a potentially large unbalanced vertical force component in the tension member that is, in turn, applied to the beam above.

Hysteretic Behavior of Unbonded Brace Damper



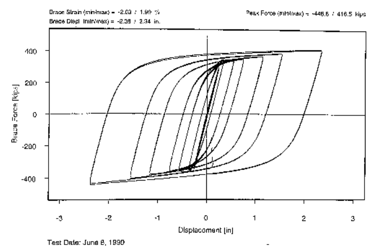
Under compressive loads, the damper behavior is similar to its behavior in tension. Buckling is prevented and thus significant energy dissipation can occur over a cycle of motion.

Testing of Unbonded Brace Damper

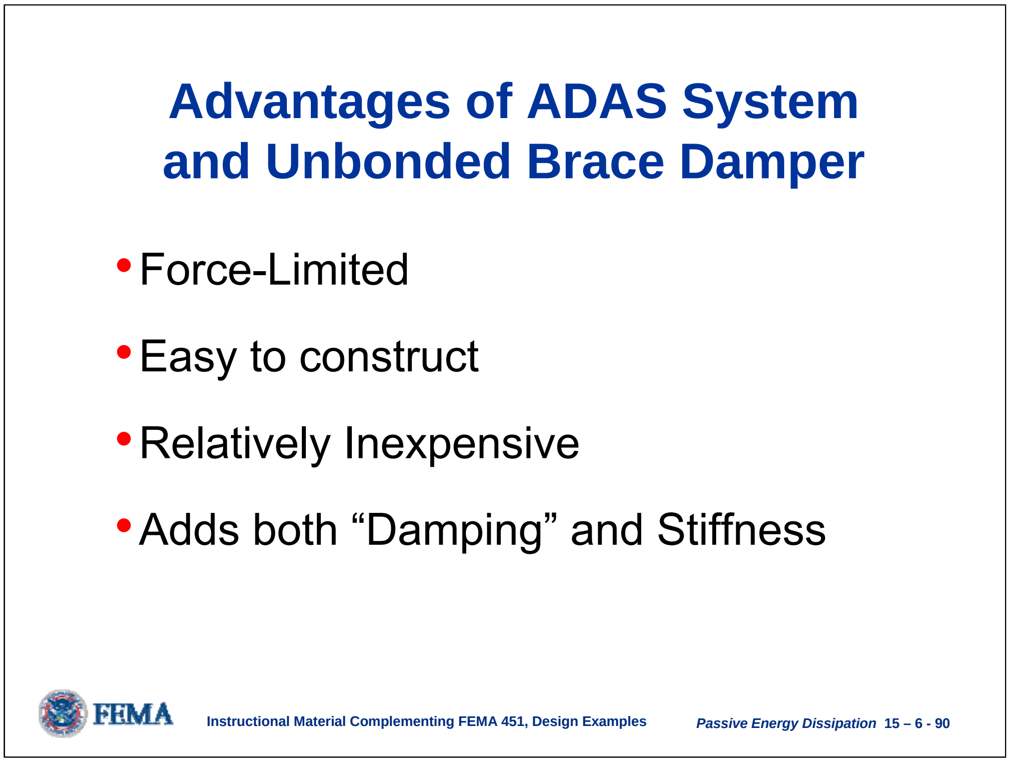


Testing Performed
at UC Berkeley

Typical Hysteresis
Loops from
Cyclic Testing



The hysteresis loops under cyclic loading reveal the ability of the damper to dissipate significant amounts of energy per cycle.



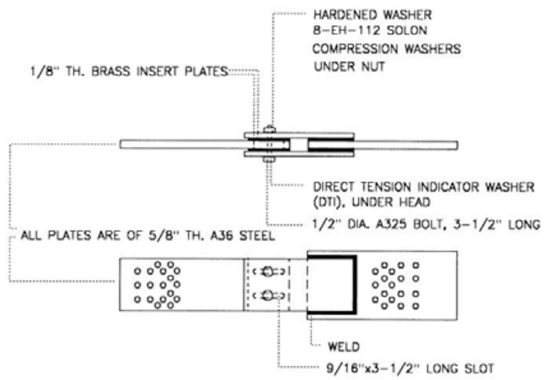
The ADAS System and Unbonded Brace Dampers have a force-limited output due to yielding of the steel plates and brace, respectively. The damping that is added to structures with these devices is due to inelastic energy dissipation.

Disadvantages of ADAS System and Unbonded Brace Damper

- Must be Replaced after Major Earthquake
- Highly Nonlinear Behavior
- Adds Stiffness to System
- Undesirable Residual Deformations Possible

The energy dissipation provided by the ADAS system and Unbonded Brace Dampers is due to inelastic energy dissipation. Thus, the steel plates and braces, respectively, may be severely damaged during a strong earthquake and need to be replaced. The ADAS system and Unbonded Brace Dampers add stiffness to the structure. The additional stiffness can attract larger seismic loads. Damage to the ADAS system and Unbonded Brace Dampers can result in permanent deformations of the structural system.

Friction Dampers: Slotted-Bolted Damper

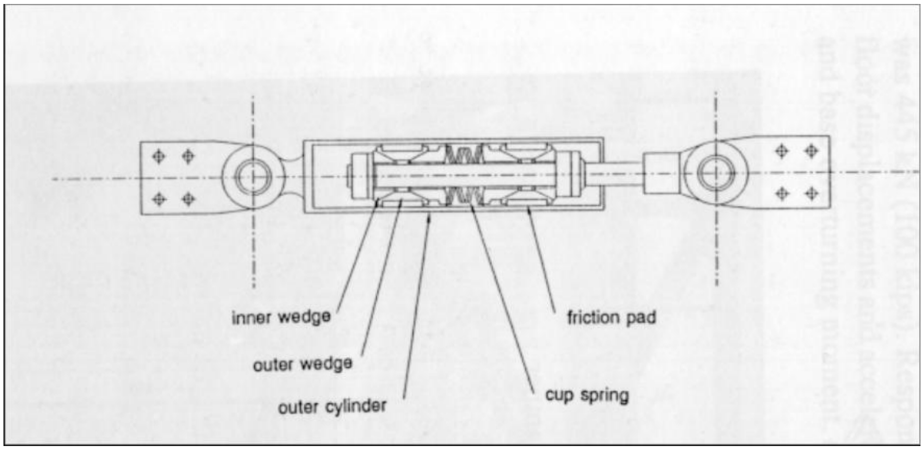


Pall Friction Damper

The slotted-bolted damper consists of steel plates that are bolted together with a specified clamping force. The clamping force is such that slip can occur at a pre-specified friction force. At the sliding interface between the steel plates, special materials are utilized to promote stable coefficients of friction. The damper shown on the right is a Pall Slotted-Bolted Friction Damper in a configuration that is different from the typical Pall Cross-Bracing Friction Damper.

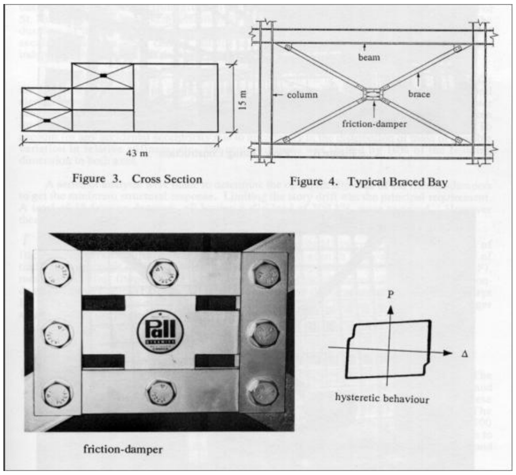
Sumitomo Friction Damper

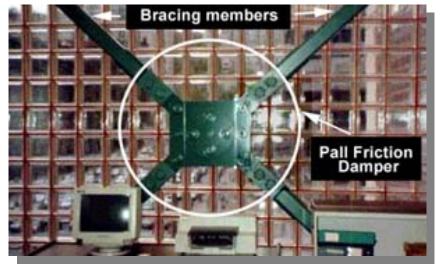
(Sumitomo Metal Industries, Japan)



The Sumitomo Friction Damper was developed by Sumitomo Metal Industries in Japan. The damper dissipates energy via sliding friction between copper friction pads and a steel cylinder. The copper pads are impregnated with graphite to lubricate the sliding surface and ensure a stable coefficient of friction.

Pall Cross-Bracing Friction Damper





Interior of Webster Library at Concordia University, Montreal, Canada

The Pall cross-bracing friction damper consists of cross-bracing that connects in the center to a rectangular damper. The damper is bolted to the cross-bracing. Under lateral load, the structural frame distorts such that two of the braces are subject to tension and the other two to compression. This force system causes the rectangular damper to deform into a parallelogram, dissipating energy at the bolted joints through sliding friction.

Implementation of Pall Friction Damper

McConnel Library at Concordia University, Montreal, Canada

- Two Interconnected Buildings of 6 and 10 Stories
- RC Frames with Flat Slabs
- 143 Cross-Bracing Friction Dampers Installed in 1987
- 60 Dampers Exposed for Aesthetics

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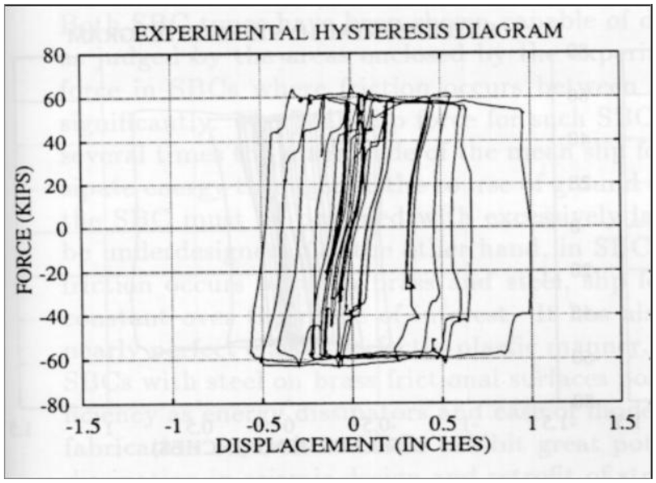
Hysteretic Behavior of Slotted-Bolted Friction Damper



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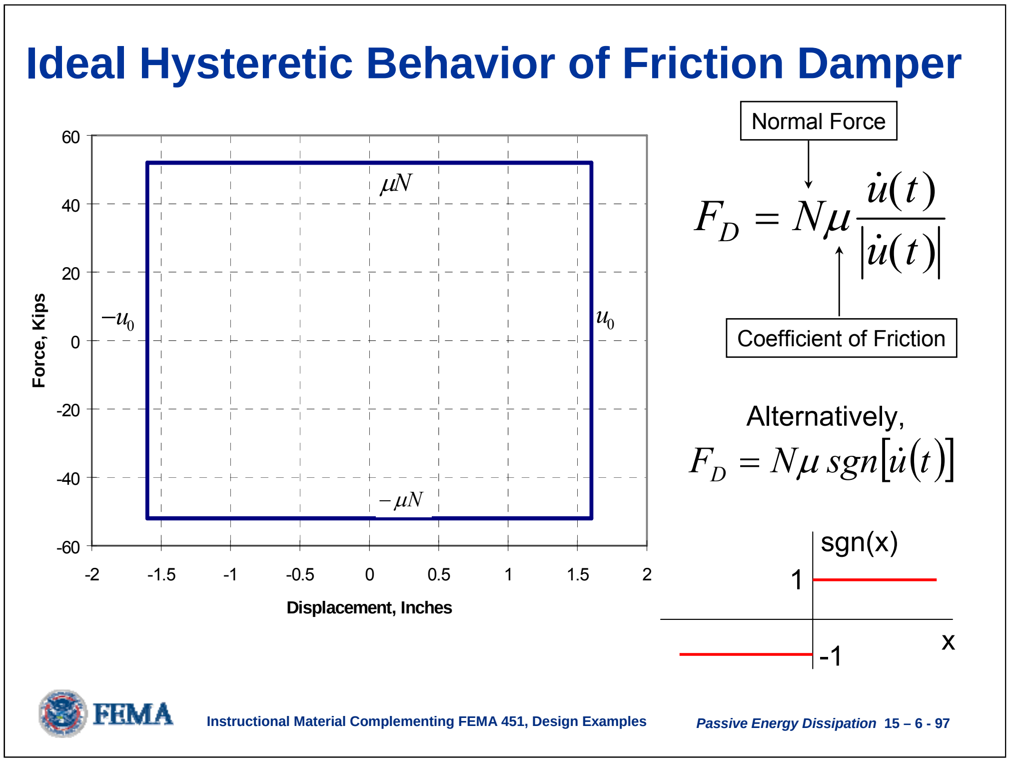
The hysteresis loops for cyclic loading reveal that the friction damper force output is bounded and has the same value for each direction of sliding. The loops are rectangular, indicating that significant energy can be dissipated per cycle of motion.



Assuming the classical Coulomb friction model applies, the damping force has a magnitude equal to the product of the normal force and the coefficient of friction and the direction is opposite to direction of sliding (i.e., opposite to the sign of the velocity). This model leads to a rectangular hysteresis loop. The model assumes that the clamping (or normal) force and the coefficient of friction can be held constant which, of course, is questionable over long durations of time. The direction of the friction force can be expressed in terms of the velocity and its magnitude or in terms of the signum function. Also, note that, according to the Coulomb friction model, a coefficient of friction can be defined for incipient slipping (static coefficient of friction) and for sliding conditions (dynamic coefficient of friction). In the mathematical model shown above, the coefficient of friction is assumed to be constant and equal to the dynamic coefficient of friction.

Advantages of Friction Dampers

- Force-Limited
- Easy to construct
- Relatively Inexpensive

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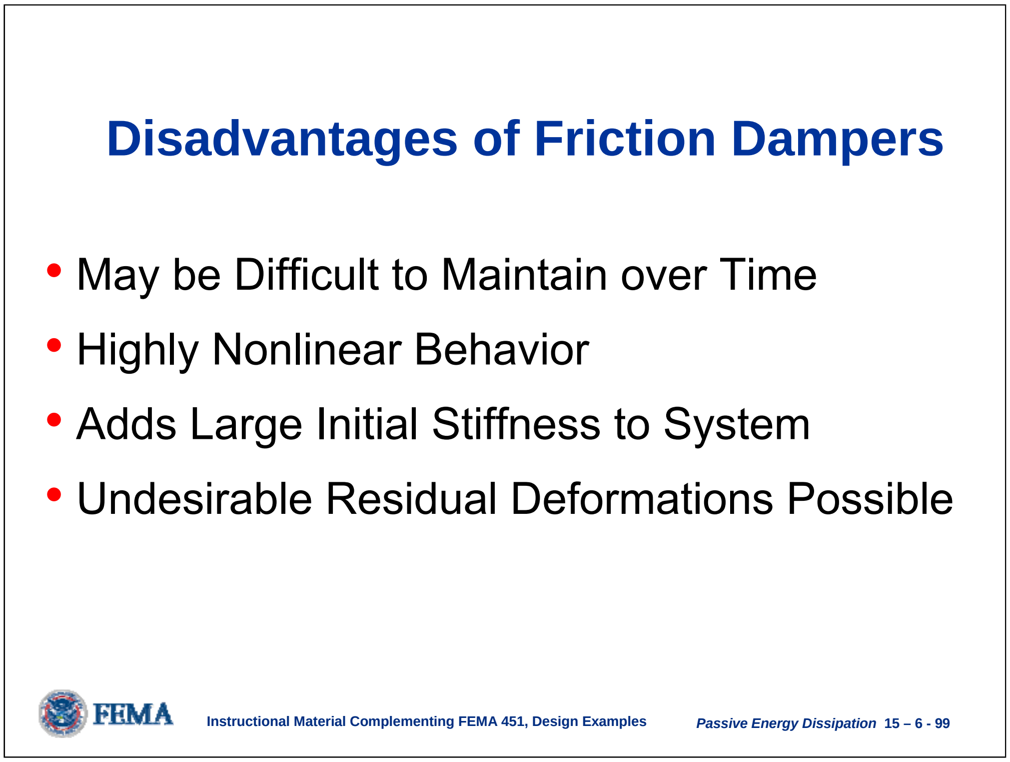

The damper friction force will tend to change over time, thus requiring periodic maintenance. The rectangular shape of the hysteresis loop indicates that the cyclic behavior of friction dampers is strongly nonlinear. The nonlinear behavior is also evident in the signum function that can be used for the mathematical modeling. The deformations of the structural framing are largely restricted until the friction force is overcome; thus, the dampers add large initial stiffness to the structural system. Since frictions dampers have no restoring force mechanism, permanent deformation of the structure may exist after an earthquake.

Outline: Part II

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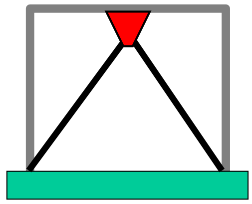
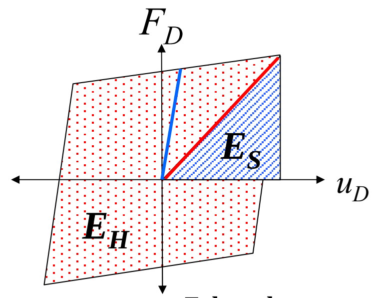
In addition to what is presented here, more will be said about this subject during the sessions on Advanced Analysis.

Equivalent Viscous Damping:
Damping System with Inelastic or Friction Behavior





E_s **based on secant stiffness**

$$\xi_H = \frac{E_H}{4\pi E_S}$$

Note: Computed damping ratio is *displacement-dependent*

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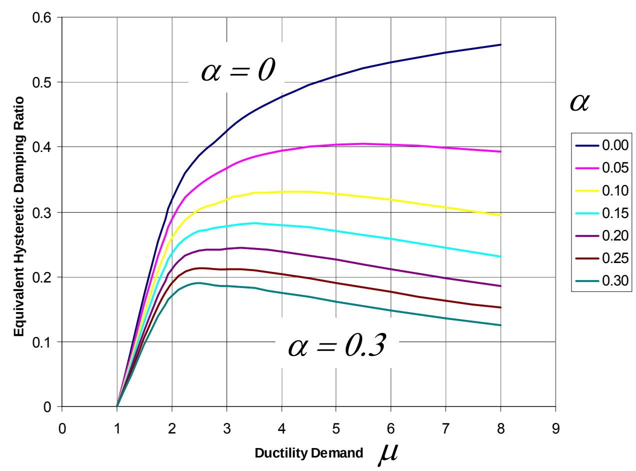
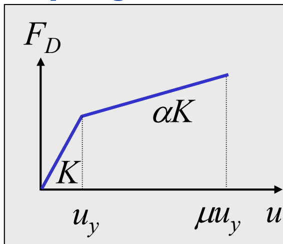
For a SDOF system with a damping system that exhibits inelastic or friction behavior, an equivalent viscous damping ratio associated with the damping system can be computed. The damping ratio is a measure of the ability of the damping system to dissipate energy and is displacement-dependent since the hysteretic behavior is displacement-dependent.

Effect of Inelastic System Post-Yielding Stiffness on Equivalent Viscous Damping





$$\xi_H = \frac{2(\mu - 1)(1 - \alpha)}{\pi\mu(1 + \mu\alpha - \alpha)}$$

Note: May be Modified **(κ)** for Other (less Robust) Hysteretic Behavior

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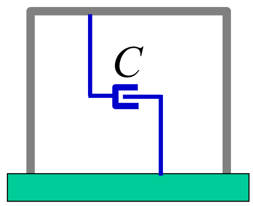
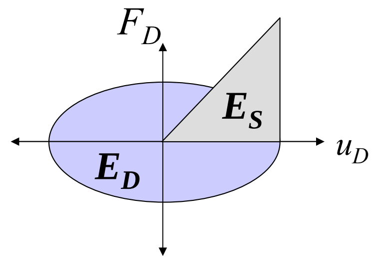
The equivalent viscous damping ratio is dependent on the displacement ductility and thus, as noted previously, the equivalent viscous damping ratio is displacement-dependent. For a given level of ductility, the equivalent damping ratio is largest for elasto-plastic systems (alpha = 0) since such systems dissipate large amounts of energy per cycle at a given level of deformation. Furthermore, for a given level of ductility, the peak elastic strain energy in elasto-plastic systems is less than for systems with strain-hardening. Surprisingly, as the ductility increases, the equivalent damping ratio for higher values of alpha can decrease. This is counterintuitive and is due to the damping ratio being simultaneously dependent on the energy dissipated per cycle and the peak elastic strain energy. Both of these quantities increase at higher ductility levels, with the peak elastic strain energy increasing faster than the energy dissipated, resulting in the aforementioned phenomenon. The reduction in equivalent damping with larger ductility ratios and strain hardening ratios is not likely a problem since the ductility demand reduces with an increase in strain hardening ratio. For systems with high strain hardening ratios, it is unlikely that the ductility demand will be high enough to result in any significant reduction in equivalent damping.

Equivalent Viscous Damping:
"Equivalent" System with Linear Viscous Damper





$$\boxed{C = 2m\omega\xi_H}$$

E_s and ω are based on Secant Stiffness of Inelastic System

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The equivalent viscous damping coefficient is given by the standard expression relating the damping coefficient to the damping ratio. This expression is derived for systems with linear viscous damping and thus does not strictly apply for systems that behave inelastically. The peak elastic strain energy and the circular frequency are based on the secant stiffness of the inelastic system. Specifically, the circular frequency is computed as the square root of the ratio of the secant stiffness to the mass of the structure.

Equivalent Viscous Damping: Caution!

- It is not possible, on a device level, to “replace” a displacement-dependent device (e.g. a Friction Damper) with a velocity-dependent device (e.g. a Fluid Damper).
- Some simplified procedures allow such replacement on a structural level, wherein a “smeared” equivalent viscous damping ratio is found for the whole structure. This approach is marginally useful for preliminary design, and should not be used under any circumstances for final design.

No annotation is provided for this slide.

