

This topic addresses structural analysis requirements in performance-based earthquake engineering. Such analysis must typically include a variety of nonlinear effects, both material and geometric. This topic provides an overview of nonlinear analysis methodologies.



This is a summary of the topics covered. It should be emphasized that this slide (third bullet) and subsequent slides use the terminology "Response History Analysis" instead of "Time History Analysis". Response history is a more accurate description of what is being done. Analyzing the history of time makes little sense, whereas the history of the response of a structure is meaningful.

Incremental Dynamic Analysis (IDA) is a relatively new approach that uses response history analyses in a systematic manner to assess the behavior of a structure subjected to a suite of ground motions. The structure is repeatedly analyzed for each motion scaled for gradually increasing intensities. For each intensity analyzed, certain damage measures are recorded and plotted against intensity to produce "IDA Curves".

We will only briefly review the probabilistic approaches because the approach is quite new, and has not been fully formulated. We will describe the probabilistic approach used by FEMA 350, and the broader approach being suggested by PEER.



It must be made very clear that the purpose of analysis (in the context of this Topic) is NOT to accurately predict the response of a certain structure to a certain ground motion. This is impossible due to the large number of uncertainties in modeling, loading, analysis, and interpretation of results. Also, what is being predicted will never occur since the actual ground motion is not known.

What we are trying to do is to use analysis to get a handle on the likely behavior of a structure, and to estimate whether or not such behavior will meet pre-established performance objectives.



In earthquake engineering it has been said that "Strength is essential, but otherwise unimportant". This is true because the basic requirement in seismic resistant design is that deformation demand must be less than or equal to the deformation capacity of the system's elements and components. The design objective is to provide sufficient strength to keep deformation demands below the capacity.

In structural analysis for performance-based engineering, therefore, the emphasis is on predicting deformation demands. Because the response is almost certainly inelastic, the analysis must explicitly include inelastic effects. Thus the first four analysis methods listed are not applicable.

Pushover analysis was originally adopted as a "practical" replacement for more time-consuming response history analysis. However, the method has its limitations, and is falling out of favor with many researchers.

While nonlinear response history analysis has certain advantages, the down side is that multiple ground motions must be considered, and that response can vary widely for the same system analyzed for a suite of reasonably scaled motions. Response can also vary considerably for minor variations of the same system responding to the same ground motion.

FEMA 368 Analysis Requirements (SDC D, E, F)			Analysis Method				
			Linear Static	Response Spectrum	Linear Resp. Hist.	Nonlinear Resp. Hist.	
		Regular Structures	YES	YES	YES	YES	
	$T \leq T_s$	Plan Irreg. 2,3,4,5 Vert. Irreg. 4, 5	YES	YES	YES	YES	
		Plan Irreg. 1a ,1b Vert. Irreg. 1a, 1b 2, or 3	NO	YES	YES	YES	
	All Other Structures		NO	YES	YES	YES	
	Nonl	inear Static Analys	sis Limit	ations r	not State	ed	
🧱 FEMA	Instructional Materia	I Complementing FEMA 451, D	esign Examp	lles	Methods of A	Analysis 15-5a - 5	

The 2003 *NEHRP Recommended Provisions* is not a performance-based document and, hence, has no requirements for nonlinear analysis. An Appendix to Chapter 5 of the *Provisions,* however, uses nonlinear analysis in the context of a FEMA 273 nonlinear static pushover approach.

Note that  $T_s$  is the point on the design spectrum where the constant acceleration portion of the spectrum crosses the constant velocity (inversely proportional to T) portion of the spectrum.  $T_s=S_{D1}/S_{DS}$ 

Requi Collapse	rements Preventi	on)	Linear Static	Linear Dynamic	Nonlinear Static	Nonlinear Dynamic
$T \leq T_s$	Deculor	Strong Column	YES	YES	YES	YES
	Regular	Weak Column	NO	NO	YES	YES
	Irregular	Any Condition	NO	NO	YES	YES
$T > T_s$	Regular	Strong Column	NO	YES	NO	YES
		Weak Column	NO	NO	NO	YES
	Irregular	Any Condition	NO	NO	NO	YES

FEMA 350 is a performance-based document, and has specific requirements for nonlinear analysis. Note the situations where nonlinear dynamic response history analysis is *required*. The first table is for short period buildings and the second is for longer period buildings.



