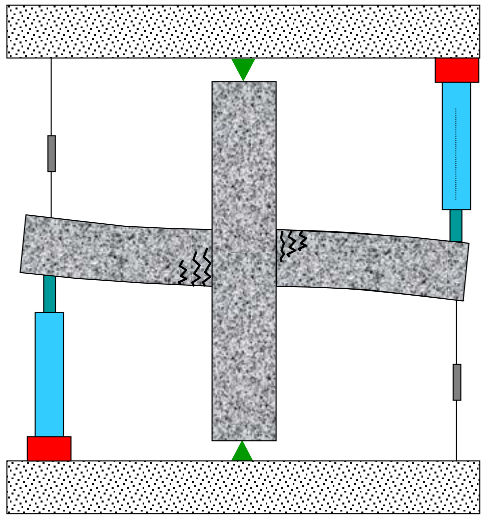
INELASTIC BEHAVIOR OF MATERIALS AND STRUCTURES



Instructional Material Complementing FEMA 451, *Design Examples* Inelastic Behaviors 6 - 1

This topic introduces the concepts of inelastic behavior, explains why the behavior is expected in seismic response, and shows how the inelastic behavior is incorporated into the *NEHRP Recommended Provisions*, ASCE 7, and the *International Building* Code (IBC).

Inelastic Behavior of Materials and Structures

- Illustrates inelastic behavior of materials and structures
- Explains why inelastic response may be necessary
- Explains the “equal displacement “ concept
- Introduces the concept of inelastic design response spectra
- Explains how inelastic behavior is built into the *NEHRP Recommended Provisions* and ASCE 7-05

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These are the main objectives of the topic.

Importance in Relation to ASCE 7-05

- Derivation and explanation of the *response reduction factor, R*
- Derivation and explanation of the *displacement amplification factor, C_d*
- Derivation and explanation of the *overstrength factor, Ω_o*

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The relevance of the current topic to the ASCE 7-05 document is provided here. Detailed referencing to numbered sections in ASCE 7 is provided in some of the slides or slide annotations.

Inelastic Behavior of Structures

From material

↓

to cross section

↓

to critical region

↓

to structure

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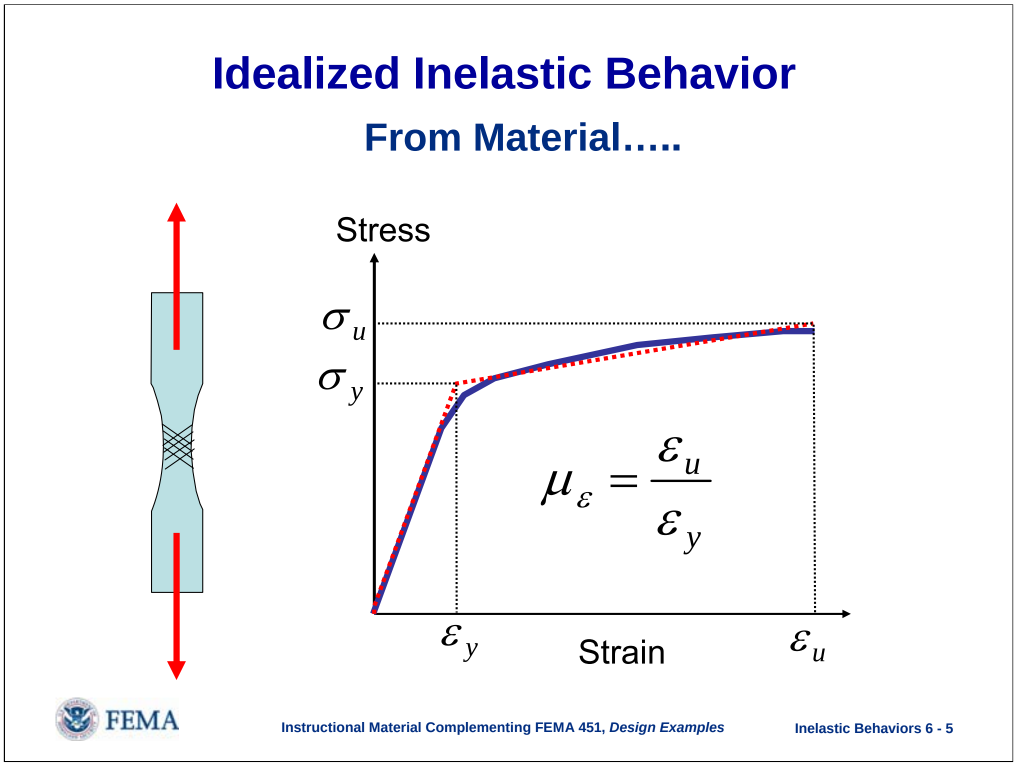
In the first part of this lesson, we work through a “hierarchy” of behavior from material, to cross-section, to critical region, and, finally, to the entire structure.