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Modeling Considerations for Structures with Passive Energy Dissipation Devices

- Damping is almost always nonclassical (Damping matrix is not proportional to stiffness and/or mass)
- For seismic applications, system response is usually partially inelastic
- For seismic applications, viscous damper behavior is typically nonlinear (velocity exponents in the range of 0.5 to 0.8)

Conclusion: This is a ***NONLINEAR*** analysis problem!

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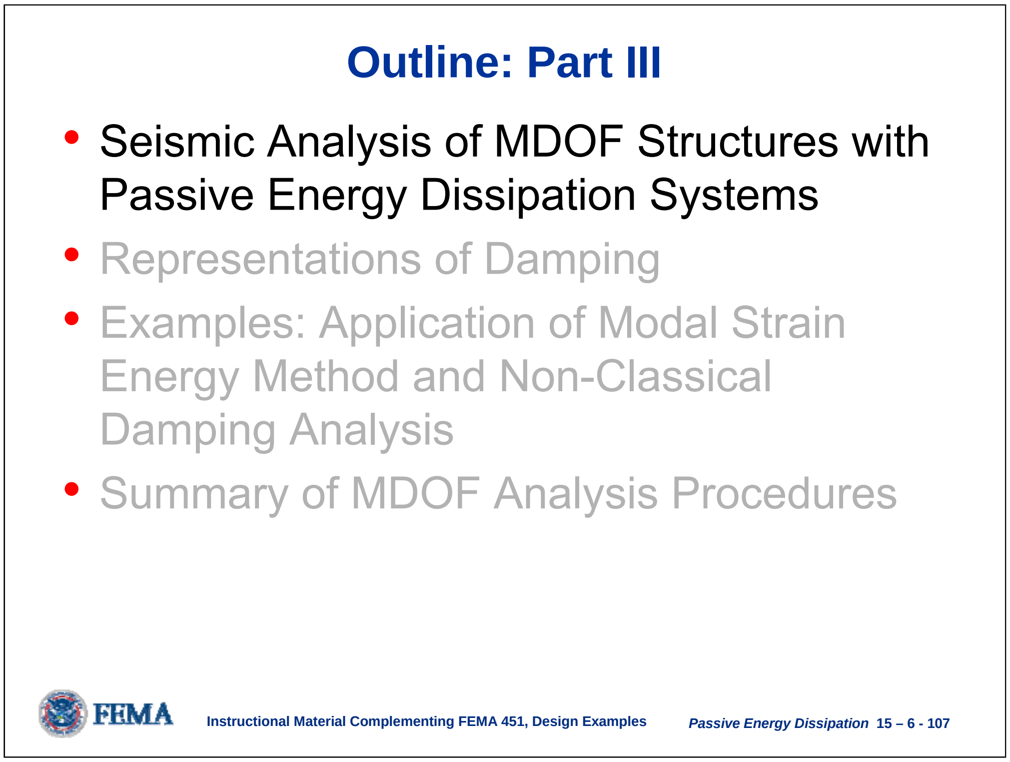

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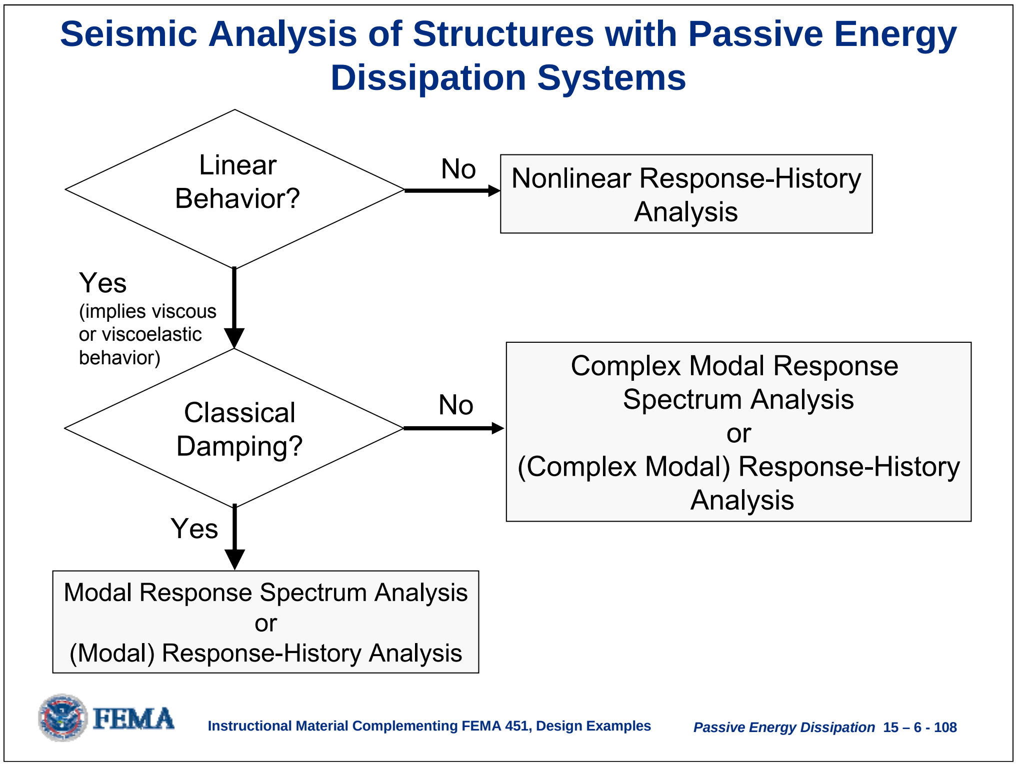

For response-history analysis, the equations of motion can be solved directly as a coupled system of equations. Alternatively, the equations can be uncoupled into modal equations through either a real or complex modal transformation. Typically, only a limited number of modes needs to be considered for a reasonably accurate solution (e.g., enough such that the effective modal mass is greater than or equal to 90% of the total mass).

Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t)$$

Inherent Damping:
Linear

Added Viscous Damping:
Linear or ***Nonlinear***

Restoring Force:
(May include Added Devices)
Linear or ***Nonlinear***

$$\boxed{C_A \neq f(\omega)}$$

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The equation of motion of a MDOF structure with passive dampers and subjected to earthquake ground motion is given above. The inherent damping in the structure is modeled as linear viscous and the restoring forces may be linear or nonlinear. The equations represent N second-order, linear or nonlinear, coupled, ordinary differential equations where N is the number of degrees of freedom. Note that, in this slide and those that follow, it is assumed that the damping coefficients in the added viscous damping matrix are independent of frequency. In cases where the added viscous damping is frequency-dependent, one approach to obtain constant damping coefficient values is to use values corresponding to the fundamental frequency of the undamped structure.

MDOF Solution Techniques

$$M\ddot{v}(t) + C_I\dot{v}(t) + C_A\dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t)$$

Explicit integration of fully coupled equations:

- Treat C_I as Rayleigh damping and model C_A explicitly.
- Use Newmark solver (with iteration) to solve full set of coupled equations.

System may be linear or nonlinear.

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There are a variety of methods for solving the equations of motion of structures with passive dampers. All methods can be categorized as either frequency-domain or time-domain methods. Frequency domain methods are applicable only to linear systems. Thus, for generality, we focus on time-domain methods that are applicable to both linear and nonlinear systems. One method is to perform a step-by-step analysis in which the fully coupled system of N equations is explicitly integrated (e.g., using a Newmark solver). In this case, the inherent damping matrix can be represented by a Rayleigh formulation and the added viscous damping matrix can be defined explicitly based on the position of the dampers within the structure. Note that, while iteration is generally desirable, it may not be necessary if a short enough integration time step (or automatic time-stepping) is used.

MDOF Solution Techniques

$$M\ddot{v}(t)+C_I\dot{v}(t)+C_A\dot{v}(t)+F_S(t)=-MR\ddot{v}_g(t)$$

Fast Nonlinear Analysis:
Treat C_I as modal damping and model C_A explicitly. Move C_A (and any other nonlinear terms) to right-hand side. Left-hand side may be uncoupled by Ritz Vectors. Iterate on unbalanced right-hand side forces.

System may be linear or nonlinear.

Another approach to solving the system of equations is to utilize the "Fast Nonlinear Analysis" method developed by Wilson and used in SAP2000 (Wilson 1993). First, the added damping force vector and the restoring force vector (if it is nonlinear) are moved to the right-hand side. The physical coordinates are transformed to a new set of coordinates through a transformation that employs stiffness and mass orthogonal load-dependent Ritz vectors. The left-hand side of the resulting equation will be uncoupled. In the transformation process, the inherent damping is represented by modal damping ratios and the forces associated with the discrete added viscous dampers are included in the right-hand side nonlinear force vector. Finally, iteration is performed on the unbalanced right-hand side forces.

Fast Nonlinear Analysis

$$M\ddot{v}(t)+C_I\dot{v}(t)+C_A\dot{v}(t)+K_E v(t)+F_H(t)=-MR\ddot{v}_g(t)$$

Nonlinear Restoring Force

Move Added Damper Forces and Nonlinear Forces to RHS:

$$\underbrace{M\ddot{v}(t)+C_I\dot{v}(t)+K_E v(t)}_{\text{Linear Terms}}=-MR\ddot{v}_g(t)\underbrace{-F_H(t)-C_A\dot{v}(t)}_{\text{Nonlinear Terms}}$$

Transform Coordinates: $v(t)=\Phi y(t)$

Orthogonal basis of Ritz vectors: Number of vectors << N

Apply Transformation:

$$\underbrace{\tilde{M}\ddot{y}(t)+\tilde{C}_I\dot{y}(t)+\tilde{K}_E y(t)}_{\text{Uncoupled}}=\underbrace{-\Phi^T MR\ddot{v}_g(t)-\Phi^T F_H(t)-\tilde{C}_A\dot{y}(t)}_{\text{Coupled}}$$

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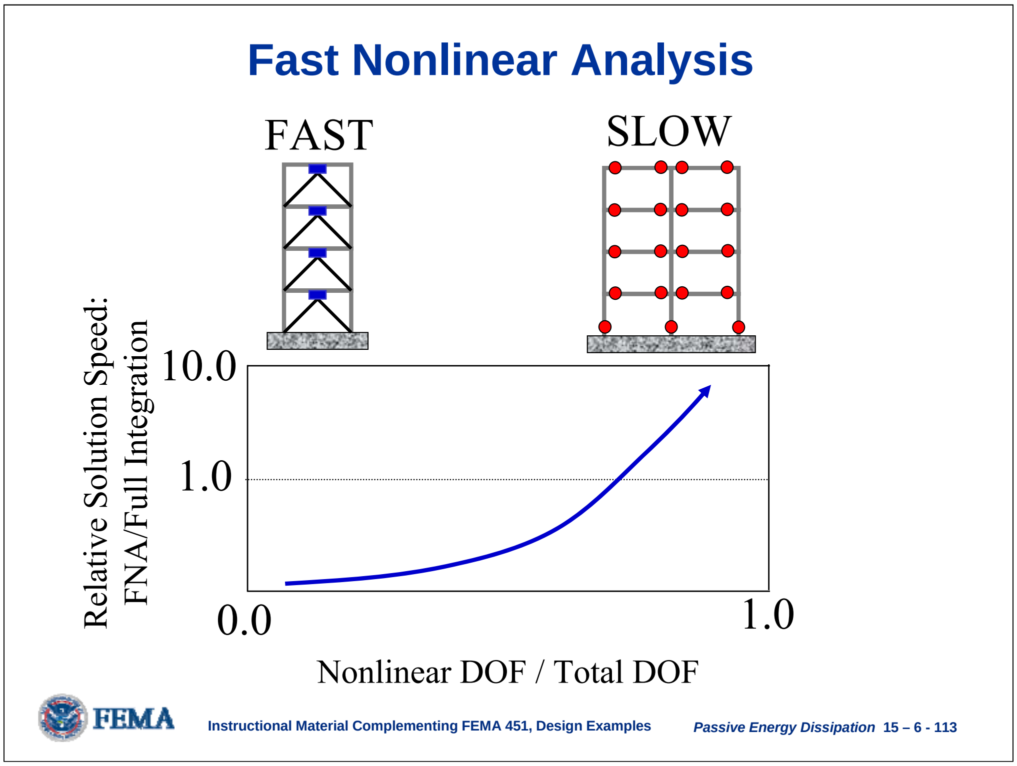
The basic steps for the Fast Nonlinear Analysis method are illustrated in this slide.



The Fast Nonlinear Analysis method is most efficient when the number of DOF associated with nonlinear behavior is small.

MDOF Solution Techniques

$$M\ddot{v}(t)+C_I\dot{v}(t)+C_A\dot{v}(t)+F_S(t)=-MR\ddot{v}_g(t)$$

Explicit integration or response spectrum analysis of first few uncoupled modal equations:

- Treat C_I as modal damping or Rayleigh damping
- Use Modal Strain Energy method to represent C_A as modal damping ratios.

System must be linear.
Applicable only to viscous (or viscoelastic) damping systems.

For a linear structural system with viscous (or viscoelastic) damping, the equations of motion can be transformed to the modal domain via a modal transformation. In some cases, this leads to a set of N uncoupled equations. Often, only the lowest modes of vibration contribute significantly to the response and thus only a few of the uncoupled modal equations need to be solved. For an uncoupled modal analysis, the inherent damping can be represented by modal damping ratios or a Rayleigh damping formulation can be used. The modal strain energy method can be employed to represent the added damping in terms of modal damping ratios.

Note: The viscoelastic case is noted parenthetically since, if the storage stiffness and damping coefficient are assumed to have constant values, viscoelastic damping can be considered to be equivalent to a "linear structural system with viscous damping."

Outline: Part III

- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
- Representations of Damping
- Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
- Summary of MDOF Analysis Procedures

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Modal Damping Ratios

$$M\ddot{v} + C\dot{v} + Kv = -MR\ddot{v}_g$$

$$v = \Phi y$$

$$\ddot{y}_i + 2\xi_i \omega_i \dot{y}_i + \omega_i^{\,2} y_i = \Gamma_i \ddot{v}_g$$

Specify modal damping values directly

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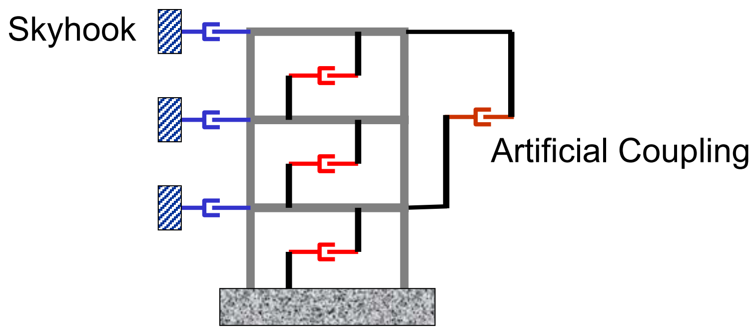
The equation of motion shown above is for a linear structural system with viscous (or viscoelastic) damping. Assuming the damping matrix is of classical form, the coupled equations can be uncoupled via a modal transformation that employs the mode shape matrix. The uncoupled equation for the i-th mode is shown and is expressed in terms of the i-th modal frequency and damping ratio. Rather than constructing a classical damping matrix and computing the modal damping ratios, the modal damping ratios can be directly specified for each mode. Note that, even if the damping matrix is not of a classical form, a modal transformation may be useful since only a limited number of modes may contribute significantly to the response (i.e., only a subset of the full set of N coupled modal equations would need to be solved).

Modal Superposition Damping

$$C = M\left[\sum_{i=1}^{n} \frac{2\xi_i\omega_i}{\phi_i^T M \phi_i}\ \phi_i^T \phi_i\right]M$$

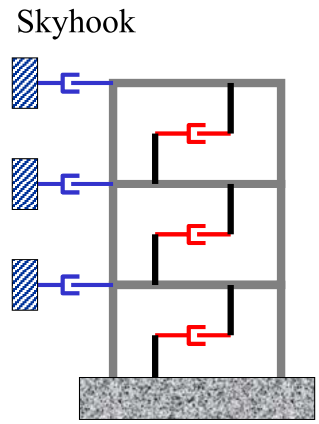


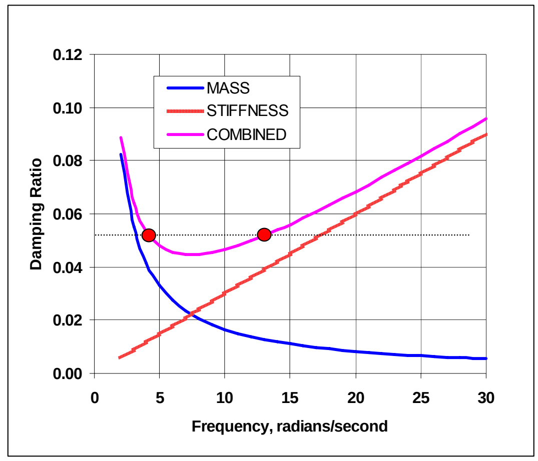
Note: There is no need to develop C explicitly.

A classical damping matrix can be constructed via a superposition of modal damping matrices. In this formulation, the damping ratios are specified for each mode. Since the damping ratios are the parameters that are needed to solve the uncoupled modal equations, the damping matrix actually does not need to be constructed. However, the damping matrix that is associated with the selected modal damping ratios may include, in addition to interstory damping, skyhook damping and dampers that produce artificial coupling of the response of two nonadjacent floor levels.

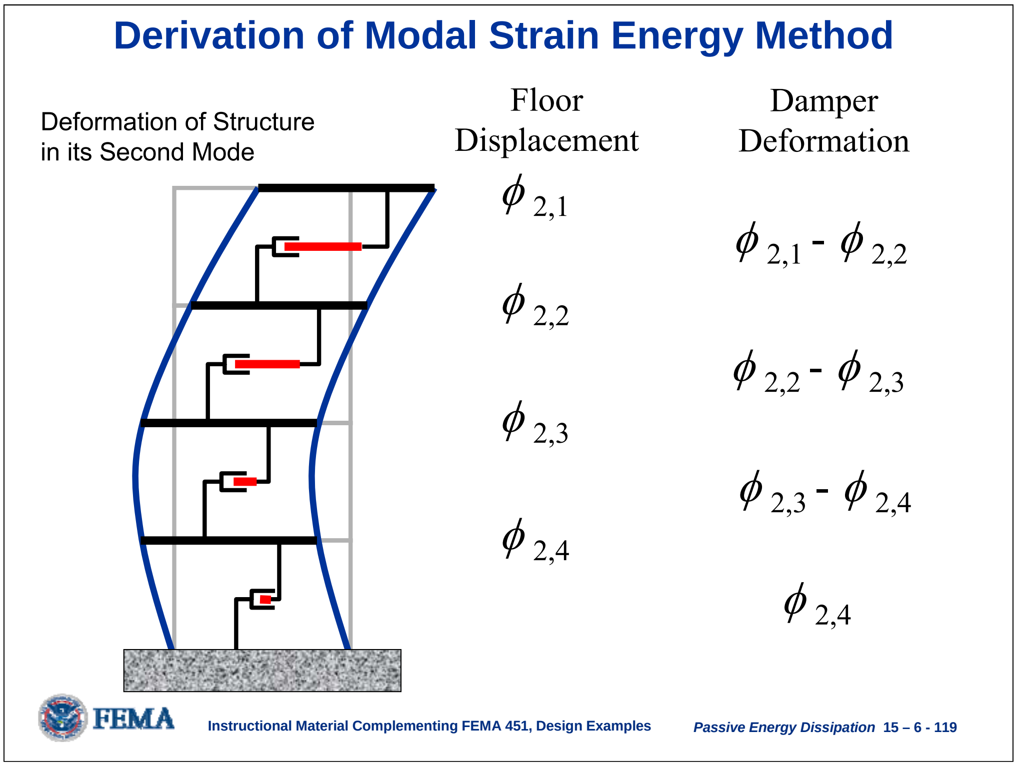
Rayleigh Proportional Damping

$$C_R = \alpha M + \beta K$$



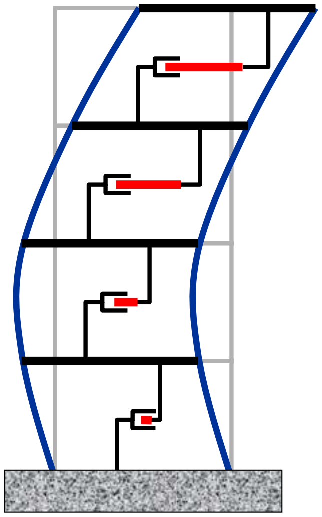


A classical damping matrix can also be constructed as a mass proportional matrix, stiffness proportional matrix, or a combination of the two. If only the mass proportional term is considered, the modal damping ratio can only be specified in one mode (usually the first mode is used) and the damping ratios decrease monotonically with frequency. In this case the higher mode contributions to the response may be overemphasized. The physical interpretation of mass proportional damping is that it corresponds to skyhook dampers at each mass level. If only the stiffness proportional term is considered, the modal damping ratio can only be specified in one mode (usually the first mode is used) and the damping ratios increase monotonically with frequency. In this case the higher mode contributions to the response may be suppressed. The physical interpretation of stiffness proportional damping is that it corresponds to interstory dampers. If both the mass and stiffness proportional terms are considered (i.e., Rayleigh damping), the modal damping ratios can be specified in two modes (as indicated above) and the variation in damping ratios is non-monotonic with frequency. The physical interpretation of Rayleigh damping is that it corresponds to both skyhook and interstory dampers. Note that Rayleigh damping formulations are commonly available in structural analysis software and thus it can be a very convenient, although not necessarily accurate, approach to accounting for added damping.



The *Modal Strain Energy Method* requires the mode shape for each mode in which the modal damping ratio is to be determined. The mode shapes are for the original undamped structure. The mode shape of the second mode is shown above. Assuming a damper is located in each story and the damper bracing assembly is perfectly rigid, the relative deformation of each story corresponds to the relative deformation of each damper. If the dampers add stiffness to the structure, the stiffness should be included in the stiffness matrix for computation of undamped natural frequencies and mode shapes.