FEMA 273 uses terminology which may be confusing. An "Element" is really a system, such as a moment frame, braced frame, and so-on. A "Component" is a particular member of the "Element". The confusion lies in the fact that the word element is commonly used to refer to an individual member in the context of a finite element analysis.



This list gives the idealized requirements of an analytical model. Unfortunately, sufficient information is often not available, and when the information is available, very significant uncertainties make choices difficult. If the certainties can be identified and quantified, several analyses with a variety of properties may be required to adequately bound the response.



Three dimensional nonlinear dynamic analysis is becoming more available with the release of SAP 2000 Version 8, as well as a 3D version of RAM Perform. However, it will still take several years for these programs to supercede DRAIN 2Dx and perhaps RAM Perform 2D as the "state of the practice".



Running a nonlinear dynamic response history analysis of a structure is one of the most complex tasks a structural engineer has to do. Many engineers are too rushed to perform the analysis, and do not take the time to perform the steps outlined on this and the next slide. The result can be a meaningless analysis.

Before any nonlinear analysis is run, a linear analysis must be performed to check the model. Similarly, before any dynamic analysis is run a static analysis must be performed on the same model. After each analysis a reasonableness check must be performed. Are the frequencies and mode shapes realistic. Do P-Delta effects make the period longer? How does the presence of gravity loads affect hinge sequencing? Do the results of the pushover analysis depend on the size of the load step? Does the dynamic pulse loading produce the appropriate free vibration response (check period and damping). This is only a short list of items that should be checked.



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The two basic element modeling types are Phenomenological and Macroscopic. Phenomenological models, explained here, are much more practical and are the norm for most nonlinear dynamic response history programs.



Phenomenological models are typically used to represent a region of a component, such as a plastic hinge in a beam. Modeled behavior may include axial-flexural-shear interaction, or may be limited to simple flexural behavior as shown here. Because all of the inelastic activity is limited to the (typically zero length) hinge, a phenomenological model may also be referred to as a "lumped" plasticity model. In the diagram shown above the expected plastic behavior at each end of the beam is modeled as a simple plastic hinge. Note that the hinge is located some distance in from the ends of the beam.

## **Basic Component Model Types**

## Macroscopic

The yielding regions of the component are highly discretized and inelastic behavior is represented at the material level. Axial-flexural interaction is handled automatically.

These models are reasonably accurate, but are very computationally expensive. Pushover analysis may be practical for some 2D structures, but nonlinear dynamic time history analysis is not currently feasible for large 2D structures or for 3D structures.

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A macroscopic model attempts to represent behavior down to the "fiber" level. These models have the advantage of automatically including some aspects of complex behavior, such as axial-flexural interaction. Unfortunately, macroscopic models are still prohibitively expensive if used on a large scale. However, it may be reasonable to use a mixture of phenomenological and macroscopic models in a single structure. For example, simple plastic hinges may be used at the ends of beams, and a more refined fiber model used at the base of a critical shear wall.

FEMA Instructional Material Complementing FEMA 451, Design Examples



Here, a plastic hinge is represented as a series of slices (along the length of the beam) and a series of layers (through the depth of the beam). For a concrete structure special layers may be used to represent both concrete and reinforcing steel with different constitutive laws used for unconfined concrete, confined concrete, and steel. This type of model automatically represents growth in the length of plastic hinges, as well as neutral axis migration. Some analysts refer to this type of model as a spread plasticity model.



Whether used to represent an entire plastic hinge or a single fiber, it is necessary to have computational rules for tracking hysteretic behavior. There are a nearly infinite number of behaviors that can be so represented. These models represent stable hysteretic behavior (left; an unbonded brace, for example) and a system with gradual strength loss (right: a plastic hinge in a wide-flange beam, for example).



Hysteresis rules may also include loss of stiffness with sustained strength (left: a well confined reinforced concrete beam) or a loss of both strength and stiffness (right: a poorly confined concrete beam).