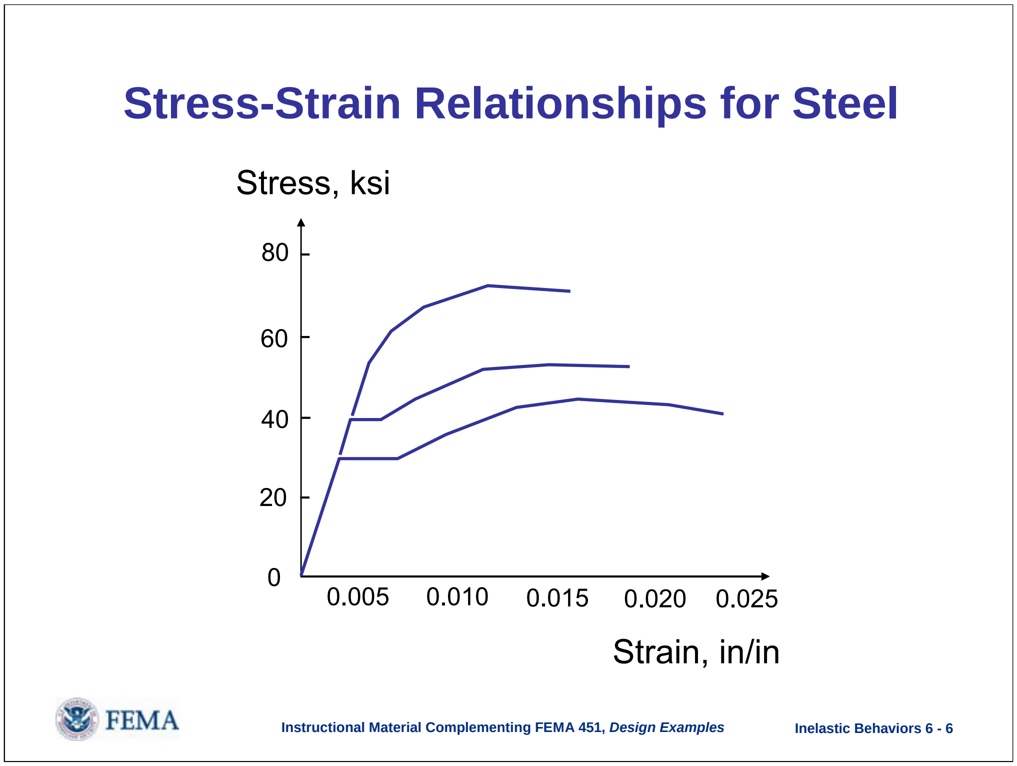
In earthquake engineering, inelastic response is expected, and survivability of structures depends on the ability of the structure to sustain several cycles of fully reversed inelastic deformation without excessive loss of stiffness or strength.