Derivation of Modal Strain Energy Method



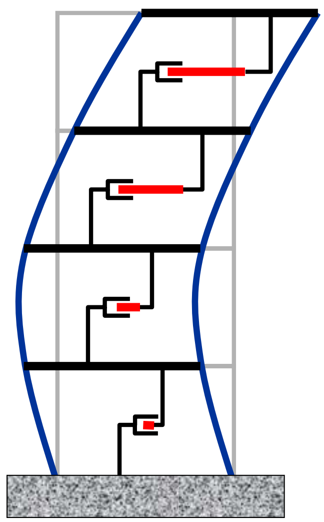
$$E_{D,i,story\,k} = \pi \omega_i C_k (\phi_{i,k} - \phi_{i,k-1})^2$$

$$E_{S,i} = \frac{1}{2}\phi_i^T K \phi_i = \frac{1}{2}\omega_i^2 \phi_i^T M \phi_i$$

$$\xi_i = \frac{\sum_{k=1}^{n\ stories} E_{D,i,story\,k}}{4\pi E_{S,i}}$$

For one cycle of vibration in the i-th mode, the energy dissipated by the k-th viscous damper is equal to the area contained within the hysteresis loop (ellipse) of that damper in that mode. The peak elastic strain energy in the i-th mode can be expressed either in terms of the stiffness or mass matrix. The damping ratio in the i-th mode due to the viscous dampers is equal to the total energy dissipated by all the dampers in the i-th mode divided by the product of 4, pi, and the peak elastic strain energy in the i-th mode.

Derivation of Modal Strain Energy Method



$$\xi_i = \frac{\sum_{k=1}^{N\ stories} C_k(\phi_{i,k} - \phi_{i,k-1})^2}{2\omega_i \phi_i^T M \phi_i}$$

$$\xi_i = \frac{\phi_i^T C_A \phi_i}{2\omega_i \phi_i^T M \phi_i} = \frac{\phi_i^T C_A \phi_i}{2m_i^* \omega_i}$$

Note: IF C_A is diagonalized by Φ, THEN

$$\xi_i = \frac{c_i^*}{2m_i^* \omega_i}$$

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The summation in the first equation can be re-written as the matrix product in the second equation. Note that the final result provides the i-th modal damping ratio from the viscous dampers. The damping ratio associated with inherent damping is not included.

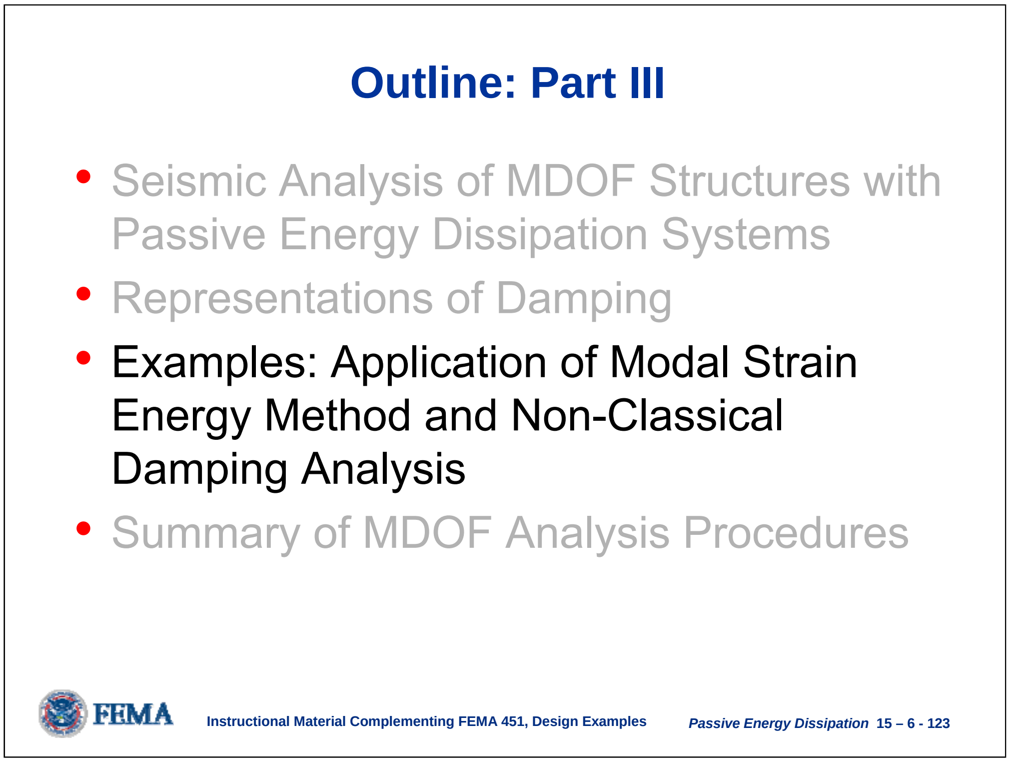
Modal Strain Energy Damping Ratio

$$\xi_i = \frac{\phi_i^T C_A \phi_i}{2 m_i^* \omega_i}$$

Note: ϕ is the *Undamped* Mode Shape

The Modal Strain Energy Method is approximate if the structure is non-classically damped since the undamped and damped mode shapes are different.

The modal strain energy method utilizes the undamped mode shapes. For structures with classical damping distributions, the damped mode shapes are identical to the undamped mode shapes and thus the modal strain energy method is applicable. For structures with non-classical damping distributions, the damped mode shapes are different from the undamped mode shapes and thus the modal strain energy method provides only an approximation to the modal damping ratios.



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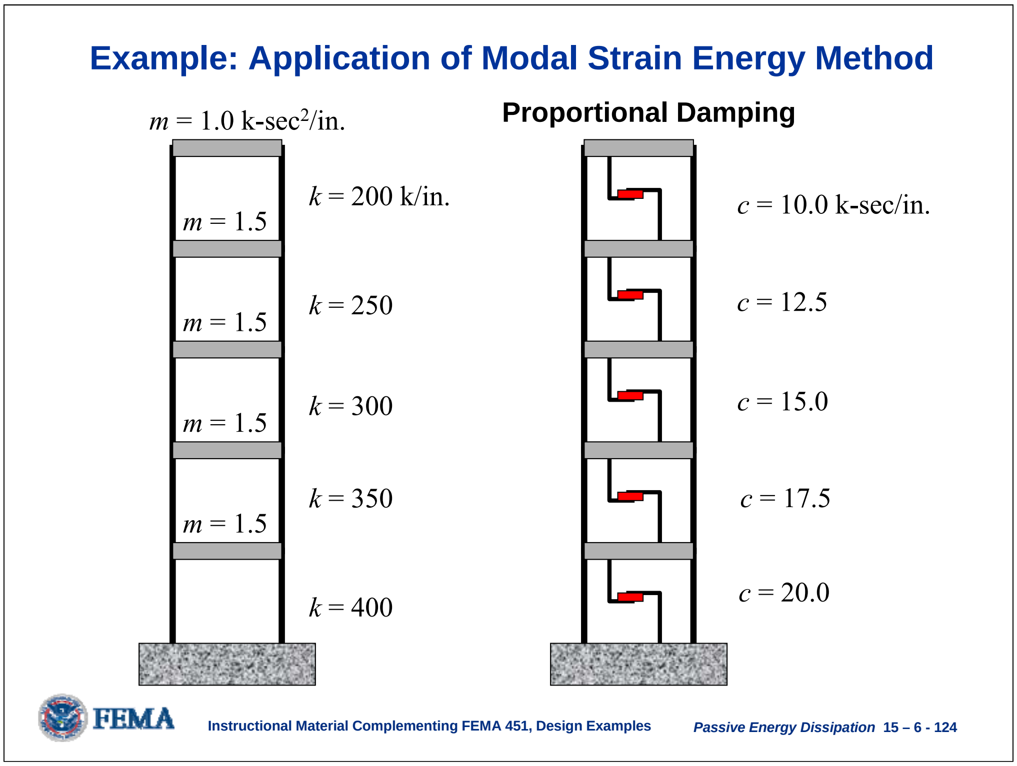
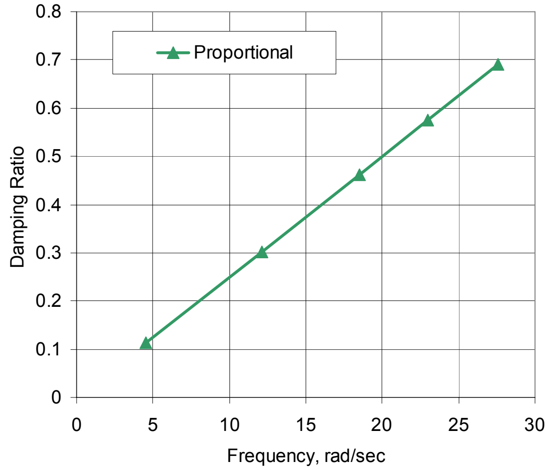


The structure shown on the left is the original undamped structure. The structure on the right has the same mass and stiffness properties as the structure on the left but includes added viscous dampers at each story. The dampers are distributed such that the damping matrix is proportional to the stiffness matrix. Thus, the damping matrix is classical and the damped mode shapes are equal to the undamped mode shapes. Therefore, the Modal Strain Energy Method is applicable. Note that inherent damping is neglected in this example.

Modal Damping Ratios from Modal Strain Energy Method for *Proportional* Damping Distribution

Frequency (rad/sec)	Damping Ratio, ξ
4.54	0.113
12.1	0.302
18.5	0.462
23.0	0.575
27.6	0.690



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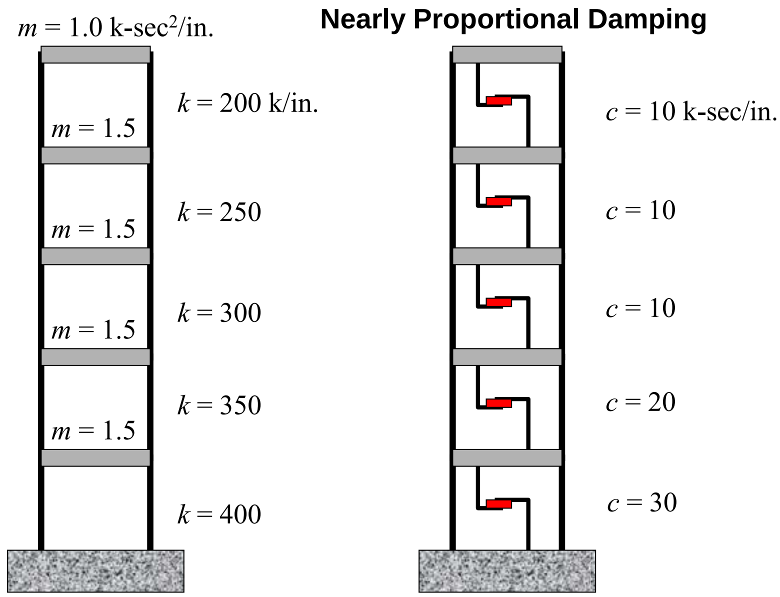
The modal damping ratios shown above were computed using the modal strain energy method. The damping ratios increase linearly with frequency due to the stiffness proportional damping distribution. The fundamental mode and highest mode damping ratios are 11.3% and 69.0%, respectively. The damping ratios are identical to those that would be obtained from calculations using the fundamental equations associated with stiffness proportional damping matrices.

Example: Application of Modal Strain Energy Method



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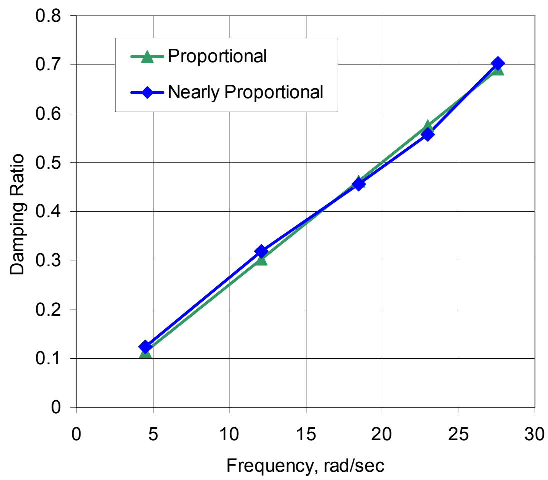
The structure shown on the left is the original undamped structure. The structure on the right includes added viscous dampers at each story. The dampers are distributed in manner that is nearly proportional to the stiffness distribution. Thus, the damped mode shapes will be similar to the undamped mode shapes and the Modal Strain Energy Method, although not strictly applicable, will give reasonable results.

Modal Damping Ratios from Modal Strain Energy Method for *Nearly Proportional* Damping Distribution

Frequency (rad/sec)	Damping Ratio, ξ
4.54	0.123
12.1	0.318
18.5	0.455
23.0	0.557
27.6	0.702



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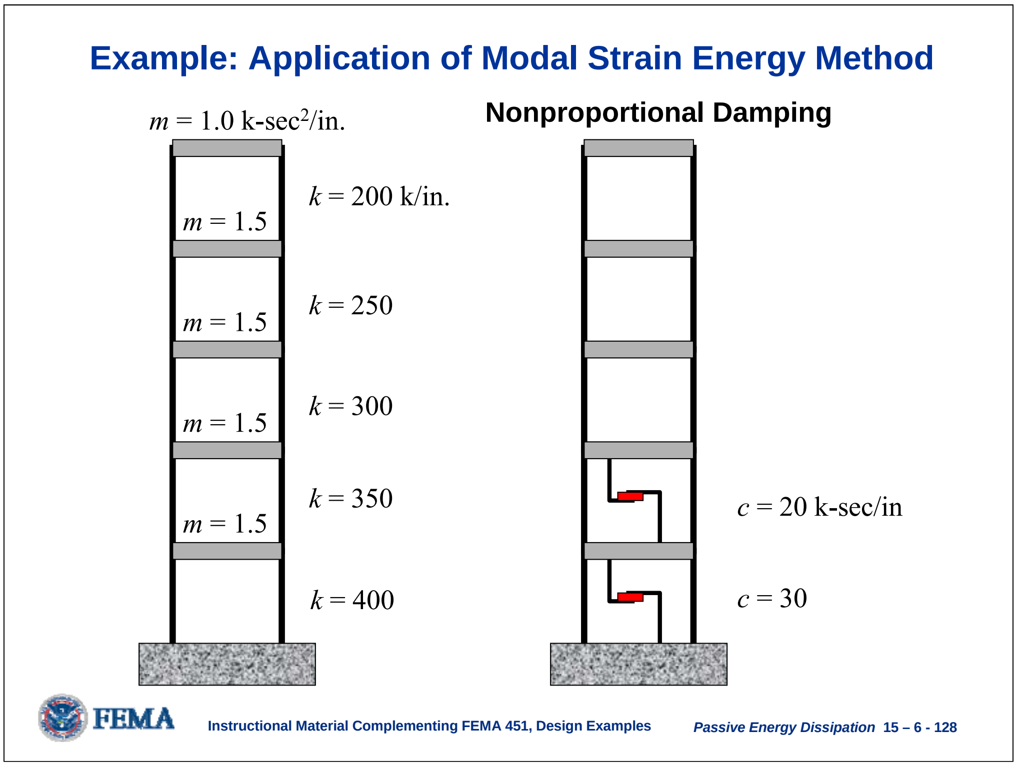
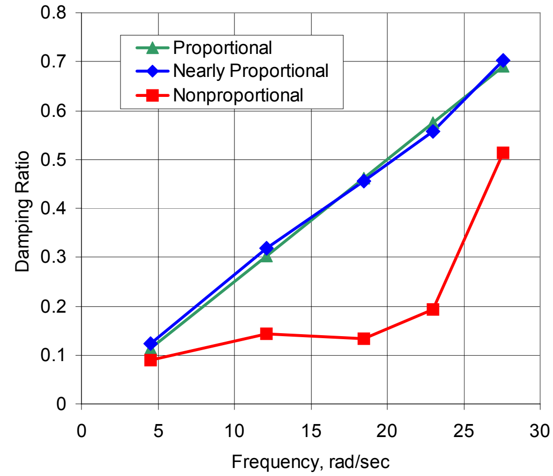
The modal damping ratios shown above were computed using the modal strain energy method. The damping ratios increase approximately linearly with frequency due to the nearly stiffness proportional damping distribution. The fundamental mode and highest mode damping ratios are estimated to be 12.3% and 70.2%, respectively, which are close to the values for the proportionally damped system.



The structure shown on the left is the original undamped structure. The structure on the right includes added viscous dampers at the first two stories only. The dampers are distributed in a strongly nonproportional manner. Since the damping distribution is non-classical, the damped mode shapes are not equal to the undamped mode shapes and the Modal Strain Energy Method is not applicable.

Modal Damping Ratios from Modal Strain Energy Method for *Nonproportional* Damping Distribution

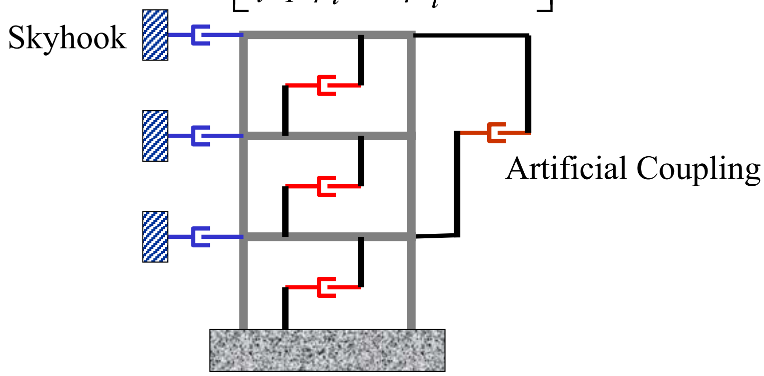
Frequency (rad/sec)	Damping Ratio. ξ
4.54	0.089
12.1	0.144
18.5	0.134
23.0	0.194
27.6	0.514



The modal damping ratios shown above were computed using the modal strain energy method. The modal damping ratios for the strongly nonproportionally damped system are generally very different from those values for the proportional or nearly proportional systems. Of course, the Modal Strain Energy Method is not applicable for non-classical systems and thus the modal damping ratios given above for the nearly proportional and nonproportional cases are incorrect. The amount of error in the modal damping ratios is illustrated in the damping matrices to be presented next. To obtain the correct modal damping ratios, the damped eigenvalue problem must be solved. This will be done soon.

Modal Superposition Damping

$$C = M\left[\sum_{i=1}^{n} \frac{2\xi_i \omega_i}{\phi_i^T M \phi_i} \ \phi_i^T \phi_i \right] M$$

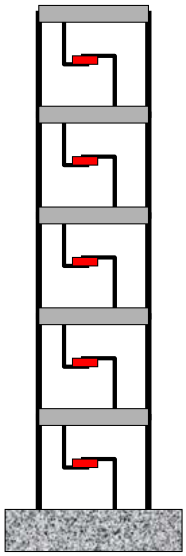


Modal Superposition Damping can be used to construct the damping matrix from the modal damping ratios obtained via the Modal Strain Energy Method

The error in the modal damping ratios obtained from the Modal Strain Energy Method can be evaluated by constructing the associated damping matrix using the Modal Superposition Damping formulation and comparing with the actual damping matrix.

Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Proportional Damping



Actual Damping Matrix

$$C_A = \begin{bmatrix} 10.0 & -10.0 & 0 & 0 & 0 \\ -10.0 & 22.5 & -12.5 & 0 & 0 \\ 0 & -12.5 & 27.5 & -15.0 & 0 \\ 0 & 0 & -15.0 & 32.5 & -17.5 \\ 0 & 0 & 0 & -17.5 & 37.5 \end{bmatrix}$$

Modal Superposition Damping Matrix Using MSE Damping Ratios

$$C = \begin{bmatrix} 10.0 & -10.0 & 0 & 0 & 0 \\ -10.0 & 22.5 & -12.5 & 0 & 0 \\ 0 & -12.5 & 27.5 & -15.0 & 0 \\ 0 & 0 & -15.0 & 32.5 & -17.5 \\ 0 & 0 & 0 & -17.5 & 37.5 \end{bmatrix}$$

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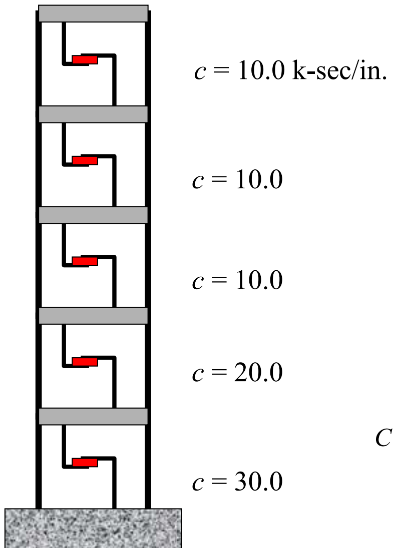
For the structure with proportional damping, the damping ratios from the Modal Strain Energy Method are correct and thus the damping matrix constructed from these damping ratios is equal to the actual damping matrix of the structure. Note that the Actual Damping Matrix is constructed in a manner similar to how the stiffness matrix is constructed.

Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Nearly Proportional Damping



Actual Damping Matrix

$$C_A = \begin{bmatrix} 10.0 & -10.0 & 0 & 0 & 0 \\ -10.0 & 20.0 & -10.0 & 0 & 0 \\ 0 & -10.0 & 20.0 & -10.0 & 0 \\ 0 & 0 & -10.0 & 30.0 & -20.0 \\ 0 & 0 & 0 & -20.0 & 50.0 \end{bmatrix}$$

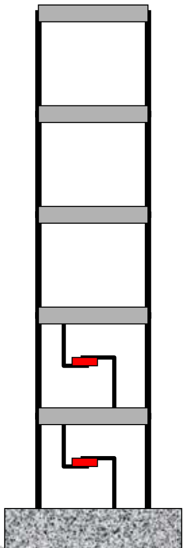
Modal Superposition Damping Matrix Using MSE Damping Ratios

$$C = \begin{bmatrix} 10.0 & -9.66 & -0.166 & -0.228 & -0.010 \\ -9.66 & 22.0 & -12.2 & -0.169 & -0.422 \\ -0.166 & -12.2 & 27.3 & -15.1 & -0.731 \\ -0.228 & -0.169 & -15.1 & 33.1 & -17.8 \\ -0.010 & -0.422 & -0.731 & -17.8 & 37.5 \end{bmatrix}$$

For the structure with nearly proportional damping, the damping ratios from the Modal Strain Energy Method are nearly correct and thus the damping matrix constructed from these damping ratios is nearly equal to the actual damping matrix of the structure. The additional entries in the damping matrix can have two effects. First, they can introduce skyhook damping and second, they can artificially couple floor levels that are not adjacent to each other.

Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Nonproportional Damping



$c = 20.0$ k-sec/in.

$c = 30.0$

Actual Damping Matrix

$$C_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20.0 & -20.0 \\ 0 & 0 & 0 & -20.0 & 50.0 \end{bmatrix}$$

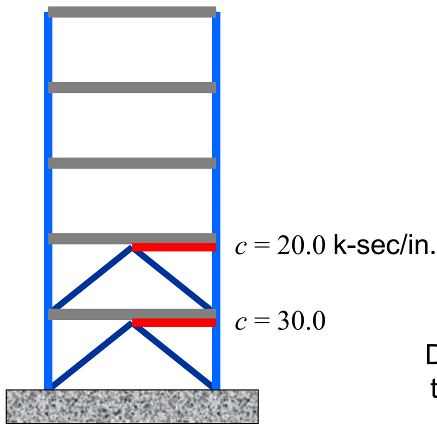
Modal Superposition Damping Matrix Using MSE Damping Ratios

$$C = \begin{bmatrix} 3.65 & -2.96 & 0.456 & -1.098 & 0.066 \\ -2.96 & 8.27 & -5.92 & 2.72 & -2.07 \\ 0.456 & -5.92 & 13.4 & -10.9 & 6.21 \\ -1.098 & 2.72 & -10.9 & 21.9 & -15.1 \\ 0.066 & -2.07 & 6.21 & -15.1 & 20.9 \end{bmatrix}$$

For the structure with a strongly nonproportional damping distribution, the damping ratios from the Modal Strain Energy Method are incorrect and thus the damping matrix constructed from these damping ratios is very different from the actual damping matrix of the structure. The additional entries in the damping matrix can have three effects. First, they can introduce skyhook damping, second they can artificially couple floor levels that are not adjacent to each other, and third, they can introduce interstory damping that does not exist.

Example: Seismic Analysis of a Structure with Nonproportional Damping

- Discrete Damping vs Rayleigh Damping
- Discrete Damping: Rigid vs Flexible Braces



MSE Results

Frequency (rad/sec)	Damping Ratio, ξ
4.54	0.089
12.1	0.144
18.5	0.134
23.0	0.194
27.6	0.514

Damping ratios in modes 1 and 4 used to construct Rayleigh damping matrix.

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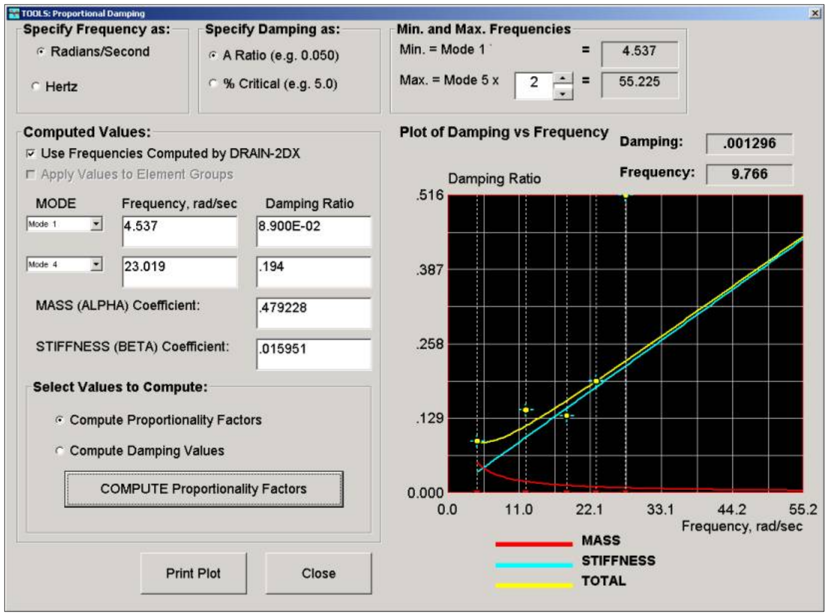
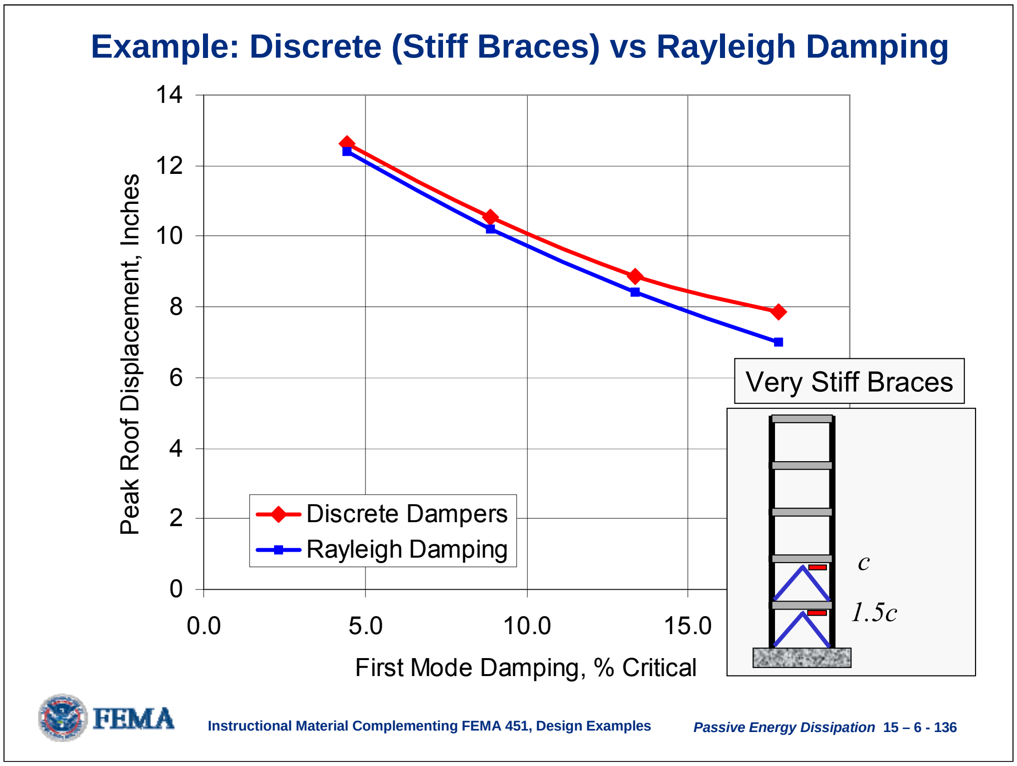
This example explores the seismic response of a structure with nonproportional damping. First, a comparison is made between the results using discrete damping (the correct damping matrix) and Rayleigh damping in which two of the modal damping ratios from the MSE method are used (the incorrect damping matrix). The Rayleigh damping approach is examined since it is commonly available in structural analysis software. Second, the influence of brace stiffness on the results is examined.

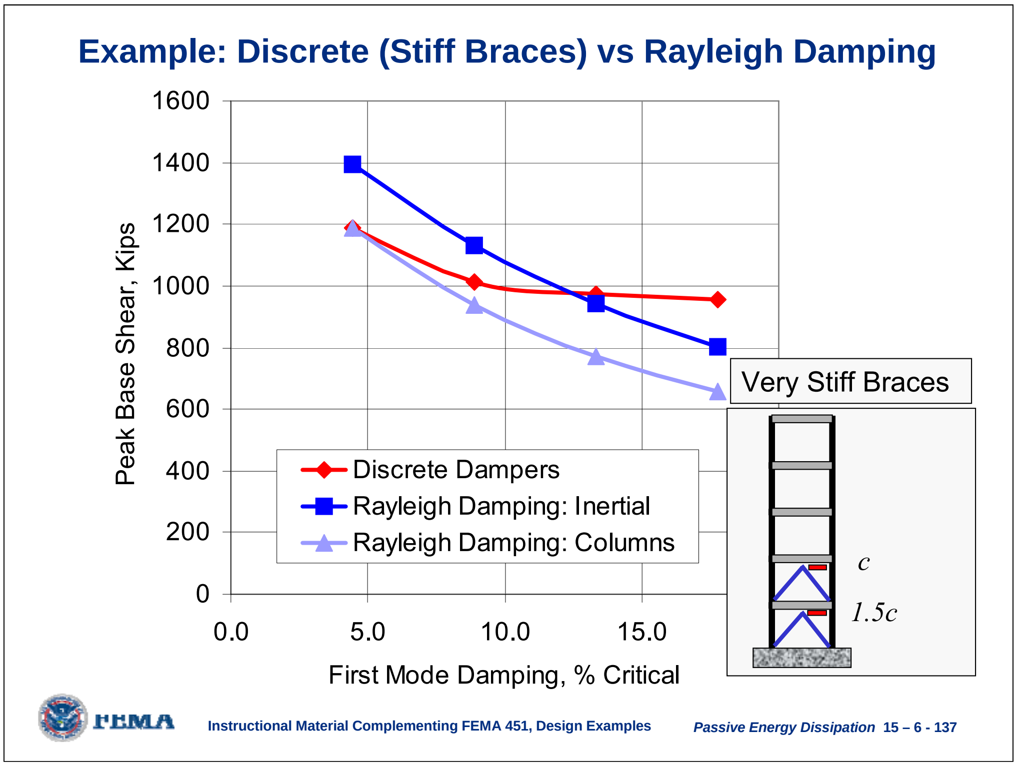
Computing Rayleigh Damping Proportionality Factors (Using NONLIN Pro)



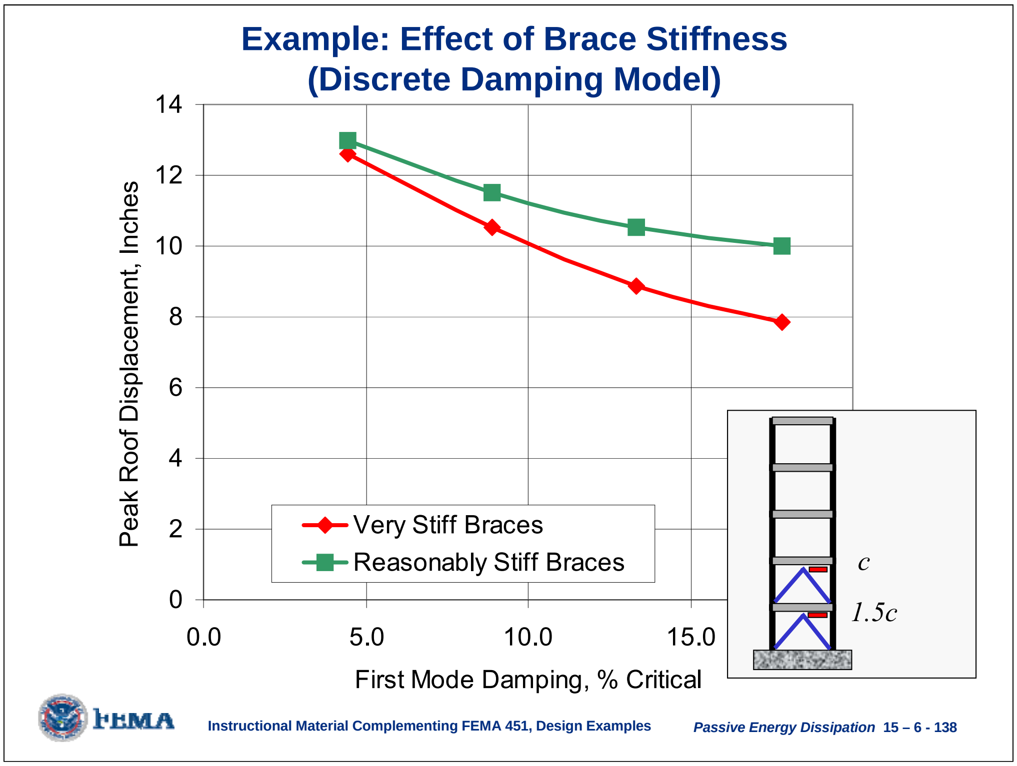
One tool available for computing Rayleigh damping proportionality factors (alpha and beta) is NONLIN Pro. In the case shown above, the first and fourth modal damping ratios from the MSE method are used as anchor points and the damping ratios in the other three modes are computed. Recall, however, that the modal damping ratios from the MSE method are not correct for this strongly nonproportionally damped structure and thus the Rayleigh damping matrix will not be correct.



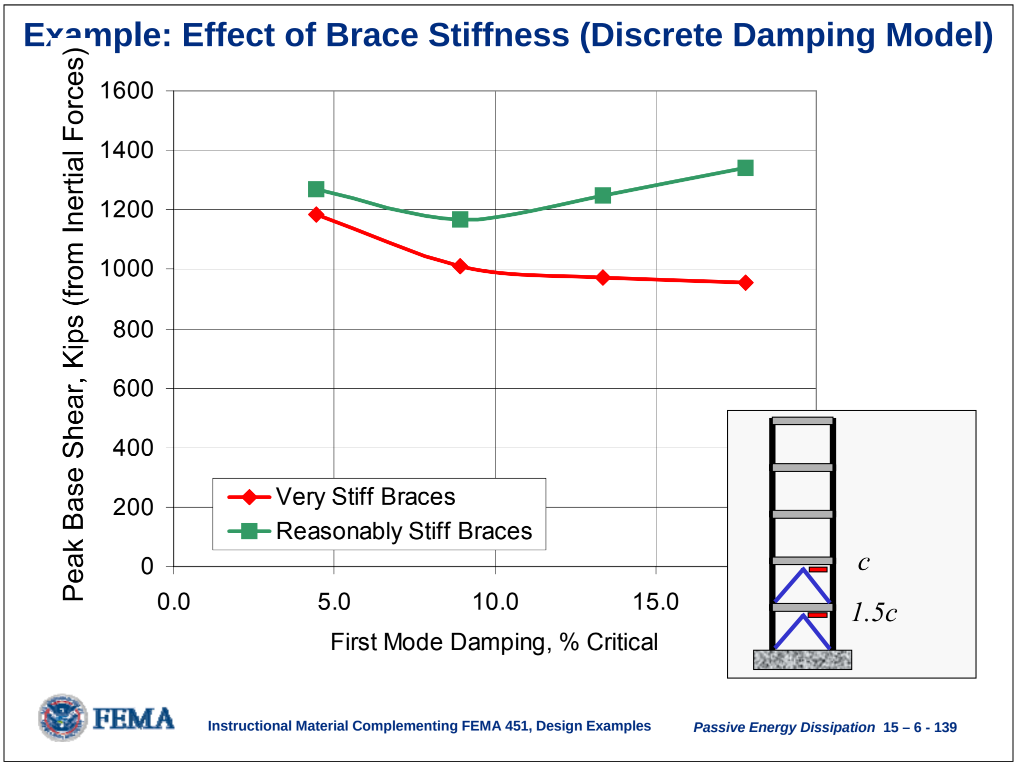
Seismic analysis of the structure was performed using two different damping matrices. The first matrix was constructed using the known location of the discrete dampers within the structure and assuming very stiff braces. This matrix is the actual damping matrix. The second matrix was constructed using the Rayleigh damping formulation in which two of the incorrect damping ratios from the MSE method were utilized. Finally, four different values of the damper damping coefficient, c, were considered. Although the Rayleigh damping matrix is known to be incorrect, the peak roof displacement is predicted quite well for each level of added damping.



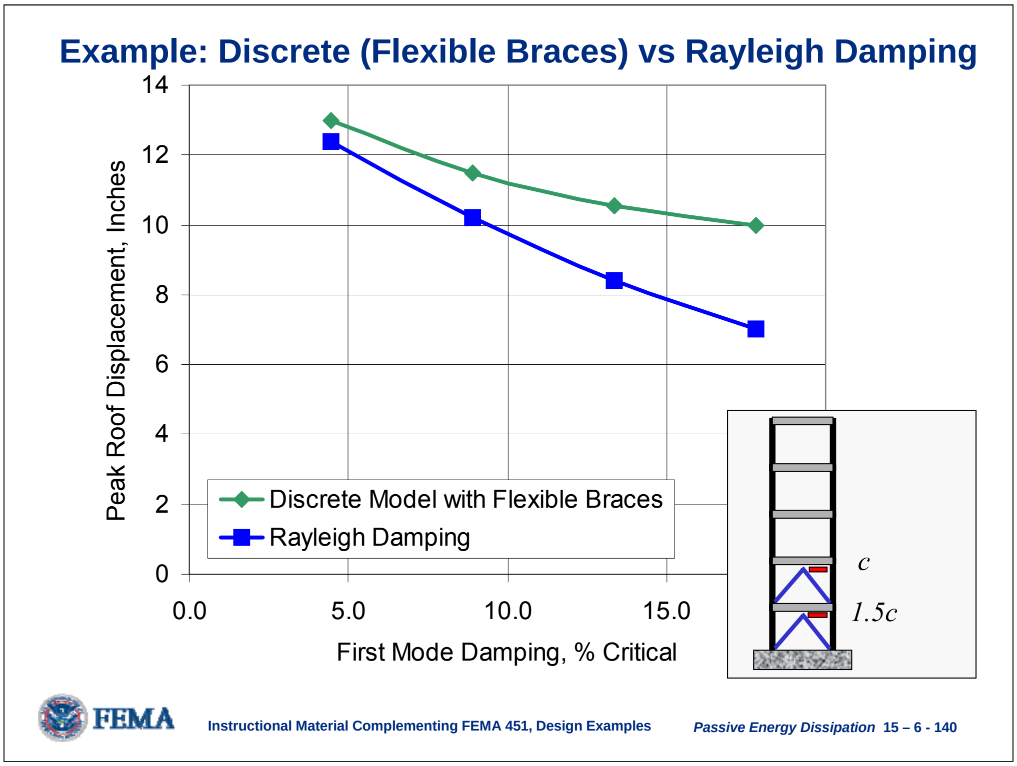
For the Rayleigh damping case, the peak base shear was calculated in two different ways. First, it was calculated as the sum of the inertial forces at each floor level. Second, it was calculated as the peak shear force in the first story columns (including the contribution from the chevron bracing). The peak base shear as calculated by either method is generally in poor agreement with the correct results from the discrete damper analysis. This is because the Rayleigh damping formulation uses damping ratios from the MSE method, a method which is not applicable for systems with nonproportional damping. The discrepancy between the results for the two different calculations is due to the skyhook damping that is present for the Rayleigh damping formulations. The skyhook dampers develop viscous forces that are not transferred to the foundation. Thus, the shear force that gets transferred to the base of the structure will be less than the sum of the inertial forces. This behavior is evident in the plot shown above.



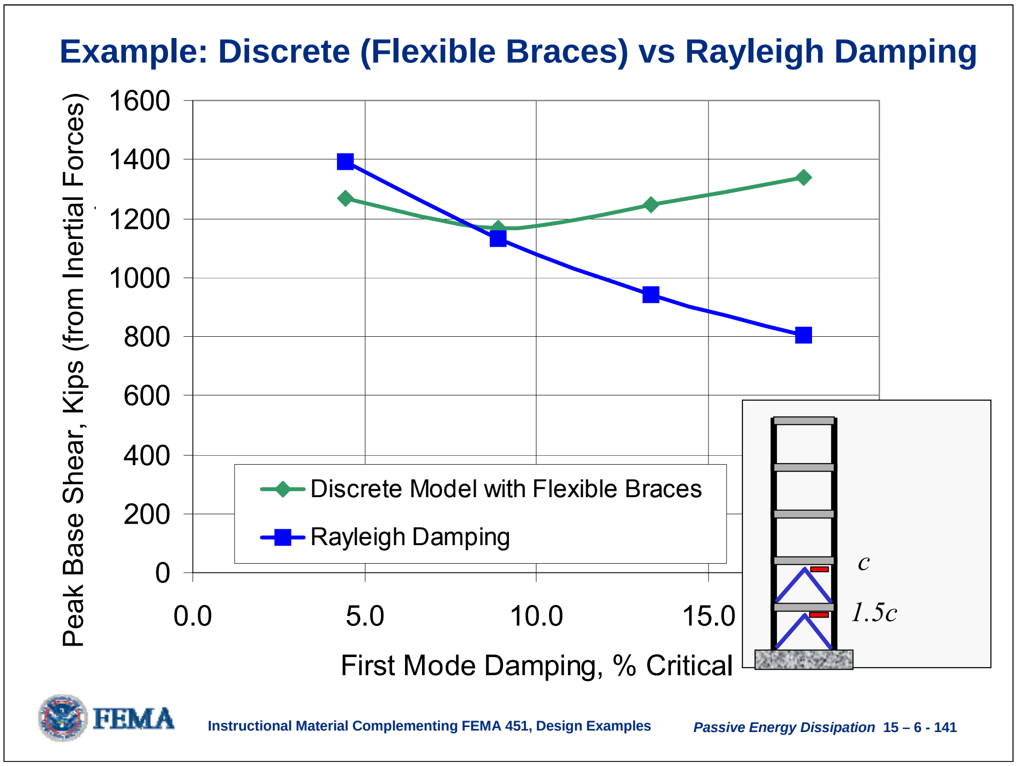
The peak roof displacement is larger if a realistic brace stiffness is used in the analysis. Thus, as expected, the effectiveness of the dampers is reduced due to the presence of the bracing. Since the potential benefit of high levels of damping are partially lost due to bracing flexibility, the influence of the brace stiffness on the roof displacement response is more significant for higher levels of damping.