Hysteresis rules may also include pure pinching (left: a self-centering device) or buckling and tension yielding (right: a slender brace).



Sivaselvan and Reinhorn have developed a nice family of multilinear models. This slide is a screen capture from NONLIN's MDOF modeler.



There are also a variety of smooth models. The one shown here is used in SAP2000. Much more elaborate models are available. The Sivaselvan/Reinhorn smooth models have been incorporated into NONLIN's MDOF model.



Basic info on NONLIN-Pro (1).



Basic info on Nonlin-Pro (2).



Basic info on Nonlin-Pro (3).



Basic info on Nonlin-Pro (4).

<b>DRAIN-2DX</b> Element Library
<b>TYPE 1: Truss Bar</b> <b>TYPE 2: Beam-Column</b> TYPE 3: Degrading Stiffness Beam-Column* <b>TYPE 4: Zero Length Connector</b> TYPE 6: Elastic Panel TYPE 6: Elastic Panel TYPE 9: Compression/Tension Link TYPE 15: Fiber Beam-Column*
* Not fully supported by Nonlin-Pro
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This in the NONLIN-Pro element library. All of the elements are provided by DRAIN 2Dx, but only those indicated elements are supported by the graphic pre- and post processors. The fiber element is supported but is not particularly dependable.



The axial truss bar element is useful for any kind of axial link, and is also used to model linear viscous damper elements. The damper element is modeled by setting a very low stiffness, and then setting a very high stiffness proportional damping constant (beta). The product of beta and the stiffness is the desired damping constant C (units = force-time/length). Note that the Type 4 element may be used in lieu of the truss element when a zero-length device is required.



This is the "standard" beam-column element provided by DRAIN. The only advantage of this element is it's ability to model axial-flexural interaction. Unfortunately, the interaction model does not work very well. For beams with flexural yielding that is independent of axial force, it is better to explicitly model the hinges using type-4 elements as explained later.



This is the theoretical axial-flexural interaction relationship for the twocomponent beam-column element. When the load path intersects the yield surface, a *flexural* hinge is placed in the yielding component of the element. While yielding, the element is required to load along the path as indicated.



For beams, the element simply yields when the moment at an end of the element reaches the flexural yield point.



Self explanatory.



The connection element is one of the most important elements in DRAIN or in any nonlinear analysis program. This is a zero length element, and is therefore connected to two nodes that share the same X-Y coordinates. These nodes are referred to as "compound nodes".



This slide illustrates the use of a pair of nodes and a connection element to model a plastic hinge. A real moment-free hinge may be modeled by elimination of the rotational spring. The last bullet item is a preliminary caution. The reason for the caution will be explained in some detail later in the topic.



Here are two applications for a compound node. In the first case, compound nodes with rotational springs are placed at each end of a beam. In the second case, four compound node sets are used to develop a "Krawinkler" beam-column joint deformation model. The Krawinkler model and a much simpler Scissors model are described in some detail later.



When modeling beams it is preferable to use a concentrated plastic hinge. This is done using rotational connection elements in DRAIN, and is done automatically in PERFORM. The key point here is that 100% of the inelastic rotation is assumed to occur in the rotational plastic hinges. The initial stiffness of the rotational spring should be set to a large number (say 1000\*4EI/L).

Note that DRAIN does not have the capability to model loss of strength after first yielding. RAM-Perform does. This capability is only important for modeling existing buildings that are not expected to perform well.



This slide is an image capture from the NONLIN-Pro program. It shows the modeling of beam-column joints using the Krawinkler model and Girder Hinges.



The program that may ultimately replace DRAIN is the OpenSees environment being developed at PEER (primarily Professor Fenves at Berkeley and Professor Deierlein at Stanford). This is an open-source object oriented C++ code. Getting into the "guts" of the program is not for the timid (even though the OpenSees web site gives a pretty good description of the development environment). The web site has a complete users manual and several examples. Pre-and post-processing still leave a lot to be desired.

The NEES equipment sites that utilize hybrid resting (e.g. UC Boulder) may rely on OpenSees as the analytical counterpart to the physical testing equipment.