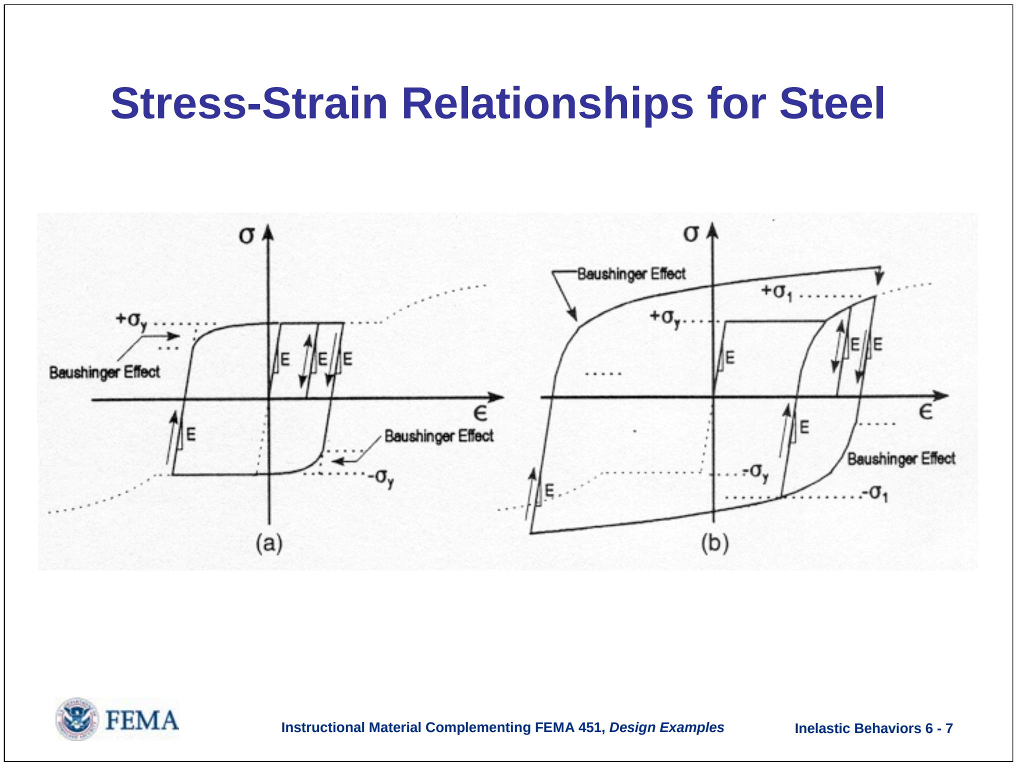
Here, a steel coupon is being tested, and the stress-strain curve is plotted. The stress and strain at yield is easy to identify for low strength steel, but may be more difficult to identify for higher strength steel.

The ultimate stress is typically taken as the maximum attainable stress, and the corresponding maximum strain is recorded. The ductility SUPPLY, in terms of strain, is equal to the maximum attainable strain divided by the yield strain. This ductility supply must be greater than that demanded by the earthquake.

Note also that the slide shows only monotonic loading for the specimen. During an earthquake, deformations (and stresses) will occur in both directions.



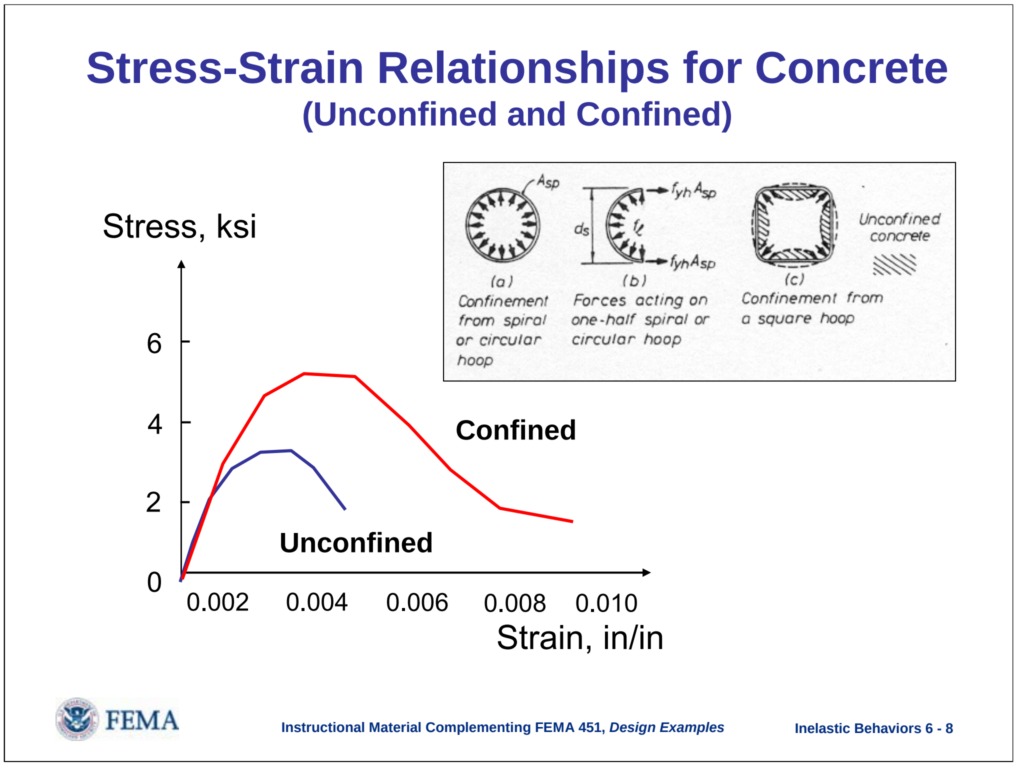
Here, a series of stress-strain curves are shown for steel. As stated previously, the yield stress is much easier to identify for the lower strength steels.