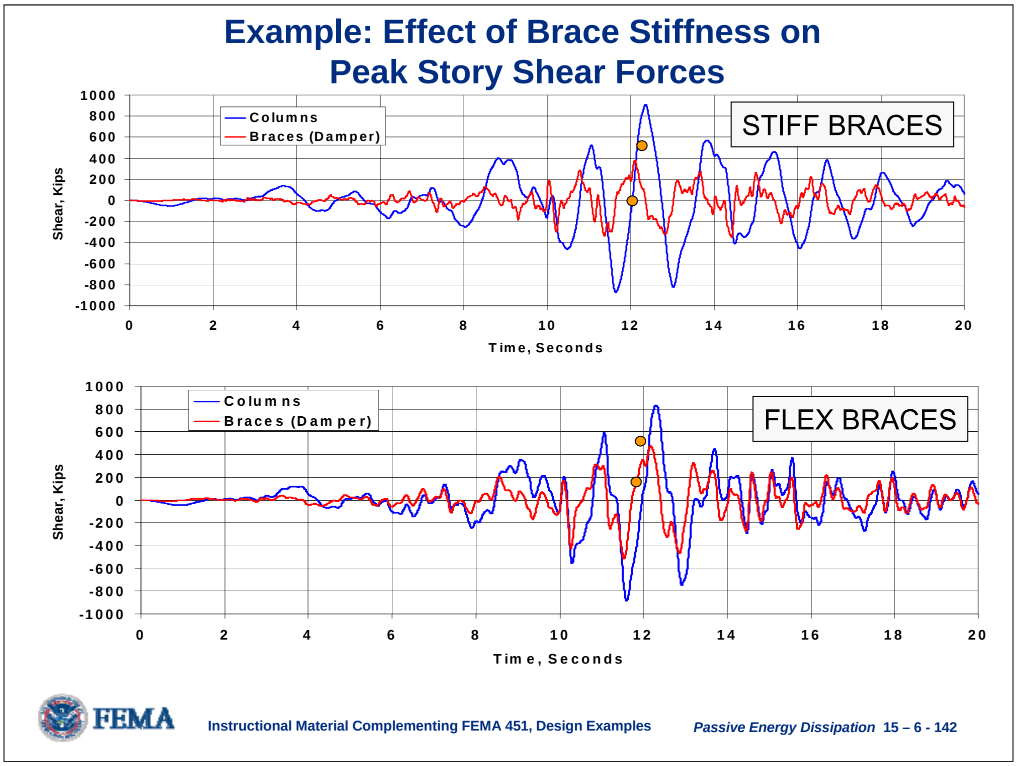
The peak base shear is larger if a realistic brace stiffness is used in the analysis. This is expected since, for realistic brace stiffness, the dampers are not as effective in limiting the lateral deformations of the structure and, in turn, the shear forces in the structure are increased. Since the potential benefit of high levels of damping are partially lost due to bracing flexibility, the influence of the brace stiffness on the base shear is more significant for higher levels of damping.



Seismic analysis of the structure was performed using two different damping matrices. The first matrix was constructed using the known location of the discrete dampers within the structure and assuming flexible (realistic stiffness) braces. This matrix is the actual damping matrix. The second matrix was constructed using the Rayleigh damping formulation in which two of the incorrect damping ratios from the MSE method were utilized. Finally, four different values of the damper damping coefficient, c, were considered. There is significant deviation in the predicted peak roof displacement using the two damping matrices, particularly for higher levels of added damping. As compared to the case where stiff bracing was assumed (slide 136), when flexible bracing is considered the Rayleigh damping formulation does not predict the peak roof displacement as well.



This slide is a continuation of the previous slide. There is significant deviation in the predicted peak base shear using the two damping matrices, particularly for higher levels of added damping.



For a structure with viscous dampers and infinitely stiff bracing, the damper forces (and thus bracing shear forces) are 90 degrees out of phase with respect to the column shear forces. This is because the damper forces are proportional to velocity whereas the column shear forces are proportional to displacement. In this case, the peak total shear force transferred to the story below does not simultaneously include contributions from both the columns and bracing. For a structure with viscous dampers and flexible (realistically stiff) bracing, the damper forces (and thus bracing shear forces) are less than 90 degrees out of phase with respect to the column shear forces. This is because the combined bracing and damper system acts as a viscoelastic damper that exhibits a phase angle of less than 90 degrees with respect to its displacement. In this case, the peak total shear force transferred to the story below includes contributions from both the columns and bracing. The different phasing between the column shear forces and bracing shear forces is shown above for stiff and flexible bracing. Note that the bracing shear force refers to a force that is parallel to the column shear force.

Outline: Part III

- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
- Representations of Damping
- Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
- Summary of MDOF Analysis Procedures

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Summary: MDOF Analysis Procedures (Linear Systems and Linear Dampers)

- Use discrete damper elements and explicitly include these dampers in the system damping matrix. Perform response history analysis of full system. **Preferred.**
- Use discrete damper elements to estimate modal damping ratios and use these damping ratios in modal response history or modal response spectrum analysis. **Dangerous.**
- Use discrete damper elements to estimate modal damping ratios and use these damping ratios in a response history analysis which incorporates Rayleigh proportional damping. **Dangerous.**

No annotation is provided for this slide.

Summary: MDOF Analysis Procedures (Linear Systems with Nonlinear Dampers)

- Use discrete damper elements and explicitly include these dampers in the system damping matrix. Perform response history analysis of full system. **Preferred.**

- Replace nonlinear dampers with "equivalent energy" based linear dampers, and then use these equivalent dampers in the system damping matrix. Perform response history analysis of full system. **Very Dangerous.**

- Replace nonlinear dampers with "equivalent energy" based linear dampers, use modal strain energy approach to estimate modal damping ratios, and then perform response spectrum or modal response history analysis. **Very Dangerous.**

No annotation is provided for this slide.

Summary: MDOF Analysis Procedures (Nonlinear Systems with Nonlinear Dampers)

- Use discrete damper elements and explicitly include these dampers in the system damping matrix. Explicitly model inelastic behavior in superstructure. Perform response history analysis of full system. **Preferred.**

- Replace nonlinear dampers with "equivalent energy" based linear dampers and use modal strain energy approach to estimate modal damping ratios. Use pushover analysis to represent inelastic behavior in superstructure. Use capacity-demand spectrum approach to estimate system deformations. **Do This at Your Own Risk!**

No annotation is provided for this slide.

Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems

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No annotation is provided for this slide.

MDOF Solution for Non-Classically Damped Structures Using Complex Modal Analysis

$$M\ddot{v}(t)+C_I\dot{v}(t)+C_A\dot{v}(t)+F_S(t)=-MR\ddot{v}_g(t)$$

Modal Analysis using Damped Mode Shapes:

- Treat C_I as modal damping and model C_A explicitly.
- Solve by modal superposition using damped (complex) mode shapes and frequencies.

System (dampers and structure) must be linear.

For structures with non-classical damping distributions, classical modal analysis can not be performed since the use of the undamped mode shapes in the modal transformation results in a generalized damping matrix that is not diagonal. Modal analysis can still be performed but the damped mode shapes must be utilized to perform the transformation. Thus, the damped eigenproblem must be solved.

Damped Eigenproblem

$$M\ddot{v}(t) + C_A\dot{v}(t) + Kv(t) = 0$$

EOM for Damped Free Vibration

Assume C_I is negligible

Linear Structure

State Vector: $Z = \begin{Bmatrix} \dot{v} \\ v \end{Bmatrix}$

State-Space Transformation: $\dot{Z} = HZ$

State Matrix: $H = \begin{bmatrix} -M^{-1}C_A & -M^{-1}K \\ I & 0 \end{bmatrix}$

The damped eigenvalue problem begins with defining the state vector which consists of the velocity and displacement vector. A state-space transformation is performed to transform the second-order ODE to a first-order ODE.

Solution of Damped Eigenproblem

Assume Harmonic Response for n-th mode: $Z_n = P_n e^{\lambda_n t}$

Substitute Response into State Space Equation: $P_n \lambda_n = HP_n$ ← Damped Eigenproblem for n-th Mode

$P\Lambda = HP$ ← Damped Eigenproblem for All Modes

Eigenvalue Matrix: (* = complex conjugate) $\Lambda = \begin{bmatrix} \lambda & \\ & \lambda^* \end{bmatrix}$ $\lambda = diag[\lambda_n]$

Eigenvector Matrix: $P = \begin{bmatrix} \Phi\lambda & \Phi^*\lambda^* \\ \Phi & \Phi^* \end{bmatrix}$

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For each mode of vibration, harmonic response is assumed in the form of a time-dependent exponential function. Substitution of the harmonic response into the state-space equation leads to the damped eigenvalue problem. Solution of the eigenvalue problem results in the eigenvalues and eigenvectors, both of which are generally complex-valued.

Extracting Modal Damping and Frequency from Complex Eigenvalues

Complex Eigenvalue for Mode n: $\lambda_n = -\xi_n \omega_n \pm i\omega_n \sqrt{1-\xi_n^2}$

Analogous to Roots of Characteristic Equation for SDOF Damped Free Vibration Problem

Modal Frequency: $\omega_n = |\lambda_n|$

Modal Damping Ratio: $\xi_n = -\dfrac{\Re(\lambda_n)}{|\lambda_n|}$

Note: $i = \sqrt{-1}$

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The complex eigenvalues contain information on both the modal frequencies and modal damping ratios. The values of the modal frequency and damping are extracted from the complex eigenvalues by recognizing the analogy between the complex eigenvalues and the roots of the characteristic equation for a SDOF damped free vibration problem.

Extracting Damped Mode Shapes

$$P = \begin{bmatrix} \Phi\Lambda & \Phi^{*}\Lambda^{*} \\ \boxed{\Phi} & \Phi^{*} \end{bmatrix}$$

Damped Mode Shapes

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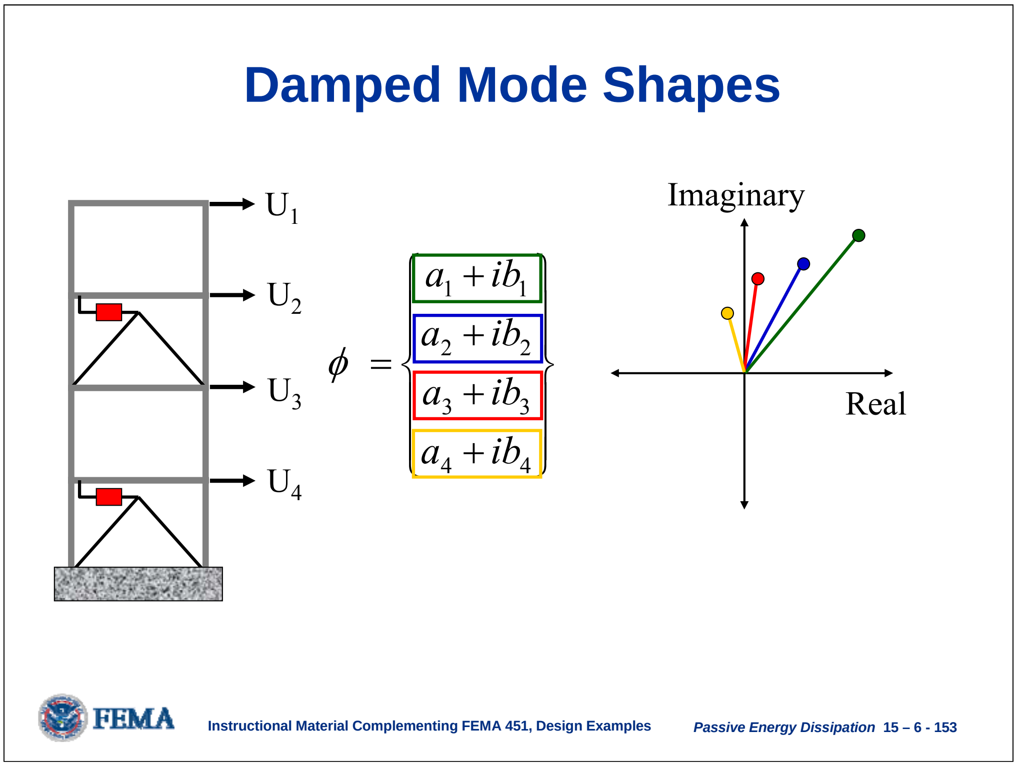
The damped mode shapes are complex-valued and are contained within the eigenvector matrix.



The damped mode shapes are complex-valued and thus can be represented as a series of vectors in the complex plane. The real part of the vectors represent the physical amplitude of the DOF's. As the vectors rotate counterclockwise, the real part changes and thus the physical amplitude changes. One cycle of motion in the complex plane is equivalent to one cycle of motion in the physical system. The angles between the rotating vectors remains constant and represents the physical phase difference between the DOF's. For systems with classical damping distributions, in each mode the phase angles between DOF's are always zero degrees (in-phase) or 180 degrees (completely out-of-phase). The result is that, as time progresses, the shape (not the amplitude) of the free vibration response in a given mode (i.e., the mode shape) remains constant. For systems with non-classical damping distributions, the phase angles generally lie between zero and 180 degrees and thus the DOF's are neither in-phase or completely out-of-phase. The result is that the shape of the free vibration response in a given mode (i.e., the mode shape) changes with time.

Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems

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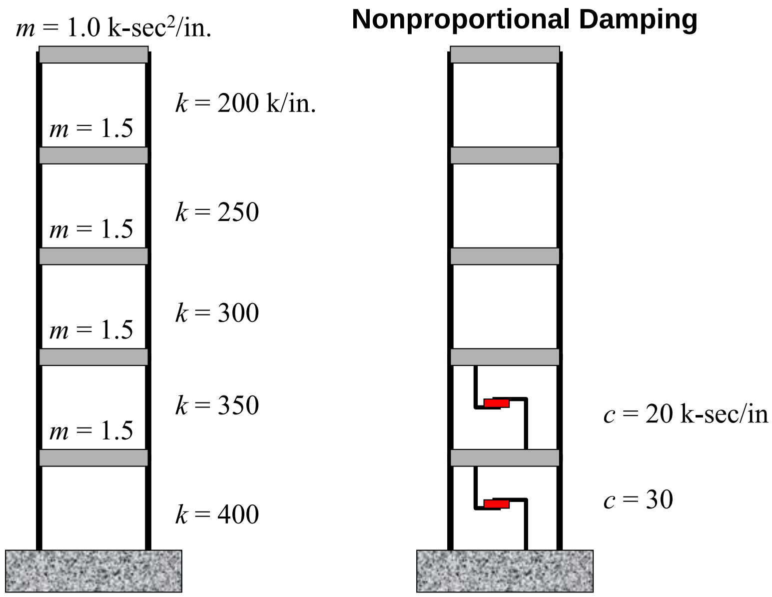
Example: Damped Mode Shapes and Frequencies



The damped mode shapes and frequencies for this structure with strongly nonproportional damping are evaluated next.

Example: Damped Mode Shapes and Frequencies
System with Non-Classical Damping

Using UNDAMPED MODE SHAPES

	Frequency (rad/sec)	Damping Ratio
1	4.54	0.089
2	12.1	0.144
3	18.4	0.134
4	23.0	0.194
5	27.6	0.516

Obtained from MSE Method

Using DAMPED MODE SHAPES*

	Frequency (rad/sec)	Damping Ratio
1	4.58	0.089
2	12.3	0.141
3	18.9	**0.064**
4	24.0	**0.027**
5	25.1	**0.770**

***Table is for model with VERY STIFF braces.**

Significant Differences in Higher Mode Damping Ratios

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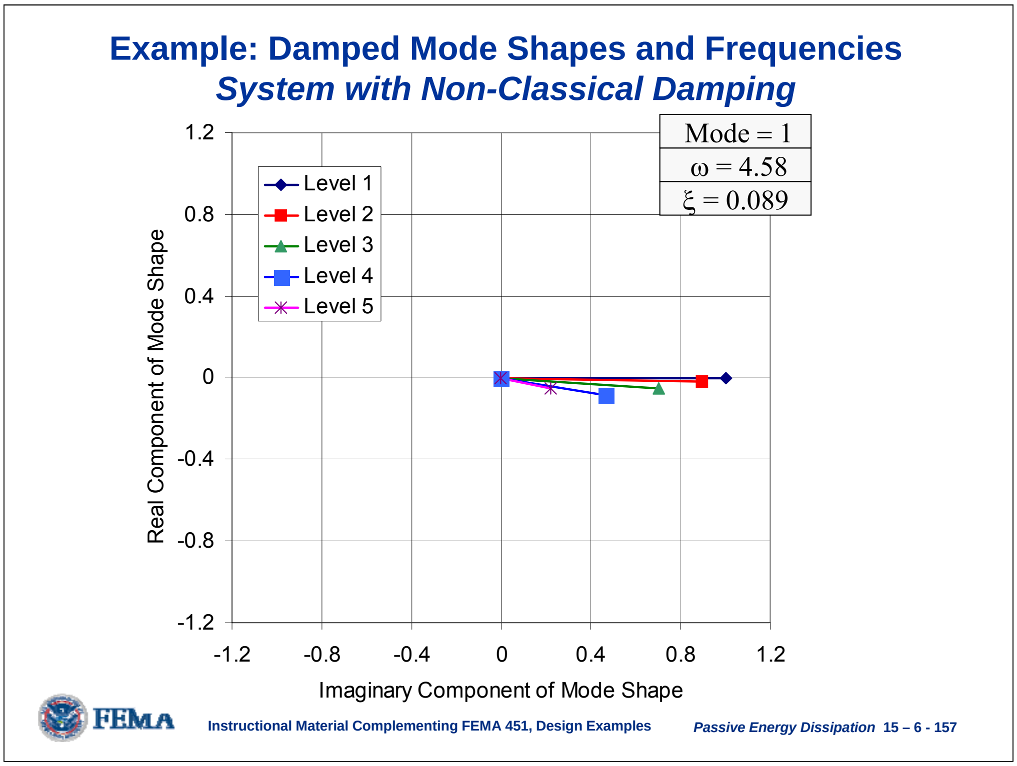
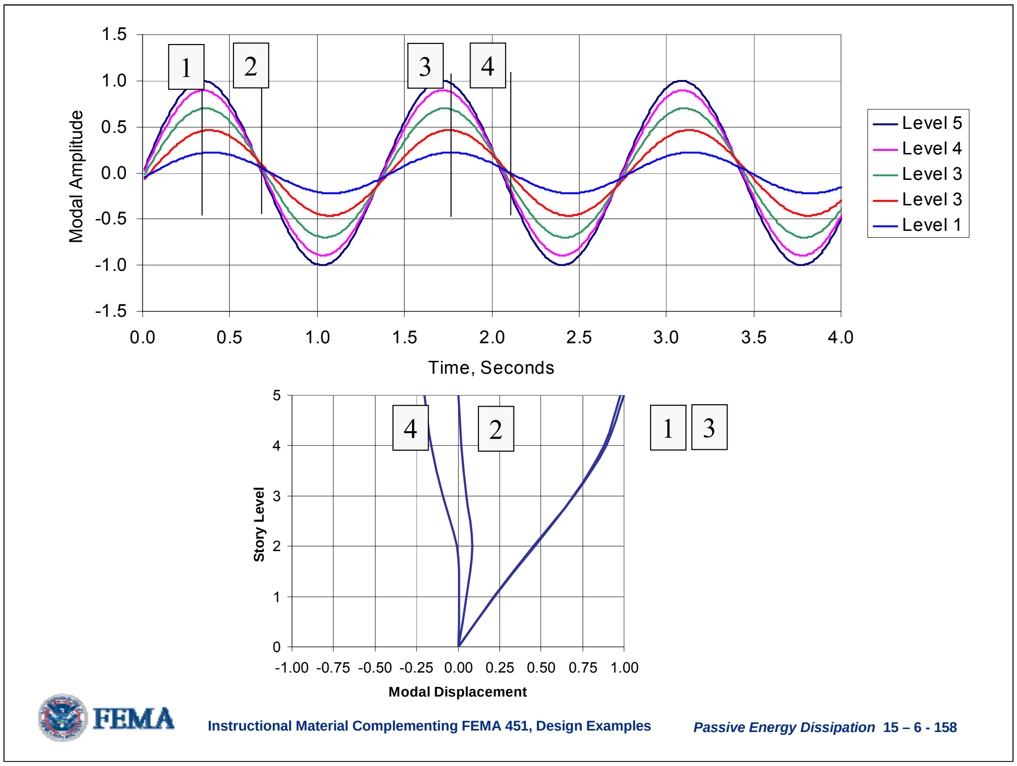
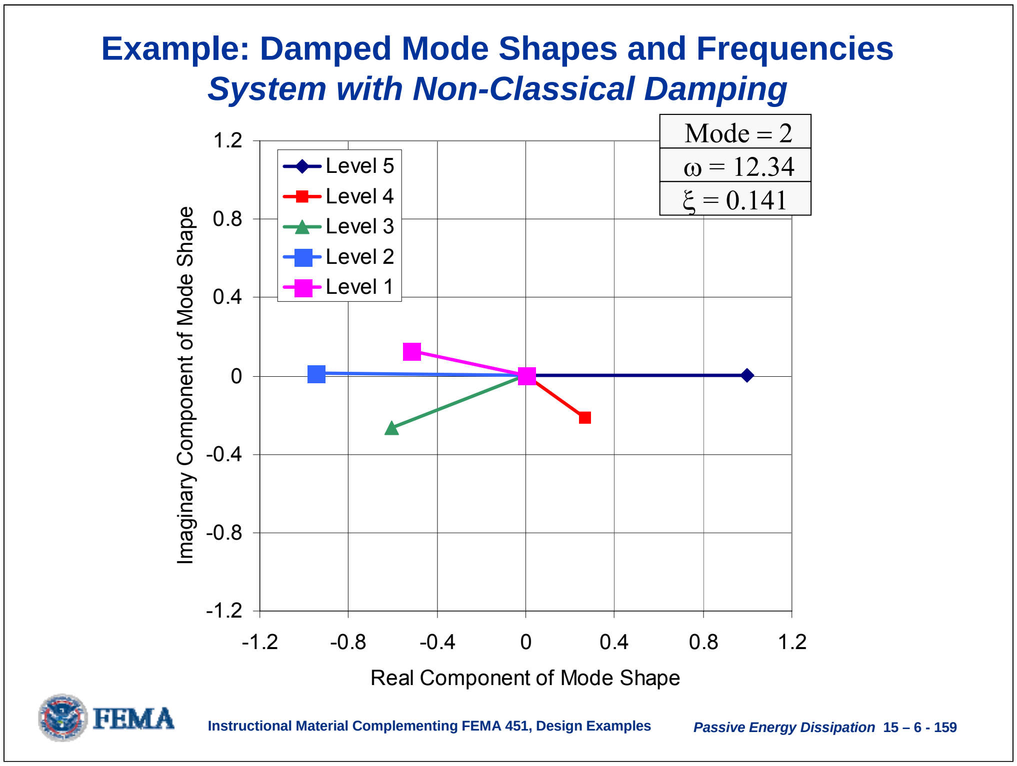
The natural frequencies and modal damping ratios were determined using two different methods. In the first method, the undamped eigenvalue problem was solved. The undamped natural frequencies are given above and the undamped mode shapes were utilized in the MSE method to estimate the modal damping ratios. Since the structure actually has strongly nonproportional damping, it is expected that the use of the MSE method will result in erroneous modal damping ratios. In the second method, the damped eigenproblem was solved, resulting in the damped natural frequencies and damping ratios provided above. As expected, there are significant differences in some of the modal damping ratios obtained by the two methods.