It is envisioned that OpenSees will replace DRAIN-2Dx as the analysis method of choice among researchers. This will happen with time, but the program has a way to go before it can be readily accepted.



One of the strengths of OpenSees is also a weakness. It is written in object oriented C++ which is good from the original programmer's prospective, but is not so good for the typical engineer that would like to get his or her hands into the code. Few engineers master C++, not to mention the object oriented programming concepts necessary for contributing to the OpenSees project.









One of the strengths of OpenSees is the large variety of solvers available.







No annotation. Provided for student reference only.

## **Other Commercially Available Programs**

## SAP2000/ETABS

Both have 3D pushover capabilities and linear/nonlinear dynamic response history analysis. P-Delta and large displacement effects may be included. These are the most powerful commercial programs that are specifically tailored to analysis of buildings(ETABS) and bridges (SAP2000).

## **RAM/Perform**

Currently 2D program, but a 3D version should be available soon. Developed by G. Powell, and is based on DRAIN-3D technology. Some features of program (e.g. model building) are hard-wired and not easy to override.

## ABAQUS, ADINA, ANSYS, DIANA, NASTRAN

These are extremely powerful FEA programs but are not very practical for analysis of building and bridge structures.



SAP2000 has grown tremendously in power with the release of Version 8. The most important new capabilities include full nonlinear dynamic response history analysis and large displacement effects.

Programs like ABAQUS are very powerful, but typically do not have capabilities to easily model building and bridge type structures. Deficiencies include lack of standard section databases, lack of phenomenological models, inability to conveniently apply ground motions, and inability to apply load combinations. Also, these programs are very expensive (ABAQUS is \$25K per year), and VERY HARD TO LEARN.



Modeling of the beam-column joint region in steel moment frames is presented in the next several slides. The image shown on this slide is an image from ANSYS showing the shear stresses in a typical subassemblage. Note the very high shear stress in the panel zone region. Such stresses and associated strains may be responsible for as much as 40 percent of the total drift in steel frame structures.



This is a typical interior subassemblage. Note the dimensionless terms alpha and beta. The effective girder and column depth is taken as the distance between flanges.

Note that the columns always pass through the floor, and that the continuity plates are almost always present. Current AISC seismic provisions call for a strong panel zone, so the doubler plate will often be present. Such plates are extremely effective in reducing beam-column joint deformations. Unfortunately the cutting and welding of the plates is very expensive.



Here the forces on the joint are determined. The main simplifying assumption is that the girder and column moments may be represented by a couple with all of the moment being resisted by flange forces.



The shear force in the panel zone is given by the upper equation. It is easy to see that the shear force in the joint may be several times the shear force in the column above and below the joint.



Points are self explanatory.



ANSYS results illustrate the previous points.



This slide is a composite of the ANSYS results of a single subassemblage. Note that the dominate source of deformation in this frame is shear deformation in the panel zones and in the webs of the beams.



One of the best idealizations for beam-column behavior is the model developed by Krawinkler. The basic model consists of four links which frame the joint. The links are connected at the corners by true hinges or by rotational springs. The stiffness and strength of the joint is represented by these springs.



This illustration shows the Krawinkler model in its deformed state (with the beams and columns remaining relatively rigid). Note how the joint "rotates" in the opposite direction than would be expected. Note also that significant "offsets" occur in the centerlines of the columns and girders. As mentioned later, this kinematic effect does not occur in the simpler "Scissors Model".



The Krawinkler Model is a phenomenological model of a beam column joint. When modeled in DRAIN it consists of a "frame" of Type-2 beam-column elements connected at the four corners by compound nodes. The upper left compound node utilizes a rotational Type 4 spring to represent the panel zone web stiffness and strength. The lower right compound node utilizes a Type-4 rotational spring to represent column flange contributions. The other two compound nodes are simple flexural hinges.



It takes twelve nodes to represent a single Krawinkler joint. Note that the corners each contain two nodes that have constrained X-Y degrees of freedom and independent rotational degrees of freedom.