This slide has two purposes. First, it shows what the (idealized) cyclic stress-strain curve looks like for lower strength steel. The curve on the left is for strains remaining within the yield plateau. The curve on the right is for larger strains. Note that the unloading “stiffness” is the same as the initial stiffness, but that the Bauschinger effect smoothes out the curve upon reverse loading. The sharp yield plateau will occur only for the first loading cycle.

The second point of the curves is to note that empirical relations can be developed to represent the inelastic behavior of steel materials.

Source of diagram unknown.



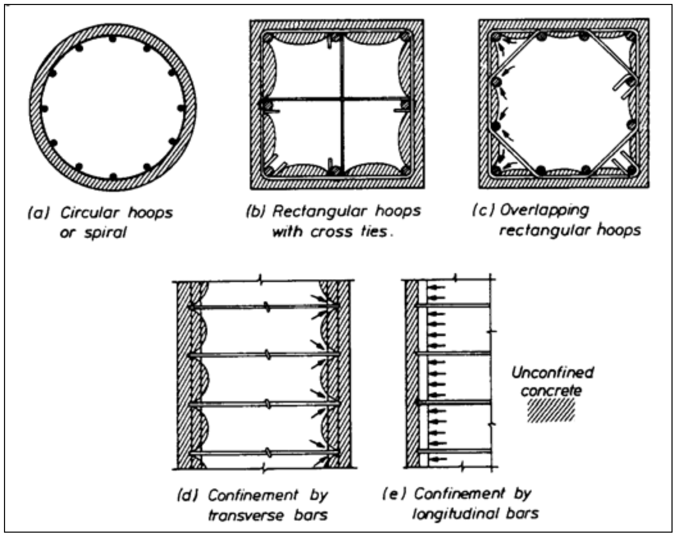
The stress-strain curve for concrete is highly dependent on the confinement of the concrete. Under high confinement (as applied by hydrostatic pressure, for example), there is an appreciable increase in strength and deformation capacity. In earthquake engineering, the large increase in deformation capacity is extremely important. If concrete strains were limited to 0.003 or 0.004, concrete could not be used in seismic areas.

In reinforced concrete structures, confinement is supplied passively by closely spaced reinforcement, usually supplied in the form of hoops. When the section is under compression, Poission's effect is to produce lateral expansion. The stiffer confining reinforcement restrains this expansion, thereby applying confining pressure.

ACI 318 has very detailed requirements for providing confinement reinforcement in the critical regions of structures.

The inset diagram is taken from Figure 3.3 of the text "Seismic Design of Reinforced Concrete and Masonry Buildings", by Paulay and Priestly, Wiley Interscience.

Concrete Confinement



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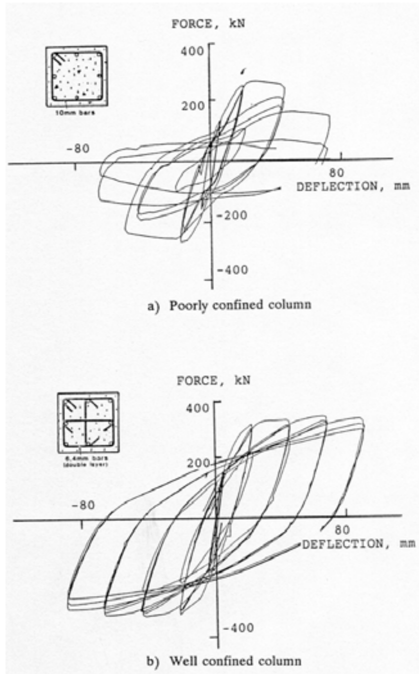
Here, different types of confinement are illustrated. The most efficient type of confining reinforcement is circular hoops or spiral reinforcement. The square section with square ties and NO crossties would provide almost no confinement. Adding crossties helps significantly.

The closer the spacing, the better. Also, the use of several longitudinal bars helps to "basket" the concrete, and maximizes the efficiency of the confinement.

The diagram is taken from Figure 3.4 of the text "Seismic Design of Reinforced Concrete and Masonry Buildings", by Paulay and Priestly, Wiley Interscience.