In the complex plane shown above, the real and imaginary axes are switched from their customary locations. Thus, as time increases, the vectors rotate clockwise rather than counterclockwise. The damped mode shape for the fundamental mode is shown. Recall that the real component of the complex vectors represent the physical amplitudes of the mode shape. In this case, for the time instant shown, level 1 (bottom) has the smallest real component whereas level 5 (at the top) has the largest real component. The phase differences between all of the vectors is relatively small, indicating that the DOF's are almost in-phase with each other. These observations regarding the first mode shape seem reasonable.



The response-history for the 5 levels of the structure are shown in the upper plot for free vibration in the first mode. A snapshot of the mode shape at four different time instants is shown in the lower plot. The response histories demonstrate that the 5 levels are out-of-phase with respect to each other. For example, time instants 1 and 3 show that all DOF do not pass through their maximum displacement at the same time whereas time instants 2 and 4 show that all DOF do not pass through zero displacement at the same time. If the structure had classical damping, the phase difference for all 5 levels would be zero (i.e., the 5 levels would vibrate completely in-phase). The snapshots demonstrate that the shape of the structure in the first mode of vibration varies with time. Again, this is due to the phase differences between the 5 levels.



In the complex plane shown above, the real and imaginary axes are in their customary locations. Thus, as time increases, the vectors rotate counter-clockwise. The damped mode shape for the second mode is shown. Recall that the real component of the complex vectors represent the physical amplitudes of the mode shape. In this case, for the time instant shown, levels 2 and 5 are approximately 180 degrees out-of-phase and are at their maximum displacements. The other three levels are out-of-phase with respect to each other and with levels 2 and 5.

Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems

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An Unexpected Effect of Passive Damping

The larger the damping coefficient ***C***, the *smaller* the damping ratio ξ.

Why?

Note:
Occurs for toggle-braced systems only.

Huntington Tower
- **111 Huntington Ave, Boston, MA**
- **New 38-story steel-framed building**
- **100 Direct-acting and toggle-brace dampers**
- **1300 kN (292 kips), +/- 101 mm (+/- 4 in.)**
- **Dampers suppress wind vibration**

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The Huntington Tower contains both direct-acting and toggle-brace dampers. During the seismic analysis of the structure, it was noted that an increase in the damping coefficient of the toggle-brace dampers tended to degrade the performance. This was unexpected and led to the analysis presented in the next few slides.

Toggle Brace Deployment

Huntington Tower

FEMA

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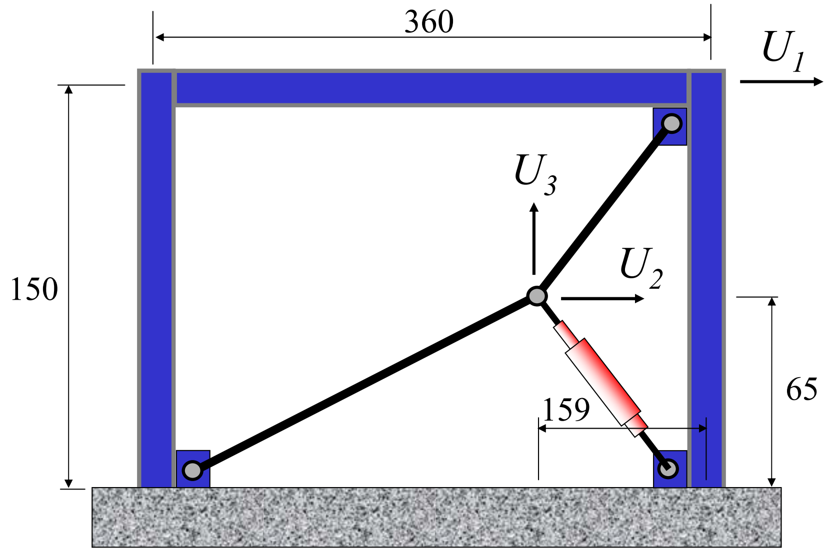
Example: Toggle Brace Damping System



A simple frame with a toggle brace damping system is shown. The damper is assumed to behave as a linear viscous dashpot. The toggle bracing arrangement amplifies the damper displacement (i.e., the displacement along the axis of the damper is larger than the lateral displacement of the frame). The amplifying effect is important for stiff structures such as reinforced concrete shear wall systems and steel-braced dual systems. The simple frame shown above was analyzed to assess the influence of the damping coefficient on the energy dissipation capacity (i.e., the damping ratio) of the system. The three DOF shown were used in the analysis.

Methods of Analysis Used to Determine Damping Ratio

- Energy Ratios for Steady-State Harmonic Loading: $\xi = E_D/4\pi E_S$
- Modal Strain Energy
- Free Vibration Log Decrement
- Damped Eigenproblem

$\boldsymbol{C}$ =10 to 40 k-sec/in (increments of 10)
$\boldsymbol{A}$ =10 to 100 in^2 (increments of 10)

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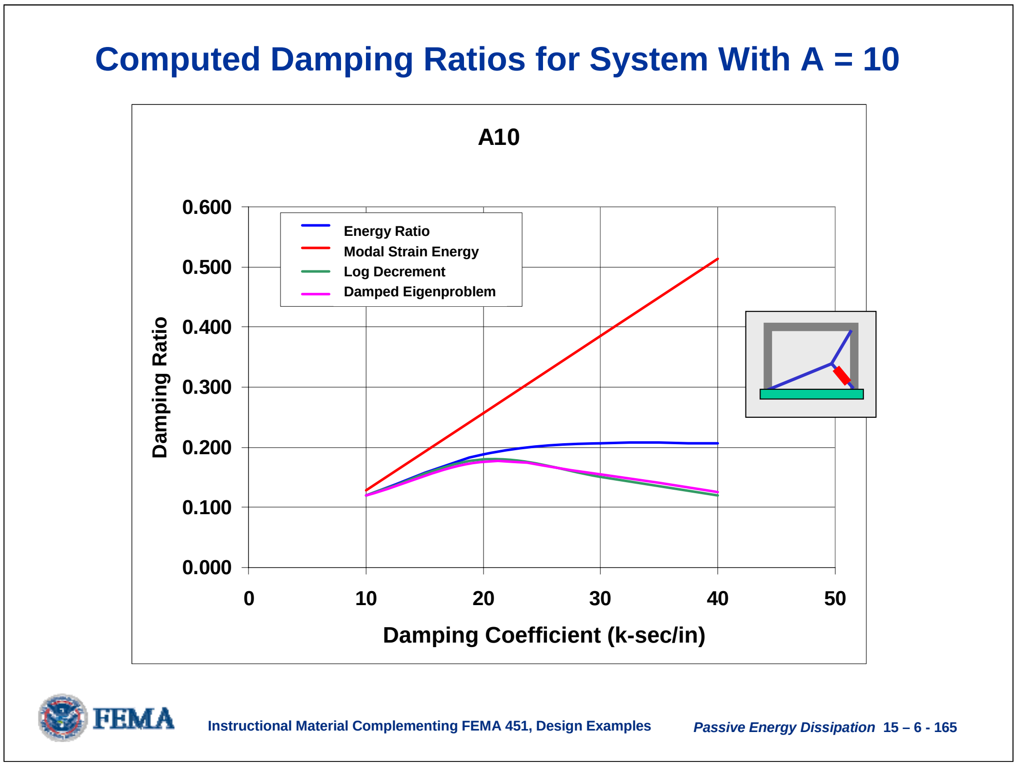
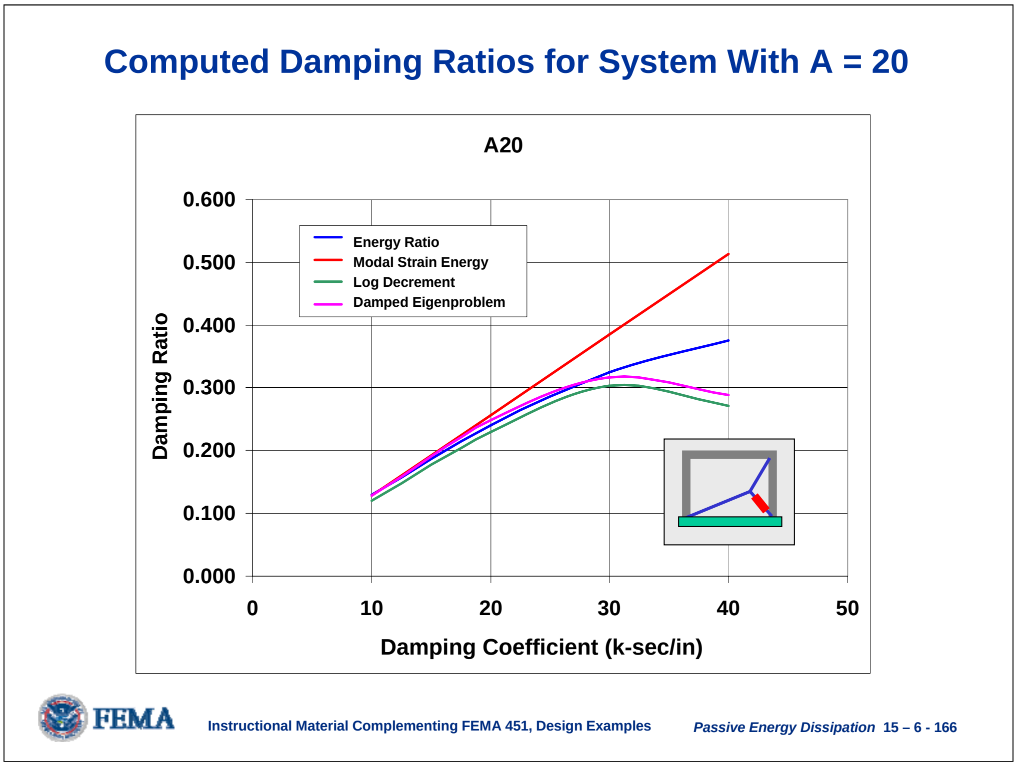
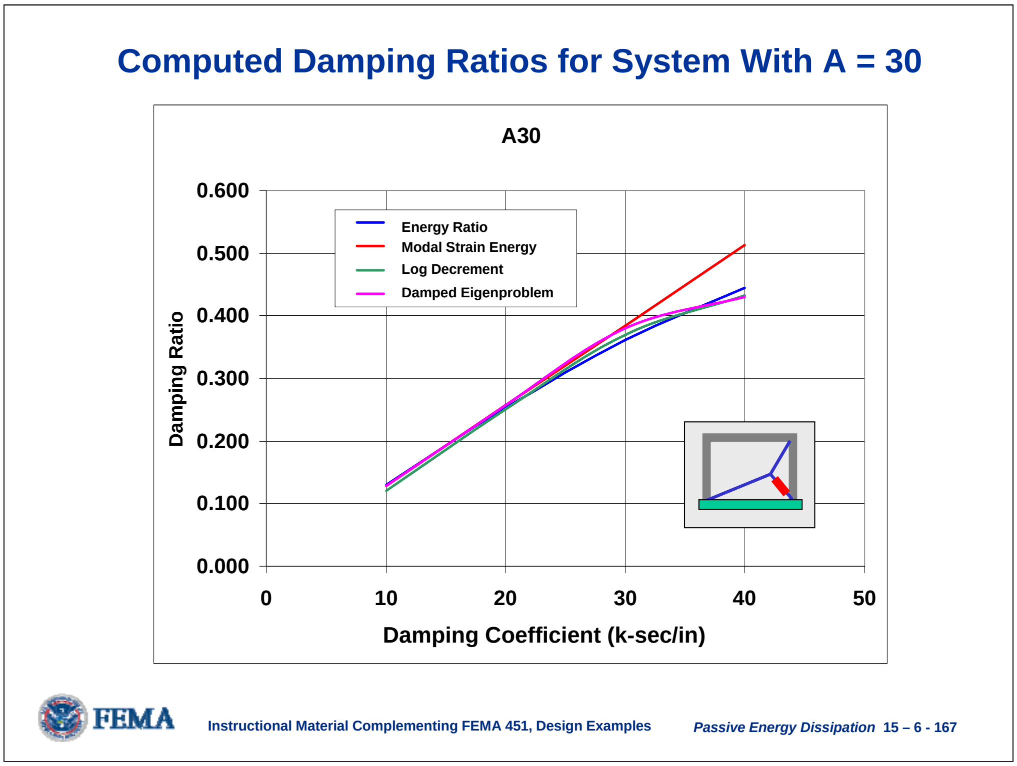
Four different methods were used to determine the damping ratio of the system. A parametric study was performed in which the damping coefficient of the damper, C, was varied over a factor of 4 and the toggle brace cross-sectional area, A, was varied over a factor of 10.



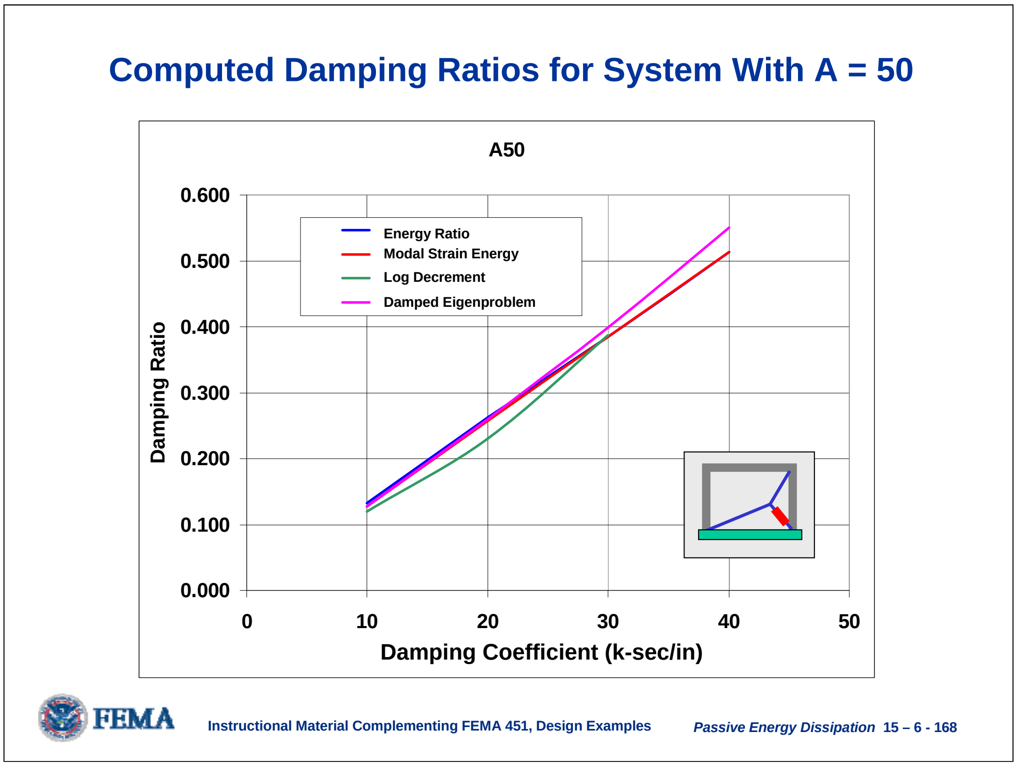
This plot shows the computed damping ratios for relatively flexible bracing (lowest cross-sectional area used in analysis; A = 10) and four different damping coefficient values. For a given damping coefficient, the damping ratio can vary significantly depending on the method used for the computation. The damped eigenproblem is the correct solution and the log decrement (free vibration) method provides a very good estimate of the damping ratios. Due to assumptions that are violated (e.g., the modal strain energy method is not strictly applicable since the system has non-classical damping), the energy ratio and modal strain energy methods generally provide poor and unconservative estimates of the damping ratio. Interestingly, for the damped eigenproblem solution (the correct solution), as the damping coefficient increases, the damping ratio can reduce. This phenomenon is not predicted by the energy ratio and modal strain energy methods. Finally, the apparent convergence of results for the lowest damping coefficient is simply due to each curve approaching the origin as the damping coefficient is reduced.



This plot shows the computed damping ratios for a higher brace stiffness (larger cross-sectional area; A = 20). For higher damping coefficients, the damping ratio can vary significantly depending on the method used for the computation. Again, for the damped eigenproblem solution (the correct solution), as the damping coefficient increases, the damping ratio can reduce. This phenomenon is not predicted by the energy ratio and modal strain energy methods.

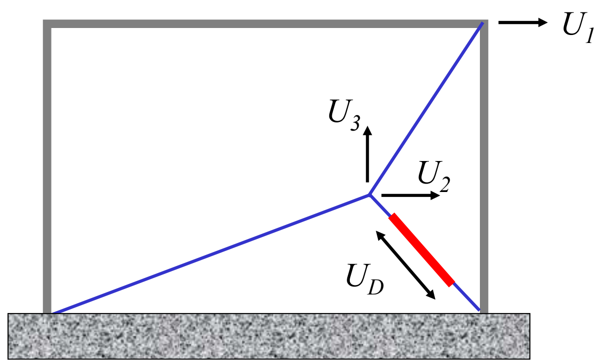


This plot shows the computed damping ratios for a higher brace stiffness (A = 30). For higher damping coefficients, the damping ratio still varies depending on the method used for the computation. The reduction in damping ratio with increasing damping coefficient is no longer predicted by the damped eigenproblem solution.



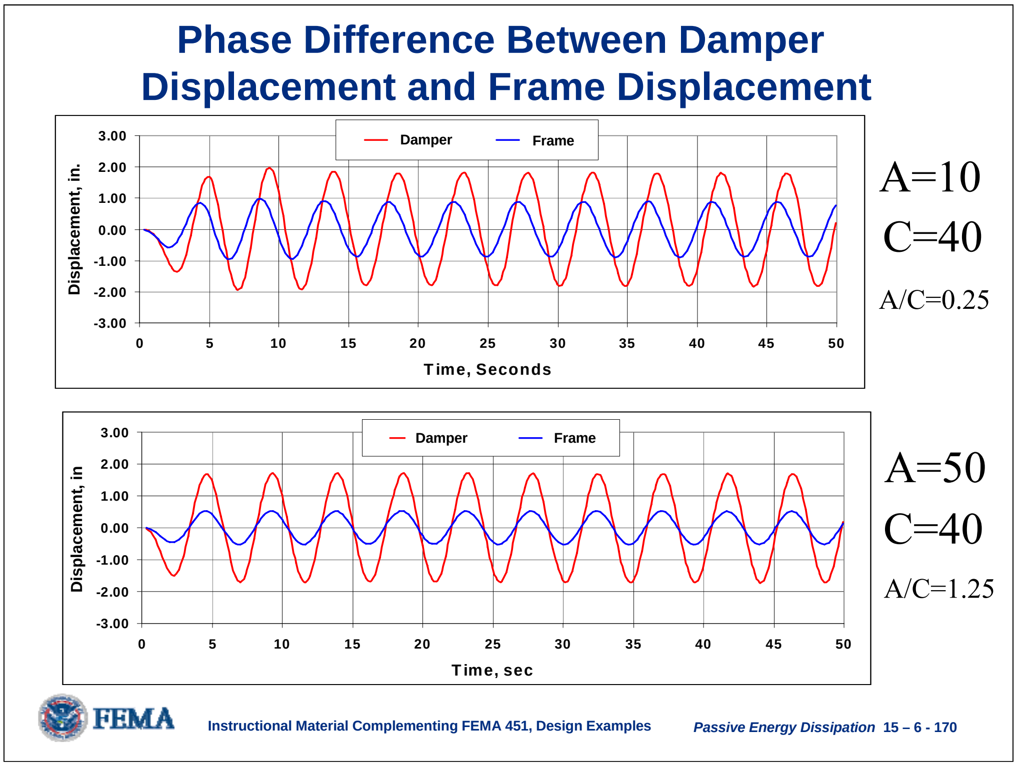
This plot shows the computed damping ratios for the highest brace stiffness (A = 50). For a given damping coefficient, the predicted damping ratio is similar for all four methods of analysis. Note that data is not provided for the log decrement method for the highest damping coefficient. In this case, the decay was so fast that it was difficult to utilize the method.

Why Does Damping Ratio Reduce for Low Brace Area/Damping Coefficient Ratios?