If no constraints are used the Krawinkler model has 28 degrees of freedom. However, only four of the degrees of freedom are truly independent. These degrees of freedom are rigid body X and Y translation, rigid body rotation, and racking. Unfortunately, DRAIN makes it difficult to impose the constraints required to minimize the number of degrees of freedom. Fortunately. experience has shown that reasonable solution times can be obtained in response history analysis of multiple story-multiple bay frames.



This slide shows the simple moment-rotation relationships for the two rotational springs in the Krawinkler Model. The Panel Component (shown in red) represents the stiffness and strength of the panel zone, including the doubler plate if present. The Flange component (shown in blue) arises from the eventual formation of plastic hinges in the flanges of the columns the panel tries to rack. In the model shown here the flange component contributes to the initial stiffness of the joint. On the basis of test results it is typically assumed that the flange components yields at four times the yield rotation of the panel component.



An alternate model assumes that the flange component has no initial stiffness, picking up force only after the panel component has yielded.



This slide shows the required properties for the Panel Component. Note that  $t_{wc}$  and  $t_d$  are the thicknesses of the web of the column and the doubler plate, respectively.



It is easy to memorize the formulas for the panel component if it is recognized that the grouped terms represent the volume of the panel zone.



Here the properties of the rotational spring used to represent the flange component are shown. The stiffness of this component is back-calculated from the rotation relationship.



This slide lists the main advantages of the Krawinkler Model.



This slide lists the main disadvantages of the Krawinkler Model.



An attractive alternative to the Krawinkler Model is the so-called Scissors Model. Here, a single compound node is used to represent the panel zone. The rigid end zones are extensions of the beams and columns that frame into the joint. A pair of rotational springs are used to represent panel zone and column flange effects.



The kinematics of the Scissors Model is quite different that that of the Krawinkler Model. This is seen more clearly on the next slide.



Here the kinematic differences between the two models can be seen more clearly. Note that the centerline offsets in the Krawinkler model are not evident in the Scissors Model. Interestingly, exhaustive testing using DRAIN 2D has shown that the kinematic differences do not have a significant effect on the response.



The properties for the Scissors model are most conveniently derived on the basis of the equivalent Krawinkler properties. It is very important to note that many engineers are under the impression that the Scissors model properties are identical to the Krawinkler properties. This is NOT TRUE as indicated by the equations on this slide.



Self explanatory. It should be noted that a method has been developed (by Professor Charney) to include panel zone flexural deformations in both the Krawinkler and Scissors models. Before this approach may be released however, careful calibration with FEA models is required, and this calibration is not yet complete.



Modeling of joint deformations in concrete structures is much more difficult than in steel structures.



It is essential that P-Delta effects be included in **any** nonlinear analysis. While such effects may have a negligible influence on an elastic response, the influence on the inelastic response of the same structure may be profound. This is particularly true if the structure has little overstrength and if the post-elastic portion of the pushover curve is nearly flat or is descending. Hence the revised recommendation.


In the simplest sense, P-Delta effects lead to a reduction in stiffness and strength of a structure. In this slide a linearized version of the P-Delta effect is shown. The term  $K_G$  refers to the "Linearized Geometric Stiffness" of the structure. The term LINEAR is used because it is assumed the column has a straight-line deflection for consideration of P-Delta effects. Hence, only the rigid-body rotation of the column is considered in the formulation. The actual deformation of the column is not included.



P-Delta effects also reduce the lateral strength of a structure. In essence, the P-Delta effects are imposing a lateral load on the structure, hence, it takes a lower additional lateral load to cause yielding.



The most profound influence of the P-Delta effect is on the dynamic response of structures. This plot shows the response history of a simple SDOF system with three different assumptions regarding the post-yield stiffness. All three systems have the same yield strength. A slightly decreased KG (to a value of -75 k/in) may have caused a complete dynamic instability of the system.



Most analyst include only the "Large" P-Delta effect as shown at the left. This model does a good job of representing the story effect, but does not represent the additional softening that can be produced by consideration of the actual deformation of the component (as shown at the right).