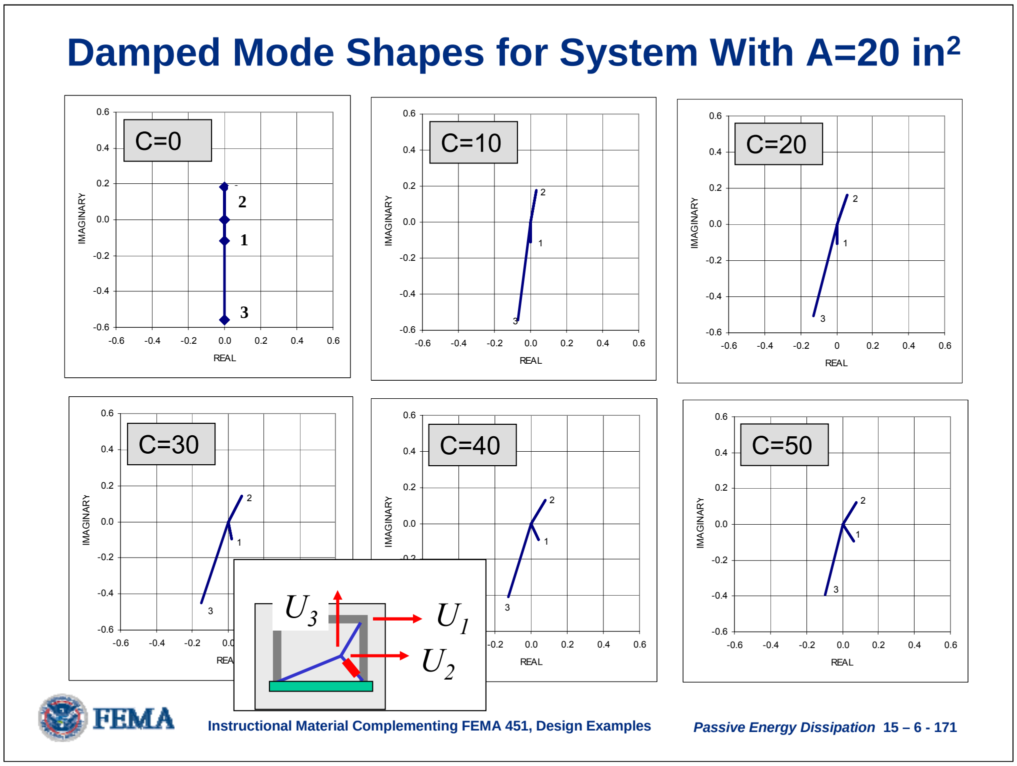


Displacement in Damper is *Out-of-Phase* with Displacement at DOF 1

For low brace area/damping coefficient ratios, some of the energy in the system goes into strain energy in the bracing instead of energy dissipation in the damper. Thus, the effectiveness of the dampers is reduced. Furthermore, an increase in the flexibility of the braces results in an increase in the phase difference between the displacement in the device and the displacement of the frame at DOF 1.



These plots show the response-history of the frame and damper displacement when the frame is subjected to sinusoidal motion having an amplitude of approximately 1 in. and a frequency of 0.2 Hz. In the top plot, the brace area/damping coefficient ratio is small and thus the displacement of the damper is out-of-phase with the frame displacement. In the bottom plot, the brace area/damping coefficient ratio is larger and thus the displacement of the damper is nearly in-phase with the frame displacement. Since it is desirable to produce damper forces (which are proportional to damper velocity) that are 90 degrees out-of-phase with respect to the column shear forces (which are proportional to frame displacements), it is desirable to design the system such that the displacements of the framing and the damper are in-phase. In this way, the velocity of the damper will be 90 degrees out of phase with respect to the frame displacements. In turn, the maximum damper forces that are transferred to the framing will occur at the instant when the framing shear forces are zero and vice versa.



The damped modes are shown above for the case of A = 20 and six different damping coefficients. As expected, for a damping coefficient of zero, DOF 1 and 3 are in-phase and DOF 1 and 3 are 180 degrees out-of-phase with DOF 2. In other words, as the frame displaces to the right (+DOF1), the toggle brace joint moves left (-DOF2) and upward (+DOF3). Since the damper offers no resistance to motion, the toggle bracing stiffness is not engaged and the toggle brace joint moves freely. As the damping coefficient increases, the damper resists motion and thus the stiffness of the bracing is engaged. As the level of damping is increased (i.e., A/C is decreased), the phase difference between the frame displacement (DOF 1) and the damper displacement (DOF 2 and 3) increasingly moves away from 180 or zero degrees, respectively. Note that observing the phase differences in the complex plane may be considered to be a more efficient visualization tool as compared to examining response-histories as was done on the previous slide.

Interim Summary Related to Modeling and Analysis (1)

- Viscously damped systems are very effective in reducing damaging deformations in structures.
- With minor exceptions, viscously damped systems are non-classical, and *must* be modeled explicitly using dynamic time history analysis.
- Avoid the use of the Modal Strain Energy method (it may provide unconservative results)

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Interim Summary Related to Modeling and Analysis (2)

- Damped mode shapes provide phase angle information that is essential in assessing and improving the efficiency of viscously damped systems. This is particularly true for linkage systems (e.g. toggle-braced systems).

- If damped eigenproblem analysis procedures are not available, use overlayed response history plots of damper displacement and interstory displacement to assess damper efficiency. (This would be required for nonlinear viscously damped systems.)

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Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
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Computer Software Analysis Capabilities

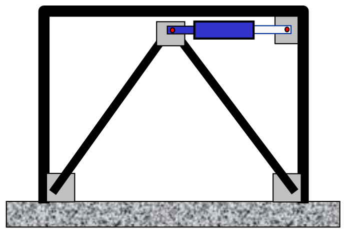
	SAP2000; ETABS	DRAIN	RAM Perform
Linear Viscous Fluid Dampers	Yes	Yes	Yes
Nonlinear Viscous Fluid Dampers	Yes	NO	Yes*
Viscoelastic Dampers	Yes	Yes	Yes
ADAS Type Systems	Yes	Yes	Yes
Unbonded Brace Systems	Yes	Yes	Yes
Friction Systems	Yes	Yes	Yes
General System Yielding	Pending	Yes	Yes

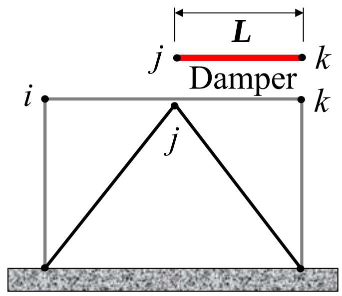
***Piecewise Linear**

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 175

A variety of software programs are available for analysis of structures with passive energy dissipation systems. Some of the programs have built-in energy dissipation elements while others require working with user-defined elements.

Modeling Linear Viscous Dampers in DRAIN





Use a Type-1 truss bar element with stiffness proportional damping:

$$K = \frac{AE}{L} \qquad C = \beta K$$

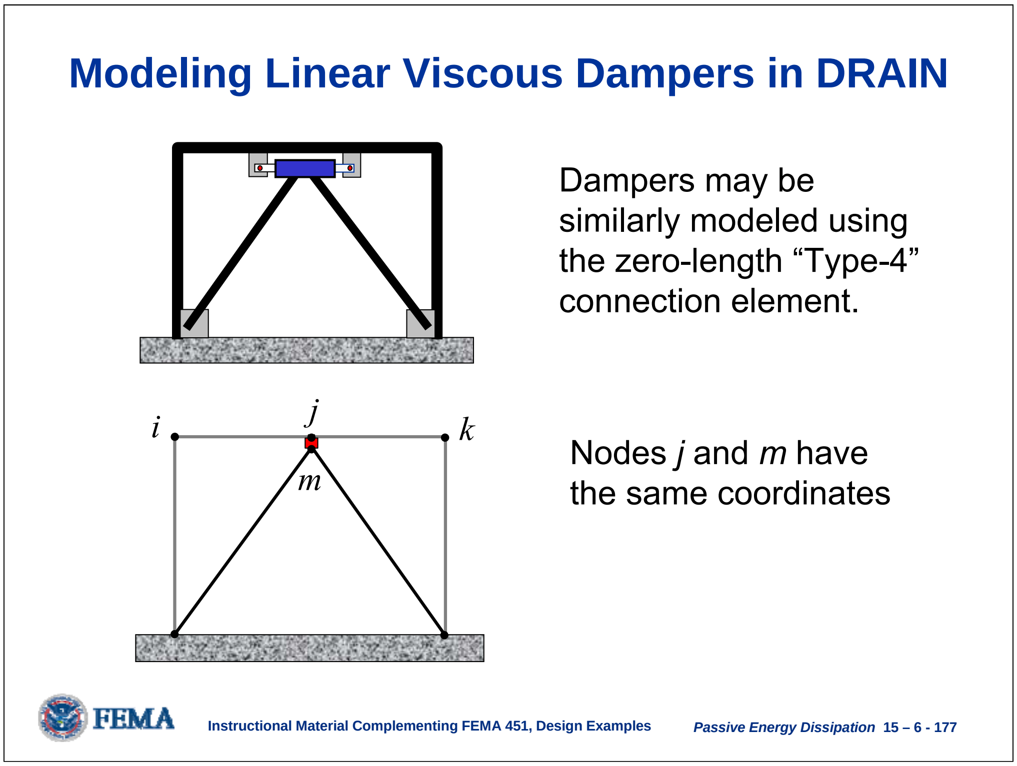
For dampers with low stiffness:
Set $A = L$, $E = 0.01$ and
$\beta = C_{Damper}/0.01$

Result:

$$K = 0.01 \qquad C = C_{Damper}$$

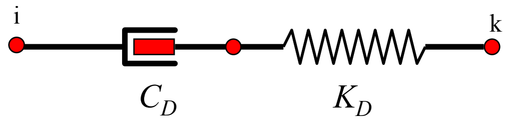
$$F = C\dot{u} = \beta K\dot{u} = C_{Damper}\dot{u}$$

Discrete linear viscous dampers may be readily modeled in DRAIN using a Rayleigh damping formulation wherein a Type-1 axial truss bar element is specified to have stiffness proportional damping. Selecting suitable values for the element parameters leads to an element with very low stiffness and linear viscous damping. If nonlinear viscous dampers need to be modeled, programs such as SAP2000, RAM Perform, or OpenSEES can be used.



Instead of the Type-1 truss bar element, a Type-4 “Simple Inelastic Connection Element” may be used. This element is a zero-length element.

Modeling Viscous/Viscoelastic Dampers Using the SAP2000 NLLINK Element



The damper is modeled as a Maxwell Element consisting of a linear or nonlinear dashpot in series with a linear spring.

To model a linear viscous dashpot, K_D must be set to a large value, but not *too* large or convergence will not be achieved. To achieve this, it is recommended that the relaxation time ($\lambda = C_D/K_D$) be an order of magnitude less than the loading time step Δt. For example, let $K_D = 100C_D/\Delta t$. Sensitivity to K_D should be checked.

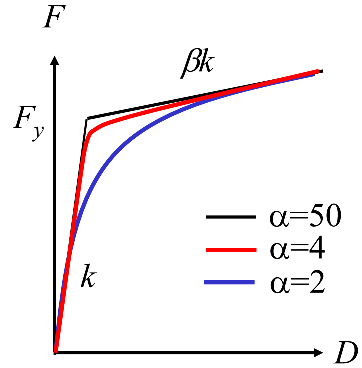
SAP2000 often has difficulty converging when nonlinear dampers are used and the velocity exponent is less than 0.4.

In SAP2000, the NLLINK (Nonlinear Link) element is used to model local nonlinearities. Nonlinear behavior of the element only occurs if nonlinear response-history analysis is performed. The NLINK element can be used to model linear viscous dampers, nonlinear viscous dampers, and viscoelastic dampers.

Modeling ADAS, Unbonded Brace, and Friction Dampers using the SAP2000 NLLINK Element

$$F = \beta k D + (1-\beta) F_y Z$$

$$\dot{Z} = \frac{k}{F_y}\begin{cases} \dot{D}\left(1-|Z|^{\alpha}\right) & if\ \dot{D}Z > 0 \\ \dot{D} & otherwise \end{cases}$$



Note: Z is an internal hysteretic variable with magnitude less than or equal to unity. The yield surface is associated with a magnitude of unity.

For bilinear behavior, use α of approximately 50. Larger values can produce strange results.

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 179

SAP2000 provides an NLLINK element for modeling plasticity. ADAS and unbonded brace dampers actually deform plastically and thus this element is directly applicable. For friction dampers, which do not deform plastically, this element can be used by setting the exponent alpha to a large value and the post-to-pre yielding stiffness ratio, β, equal to zero.

Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 180

No annotation is provided for this slide.

1993 - Tentative General Requirements for the Design and Construction of Structures Incorporating Discrete Passive Energy Dissipation Devices (1 of 3)

- Draft version developed by Energy Dissipation Working Group (EDWG) of Base Isolation Subcommittee of Seismology Committee of SEAONC (Not reviewed/approved by SEAOC; used as basis for 1994 NEHRP Provisions)

- Philosophy: For Design Basis Earthquake (10/50), confine inelastic behavior to energy dissipation devices (EDD); gravity load resisting system to remain elastic

- Established terminology and nomenclature for energy dissipation systems (EDS)

- Classified systems as rate-independent or rate-dependent (included metallic, friction, viscoelastic, and viscous dampers)

- Required at least two vertical lines of dampers in each principal direction of building; dampers to be continuous from the base of the building

- Prescribed analysis and testing procedures

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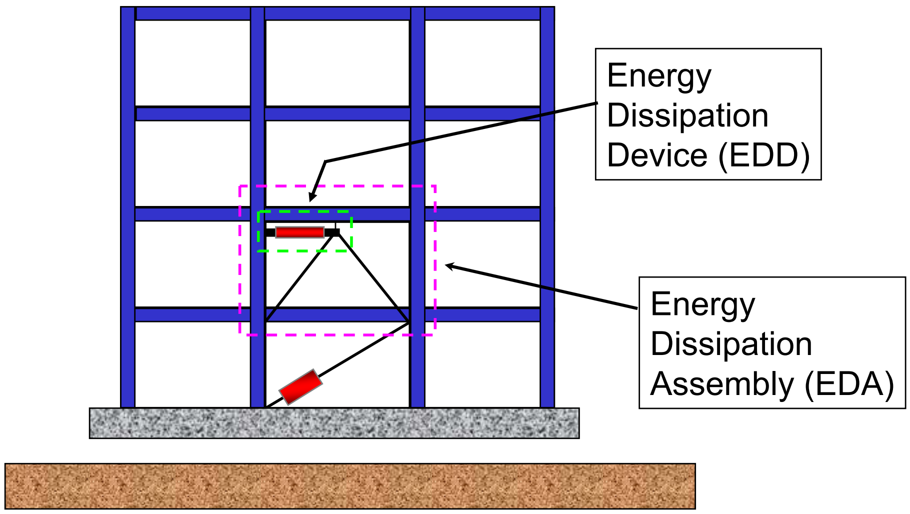
1993 - Tentative General Requirements for the Design and Construction of Structures Incorporating Discrete Passive Energy Dissipation Devices (2 of 3)



Energy Dissipation Nomenclature

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 182

No annotation is provided for this slide.

1993 - Tentative General Requirements for the Design and Construction of Structures Incorporating Discrete Passive Energy Dissipation Devices (3 of 3)

- Elastic structures with rate-dependent devices: Linear dynamic procedures (response spectrum or response history analysis)

- Inelastic structures or structures with rate-independent devices: Nonlinear dynamic response history analysis

- Prototype tests on full-size specimens (not required if previous tests performed and documented by ICBO)

- General acceptability criteria for energy dissipation systems:
    - Remain stable at design displacements
    - Provide non-decreasing resistance with increasing displacement (for rate-independent systems)
    - Exhibit no degradation under repeated cyclic load at design displ.
    - Have quantifiable engineering parameters

- Independent engineering review panel required to oversee design and testing

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 183

No annotation is provided for this slide.

1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (1 of 4)

Part 1 – Provisions & Part 2 – Commentary (FEMA 222A & 223A)

- Includes Appendix to Chapter 2 entitled: Passive Energy Dissipation Systems

- Material is based on:
  - 1993 draft SEAONC EDWG document
  - Proceedings of ATC 17-1 Seminar on Seismic Isolation, Passive Energy Dissipation, and Active Control (March 1993)
  - Special issue of Earthquake Spectra (August 1993)

- Applicable to wide range of EDD's; therefore requires EDD performance verification via prototype testing

- Performance objective identical to conventional structural system (i.e., life-safety for design EQ)

- At least two EDD per story in each principal direction, distributed continuously from base to top of building unless adequate performance (drift limits satisfied and member curvature capacities not exceeded) with incomplete vertical distribution can be demonstrated

- Members that transmit damper forces to foundation designed to remain elastic

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 184

No annotation is provided for this slide.

1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (2 of 4)

Part 1 – Provisions & Part 2 – Commentary (FEMA 222A & 223A)

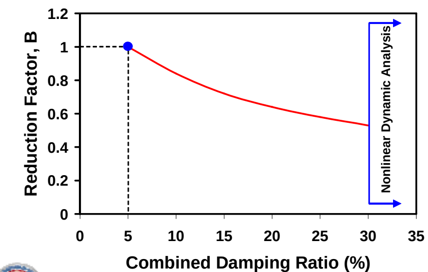
Analysis/Design Procedure for Linear Viscous Energy Dissipation Systems

$$V_{min} = BV = BC_S W$$

V_{min} = Minimum base shear for design of structure with EDS
[Use for linear static (ELF) or linear dynamic (Modal) analysis]

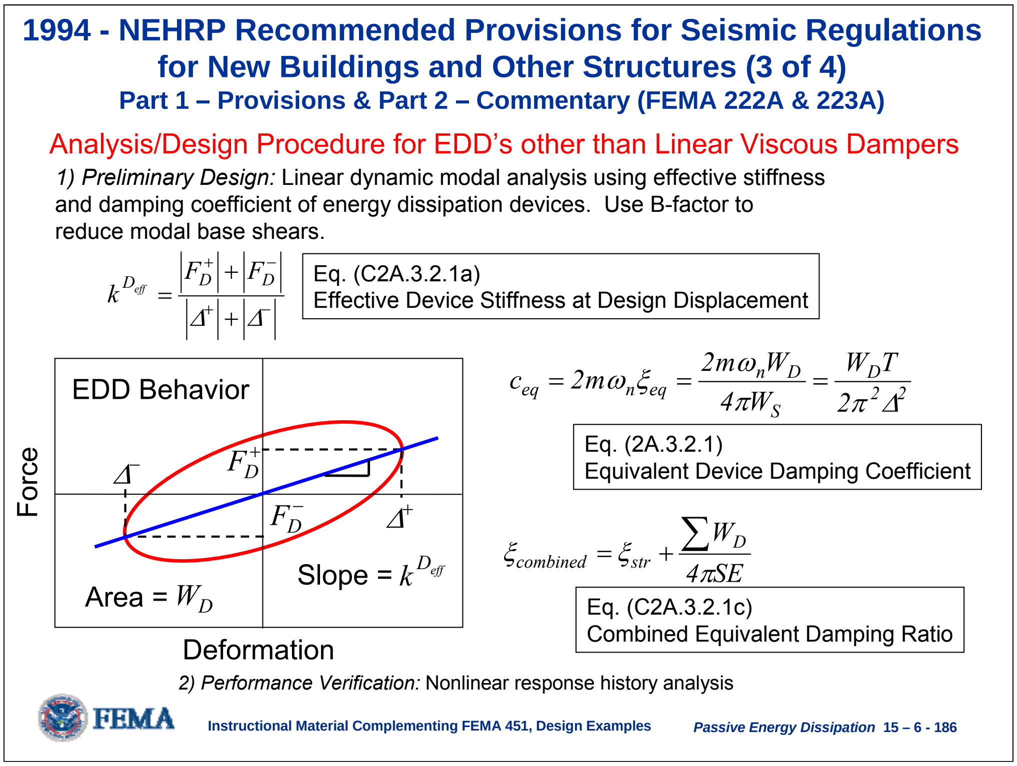
V = Minimum base shear for design of structure without EDS

B = Reduction factor to account for energy dissipation provided by EDS
(based on combined, inherent plus added damping, damping ratio)



Note: *After publication, it was recognized that this procedure may not be appropriate since it allows reduction in forces due to both inelastic structural response (R-factor) and added damping (B-factor). For yielding structures, added damping will not reduce forces.*

The Seismic Response Coefficient, C_s, includes the R-factor which reduces the elastic design forces to account for inelastic response in the structural framing. The weight, W, is the seismic weight which includes the dead load and applicable portions of other loads. The inherent damping ratio is typically taken as 5%. Thus, the reduction factor for a structure with no added damping is unity. For a structure with added damping, the reduction factor is smaller than unity to account for response reductions due to the added damping. However, after publication, it was recognized that the two force reduction factors, R and B, may produce unrealistically large force reductions.



For structures with added EDD's other than linear viscous dampers, a two-step procedure is used. The first step is a preliminary design in which linear dynamic modal analysis is performed using the effective EDD properties. The equivalent device stiffness is combined with the structural framing stiffness to obtain the combined stiffness. The equivalent device damping coefficient is combined with the inherent damping to obtain the combined equivalent damping ratio. Using the modal damping ratios, B-factors are determined and used to reduce the modal base shears. After the preliminary design is completed, the expected performance is verified via nonlinear response history analysis.