DRAIN 2D and Perform include only the linearized geometric stiffness. This is probably sufficient because most of the structural deformation (interstory drift) is due to rotation in plastic hinges... the columns stay relatively straight between ends and are in double curvature (minimizing the magnitude of the small delta).

The equilibrium equations are formulated on the basis of the undeformed geometry of the structure. Hence, large displacement effects are not considered. This is not a significant source of error in the analysis of framed structures.

Iteration is not required when linearized geometric effects are included because it is the total story (gravity) load that induces the instability. Under lateral load the sum of column forces in a story is zero, hence there is no story P-delta effect.

Linearized geometric stiffness should never be used when computing elastic buckling loads in structures. Buckling loads so predicted may be much higher than the actual buckling load. Improved accuracy may be obtained by subdividing the columns into several (at least four) segments.



Consistent geometric stiffness uses cubic polynomials to represent the displaced shape between element ends. The word "consistent" arises from the fact that exactly the same polynomials are used in the (virtual displacement) derivation of the element elastic stiffness. As with linearized geometric stiffness, the equilibrium equations are formulated on the basis of the undeformed geometry of the structure.

For buckling analysis, it is still required to subdivide columns. However, only two segments are needed when consistent geometric stiffness is used.



When using 2D analysis to analyze a single frame in a 3D structure it is very important to accurately represent the TOTAL P-Delta effects on the frame. In the frame shown here, moment resisting frames are deployed on Lines A and D only. Frames on lines B and C have only simple gravity loads, and are referred to a "leaner" columns. For the structure shown, Frame A is to be analyzed alone using a program like DRAIN 2D. The area shown in green is the tributary gravity load for the frame. However, the tributary load for modeling P-delta effects is much larger as shown on the following slide.



When Frame A drifts laterally the entire structure drifts with it. Hence the entire gravity load is producing the P-Delta effect. Thus, the geometric stiffness of the columns in Frame A must be based on the shaded blue area.



Because different tributary loads are required for frame gravity effects and for system P-Delta effects, it is necessary to model P-Delta effects through the use of a special outrigger or "ghost" frame. In DRAIN, these columns would likely be Type-1 truss elements, with one element being used for each story. The lateral DOF at each story of the ghost frame are slaved to the appropriate story in the main frame. Story gravity loads are applied as shown.

If all of the story gravity load is applied to the ghost column, the P-Delta effect would be TURNED OFF in the main frame columns.



How much gravity load to include for P-Delta analysis? This slide gives some recommendations.

It is certainly overly conservative to include full live load. Even fully reduced live load may be too conservative. See for example Table C4-2 of ASCE7-02 which suggests an average of 10.9 psf for offices, with a standard deviation of 5.9 psf.

Vertical accelerations may have an important effect on system stability, depending on phasing effects. It is not known if any research has been done in this area.



When running a pushover analysis the user has the option of performing the analysis under "Force Control" or under "Displacement Control". If the analysis is being executed under force control the analysis will terminate with an error as soon as the incremental tangent stiffness is negative. (Actually, the determinant of the tangent stiffness will be negative.) If it is desired to track behavior beyond the point where the tangent stiffness is negative, it is necessary to use a displacement controlled analysis.



In essence, a displacement controlled analysis uses a stiff spring, which when added to the system's tangent stiffness, results in a positive definite system stiffness. Displacement control algorithms maintain the desired lateral force pattern for all analysis steps. It must be noted, however, that the response of a statically loaded system beyond the point where the tangent stiffness of the original system goes negative is completely fictitious. A real structure, statically loaded, would immediately collapse at that point.

Under dynamic loads, the system tangent stiffness may be negative, but the effective system stiffness may be positive due to inertial effects. This is evident from the fact that the incremental tangent stiffness is the actual tangent stiffness + Mass/ $\Delta T^2$  + Damping/ $\Delta T$ . The Mass term is always positive. The damping term will also be positive if the damping matrix is based on mass and initial system stiffness.



When using response history analysis or displacement controlled static analysis, the system base shear must be recovered from the sum of the column shears PLUS the story wise P-Delta shears. Under force controlled static analysis, the base shear can be computed directly from the applied loads or from the process shown above (which should give the same answer).

Proceed to Topic 15-5b.