1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (4 of 4)

Part 1 – Provisions & Part 2 – Commentary (FEMA 222A & 223A)

- **For nonlinear response-history analysis, mathematical modeling should account for:**
  - **Plan and vertical spatial distribution of EDD's**
  - **Dependence of EDD's on loading frequency, temperature, sustained loads, nonlinearities, and bilateral loads**

- **Prototype Tests on at least two full-size EDD's (unless prior testing has been documented)**
  - **200 fully reversed cycles corresponding to wind forces**
  - **50 fully reversed cycles corresponding to design earthquake**
  - **10 fully reversed cycles corresponding to maximum capable earthquake**

- **Acceptability criteria from prototype testing of EDD's:**
  - **Hysteresis loops have non-negative incremental force-carrying capacities (for rate-independent systems only)**
  - **Exhibit limited effective stiffness degradation under repeated cyclic load**
  - **Exhibit limited degradation in energy loss per cycle under repeated cyclic load**
  - **Have quantifiable engineering parameters**
  - **Remain stable at design displacements**

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 187

No annotation is provided for this slide.

1997 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures

Part 1 – Provisions & Part 2 – Commentary (FEMA 302 & 303)

- Includes an appendix to Chapter 13 entitled:
Passive Energy Dissipation

- The appendix in the 1994 NEHRP Provisions was deleted since it was deemed to be insufficient for design and regulation. It was replaced with 3 paragraphs that provide very general guidance on passive energy dissipation systems.

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 188

No annotation is provided for this slide.

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (1 of 9)

- Chapter 9 entitled: Seismic Isolation and Energy Dissipation
  (Developed by New Technologies Team under ATC Project 33)

- Performance-based document
  - Rehabilitation objectives based on desired performance levels for selected hazard levels

- Global Structural Performance Levels
  - Operational (OP)
  - Immediate Occupancy (IO)
  - Life-Safety (LS)
  - Collapse Prevention (CP)

  } Most Applicable Performance Levels

- Hazard levels
  - Basic Safety Earthquake 1 (BSE-1): 10/50 event
  - Basic Safety Earthquake 2 (BSE-2): 2/50 event (Maximum Considered EQ - MCE)

- Rehabilitation Objectives
  - Limited Objectives (less than BSO)
  - Basic Safety Objective (BSO): LS for BSE-1 and CP for BSE-2
  - Enhanced Objectives (more than BSO)

  } Applicable Rehabilitation Objectives

Energy dissipation systems will generally have limited applicability for the Operational and Collapse Prevention Structural Performance Levels. The operational performance level may be difficult to achieve with an energy dissipation system since structures with energy dissipation systems are likely to experience some inelastic behavior and thus to not remain completely operational. The Collapse Prevention performance level may not warrant the cost of the energy dissipation system.

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (2 of 9)

-Simplified vs. Systematic Rehabilitation
- Simplified: For simple structures in areas of low to moderate seismicity
- Systematic: Considers all elements needed to attain rehabilitation objective

- Systematic Rehabilitation methods of analysis:
  - Linear static procedure (LSP)
  - Linear dynamic procedure (LDP)
  - Nonlinear static procedure (NSP) → Coefficient Method / Capacity Spectrum Method
  - Nonlinear dynamic procedure (NDP)

The Coefficient Method and Capacity Spectrum Method are discussed in detail within the Advanced Analysis slides.

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273) 1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (3 of 9)

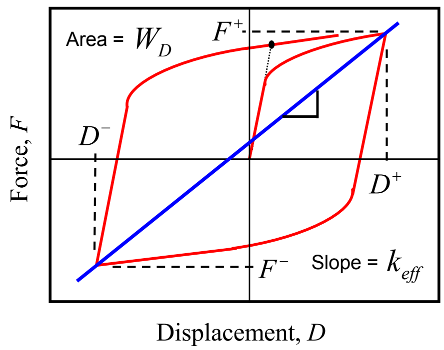
- **Basic Principles:**
  - Dampers should be spatially distributed (at each story and on each side of building)
  - Redundancy (at least two dampers along the same line of action; design forces for dampers and damper framing system are reduced as damper redundancy is increased)
  - For BSE-2, dampers and their connections designed to avoid failure (i.e, *not* weak link)
  - Members that transmit damper forces to foundation designed to remain elastic

- **Classification of EDD's**
  - Displacement-dependent
  - Velocity-dependent
  - Other (e.g., shape memory alloys and fluid restoring force/damping dampers)

Manufacturing quality control program should be established along with prototype testing programs and independent panel review of system design and testing program

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (4 of 9)

Mathematical Modeling of Displacement-Dependent Devices



$$F = k_{eff} D$$

Eq. (9-20) Force in Device

$$k_{eff} = \frac{|F^+| + |F^-|}{|D^+| + |D^-|}$$

Eq. (9-21) Effective Stiffness of Device

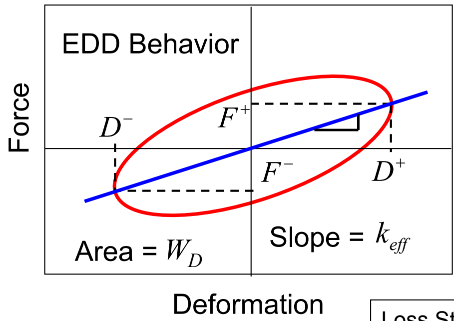
$$\beta_{eff} = \frac{1}{2\pi} \frac{W_D}{k_{eff} D_{ave}^2}$$

Eq. (9-39) Equivalent Viscous Damping Ratio of Device

The effective stiffness at the design displacement is taken as the average peak force divided by the average peak displacement. The equivalent viscous damping ratio as written above uses a different symbol for the average displacement than that given in the NEHRP document.

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (5 of 9)

Mathematical Modeling of Solid Viscoelastic Devices



$$F = k_{eff} D + C\dot{D}$$

Eq. (9-22) Force in Device

$$k_{eff} = \frac{|F^+| + |F^-|}{|D^+| + |D^-|} = K'$$

Eq. (9-23) Effective Stiffness of Device

Storage Stiffness

Loss Stiffness

$$C = \frac{W_D}{\pi \omega_1 D_{ave}^2} = \frac{K''}{\omega_1}$$

Eq. (9-24) Damping Coefficient of Device

Average Peak Displ.

Circular frequency of mode 1

A viscoelastic damper is modeled as a Kelvin element. The effective stiffness at the design displacement is taken as the average force at the peak displacement divided by the average peak displacement. The effective stiffness is equal to the storage stiffness at the frequency of the test (usually the fundamental frequency of the building, with due account given to the additional stiffness provided by the dampers). The damping coefficient is equal to the loss stiffness divided by the circular frequency of the fundamental mode of vibration.

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (6 of 9)

Mathematical Modeling of Fluid Viscoelastic and Fluid Viscous Devices

Fluid Viscoelastic Devices:

$$F + \lambda \dot{F} = C\dot{D}$$

Maxwell Model

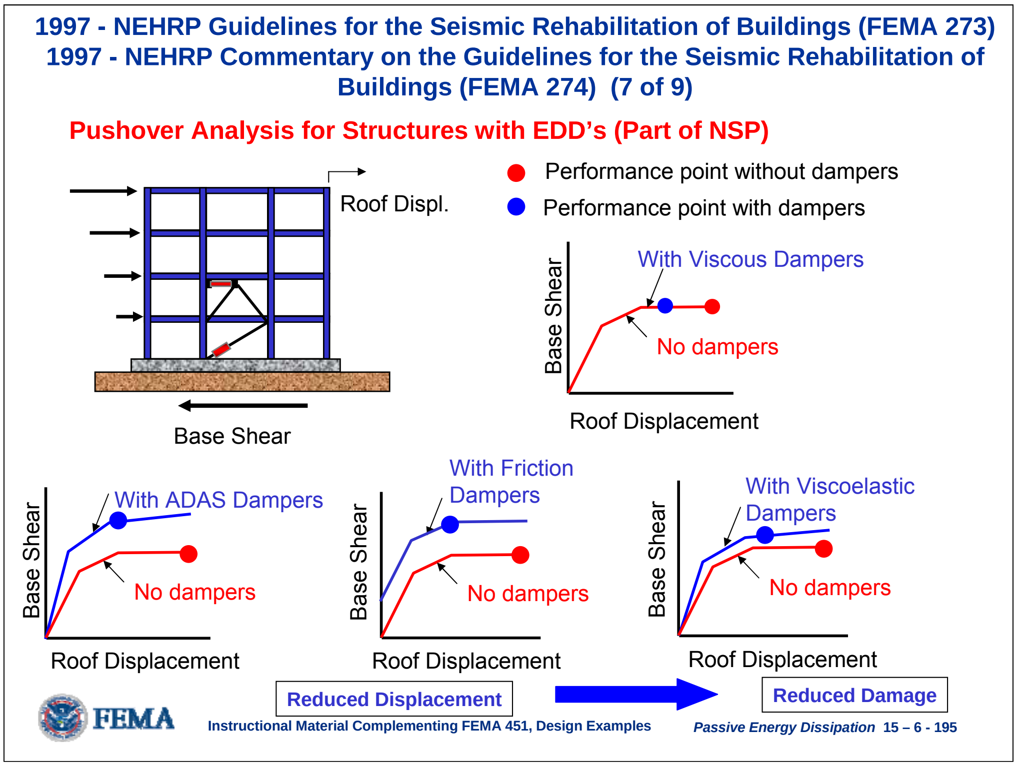
Fluid Viscous Devices:

$$F = C_0 \left| \dot{D} \right|^{\alpha} sgn\left(\dot{D} \right)$$

Eq. (9-25)
Linear or Nonlinear Dashpot Model

<u>Caution:</u> Only use fluid viscous device model if $K' = 0$ for frequencies between 0.5 f_1 and 2.0 f_1; Otherwise, use fluid viscoelastic device model.

If the storage stiffness is equal to zero over a sufficiently wide frequency range around the fundamental frequency of the building, a fluid viscous damper model may be used. Otherwise, a Maxwell model of viscoelasticity must be used.



For the Nonlinear Static Analysis Procedure (a procedure that is emphasized in the NERHP Guidelines), the capacity of the structural system is assessed via a nonlinear static pushover analysis. The pushover curve is affected by EDD's in accordance with the EDD's hysteretic response in the first quadrant of the force-displacement plane. For elastic systems, EDD's reduce both displacements and forces whereas, for systems responding inelastically, displacements are reduced but forces may be increased.

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) 8 of 9)

Design Process for Velocity-Dependent Dampers using NSP

Steps
1) Estimate Target Displacement (performance point)
2) Calculate Effective Damping Ratio and Secant Stiffness of building with dampers at Target Displacement
3) Use Effective Damping and Secant Stiffness to calculate revised Target Displacement
4) Compare Target Displacement from Steps 1 and 4.
   If within tolerance, stop. Otherwise, return to Step 1.

$$\beta_{eff} = \beta + \frac{\sum_j W_j}{4\pi W_k}$$

Effective damping ratio of building with dampers at Target Displ.; j = index over devices

$$W_k = \frac{1}{2}\sum_i F_i \delta_i$$

Maximum strain energy in building with dampers at Target Displ.; i = index over floor levels

The basic steps in the design process for buildings with velocity-dependent dampers using the Nonlinear Static Procedure is shown above.

1997 - NEHRP Guidelines for the Seismic Rehabilitation of Buildings (FEMA 273)
1997 - NEHRP Commentary on the Guidelines for the Seismic Rehabilitation of Buildings (FEMA 274) (9 of 9)

Design Process for Velocity-Dependent Dampers using NSP (2)

$$W_j = \frac{2\pi^2}{T_S} C_j \delta_{rj}^2$$

Work done by j-th damper with building subjected to Target Displacement (assumes harmonic motion with amplitude equal to Target Displacement and frequency corresponding to Secant Stiffness at Target Displacement)

$$\beta_{eff} = \beta + \frac{T_S \sum_j C_j \cos^2 \theta_j \phi_{rj}^2}{4\pi \sum_i m_i \phi_i^2}$$

Alternate expression for Effective Damping Ratio that uses modal amplitudes of first mode shape

Checking Building Component Behavior (Forces and Deformations)

For velocity-dependent dampers, must check component behavior at three stages:
1) Maximum Displacement
2) Maximum Velocity
3) Maximum Acceleration

For buildings with velocity-dependent dampers, must check component behavior at three stages since the maximum force output of a velocity-dependent damper may occur at any one of these three stages. In contrast, for a building with displacement-dependent dampers, the component behavior is only checked at the maximum displacement since the maximum force coincides with the maximum displacement.

2000 – Prestandard and Commentary for the Seismic Rehabilitation of Buildings (FEMA 356)

- Prestandard version of 1997 NEHRP Guidelines and Commentary for the Seismic Rehabilitation of Buildings (FEMA 273 & 274)
- Prepared by ASCE for FEMA
- Prestandard = Document has been accepted for use as the start of the formal standard development process (i.e., it is an initial draft for a consensus standard)

No annotation is provided for this slide.

2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (1 of 8)

Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

- **Appendix to Chapter 13 entitled *Structures with Damping Systems***
  (completely revised/updated version of 1994 and 1997 Provisions; Brief commentary provided)

- **Intention:**
  - **Apply to all energy dissipation systems (EDS)**
  - **Provide design criteria compatible with conventional and enhanced seismic performance**
  - **Distinguish between design of members that are part of EDS and members that are independent of EDS.**

- **The seismic force resisting system must comply with the requirements for the system's Seismic Design Category, except that the damping system may be used to meet drift limits.**

  No reduction in detailing is thereby allowed, even if analysis shows that the damping system is capable of producing significant reductions in ductility demand or damage.

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 199

No annotation is provided for this slide.

2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (2 of 8)

Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

- **Members that transmit damper forces to foundation designed to remain elastic**

- **Prototype tests on at least two full-size EDD's (reduced-scale tests permitted for velocity-dependent dampers)**

- **Production testing of dampers prior to installation.**

- **Independent engineering panel for review of design and testing programs**

- **Residual mode concept introduced for linear static analysis. This mode, which is in addition to the fundamental mode, is used to account for the combined effects of higher modes. Higher mode interstory-velocities can be significant and thus are important for velocity-dependent dampers.**

No annotation is provided for this slide.

2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (3 of 8)

Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

Methods of Analysis:

- Linear Static (Equivalent Lateral Force*)
  - *OK for Preliminary Design*
- Linear Dynamic (Modal Response Spectrum*)
  - *OK for Preliminary Design*
- Nonlinear Static (Pushover*)
  - *May Produce Significant Errors*
- Nonlinear Dynamic (Response History)
  - *Required if S_1 > 0.6 g and may be used in all other cases*

*The *Provisions* allow final design using these procedures, but only under restricted circumstances.

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 201

No annotation is provided for this slide.

2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (4 of 8)

Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

Effective Damping Ratio
(used to determine factors, B, that reduce structure response)

$$\beta_m = \beta_I + \underbrace{\beta_{Vm}\sqrt{\mu}} + \beta_H$$

Hysteretic Damping Due to Post-Yield Behavior in Structure

Equivalent Viscous Damping of EDS in the m-th Mode

Inherent Damping Due to Pre-Yield Energy Dissipation of Structure
(β_I = 5% or less unless higher values can be justified)

Effective Damping Ratio in m-th mode of vibration

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 202

No annotation is provided for this slide.

2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (5 of 8)

Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

Equivalent Viscous Damping from EDS

$$\beta_m = \beta_I + \boxed{\beta_{Vm}\sqrt{\mu}} + \beta_H$$

$$\beta_{Vm} = \frac{\sum_j W_{mj}}{4\pi W_m}$$

Equivalent Viscous Damping in m-th mode (due to EDS)

$$W_m = \frac{1}{2}\sum_i F_{im}\delta_{im}$$

Maximum Elastic Strain Energy of structure in m-th mode

$$\sqrt{\mu}$$

Adjustment factor that accounts for dominance of post-yielding inelastic hysteretic energy dissipation

No annotation is provided for this slide.

2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (6 of 8)

Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

Base Shear Force

Minimum base shear for design of structure without EDS

$$V_{min} = max\left\{ \frac{V}{B_{V+I}} ;\ 0.75V \right\}$$

Minimum base shear for design of seismic force resisting system

Spectral reduction factor based on the sum of viscous and inherent damping

To protect against damper system malfunction, maximum reduction in base shear over a conventional structure is 25%

FEMA Instructional Material Complementing FEMA 451, Design Examples *Passive Energy Dissipation* 15 – 6 - 204

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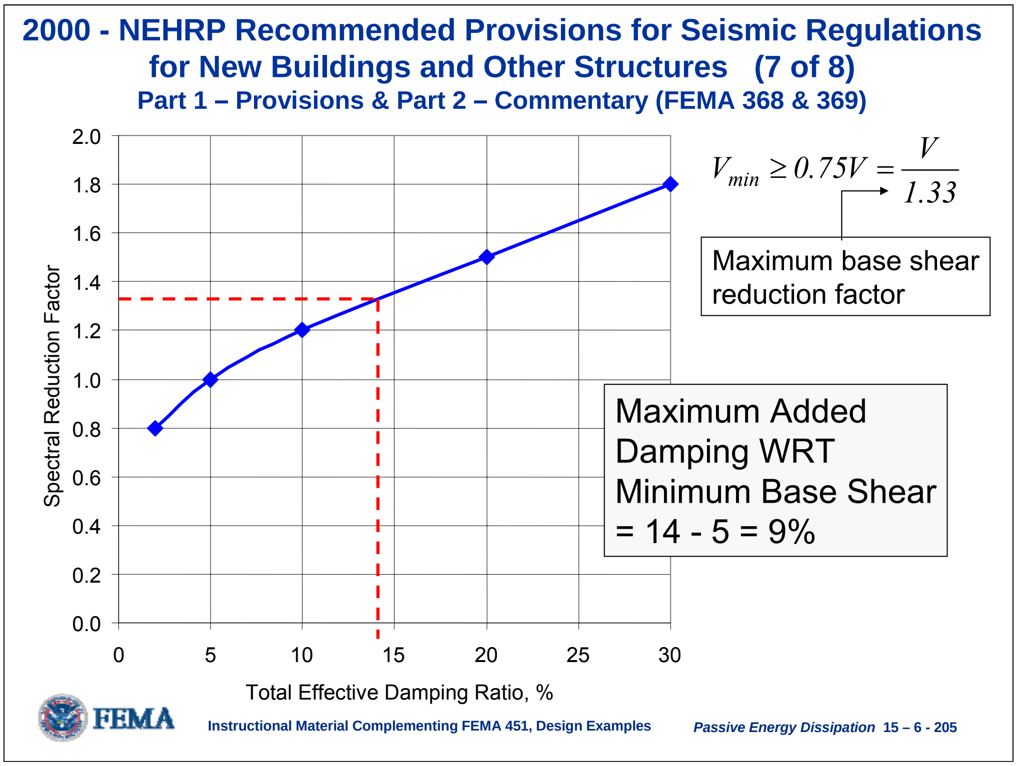
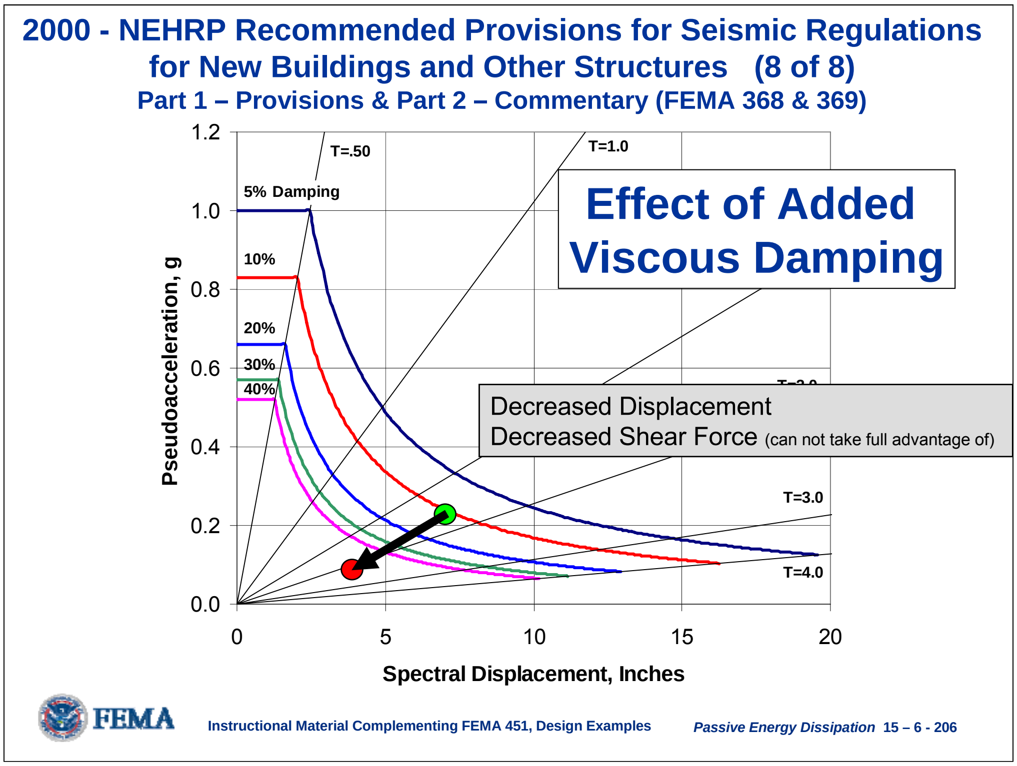


The added damping in a structure allows the base shear force to be reduced by up to 25%. The limit is placed on the maximum reduction to ensure that the damped structural system performs as intended with limited possibility of damper malfunction. The 25% reduction is equivalent to a base shear reduction factor (or spectral reduction factor) of 1.33. This reduction factor corresponds to a total effective damping ratio of about 14%. If 5% inherent damping is assumed, the added damping (assuming elastic behavior and thus no inelastic hysteretic damping) associated with the absolute minimum base shear is 9%. Of course, higher levels of damping can be added to structures, the major benefit being a reduction in displacement demands and, thus, the ability to meet the allowable story drift limits.



The effect of increasing viscous damping from 5% (inherent) to 30% for an elastic system is shown above using an ADRS demand spectrum. In general, the additional damping reduces both force and displacement demands on the structure. The NEHRP Provisions limit the amount of reduction in force that may be used for design. The reduced spectral displacement, however, is useful for meeting the allowable story drift limits.