Unconfined

Confined



a) Poorly confined column

b) Well confined column

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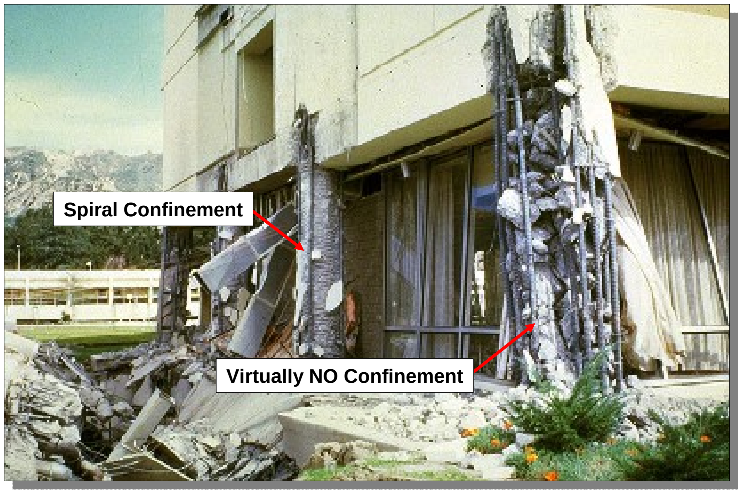
Here, the behavior of two identical cross-sections is compared. The sections are columns loaded in compression. The section on the top has a tie with a 10 mm diameter (area of 2 legs = 157 mm squared) and the section on the bottom has two layers of 6.4 mm bars (for a total area of 192 mm squared). Hence, the total area of transverse reinforcement is similar. For the section on the top, where no crossties are provided there is significant loss of stiffness and strength under cyclic loading. The addition of crossties, one in each direction, provides a tremendous benefit. There is some loss of stiffness at each cycle of loading, but strength is maintained.

The behavior shown on the top is unacceptable in high seismic regions. The behavior shown at the bottom is ideal.

The diagram is taken from Ozcebe and Saatcioglu, “Confinement of Concrete Columns for Seismic Loading”, *ACI Structural Journal*, Vol. 84, No. 4, April 1987.

Benefits of Confinement



Olive View Hospital, 1971 San Fernando Valley earthquake

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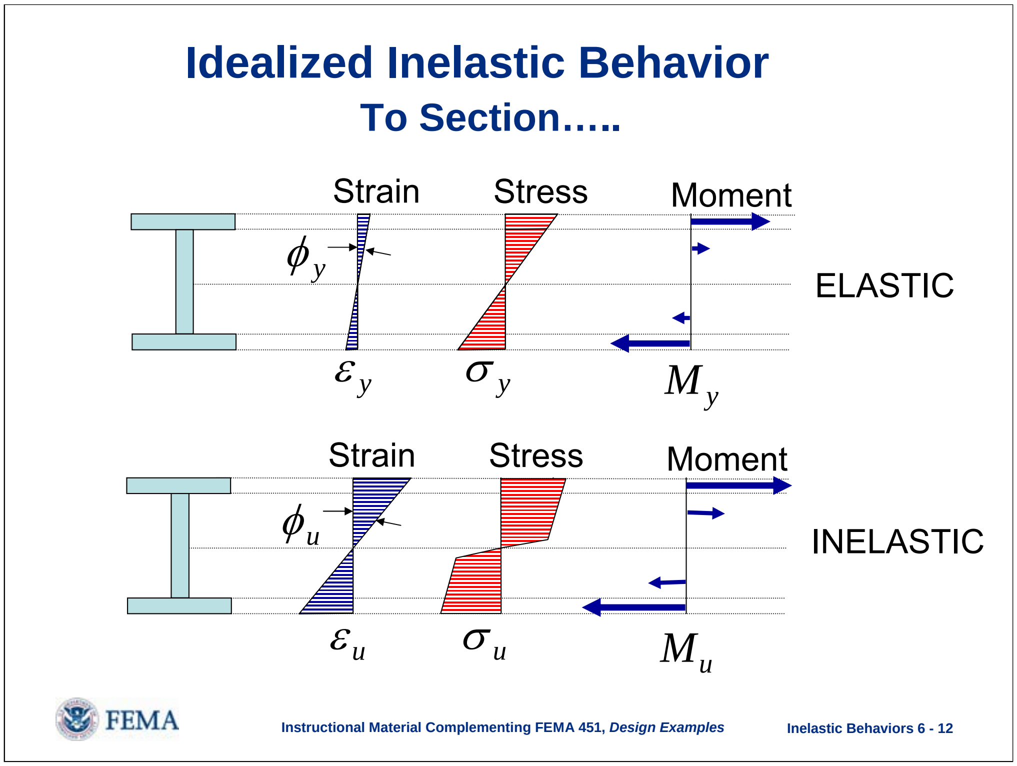
The benefits of confinement are clearly illustrated on this slide. This relatively new building (designed according to the building code) has to be demolished after the earthquake.

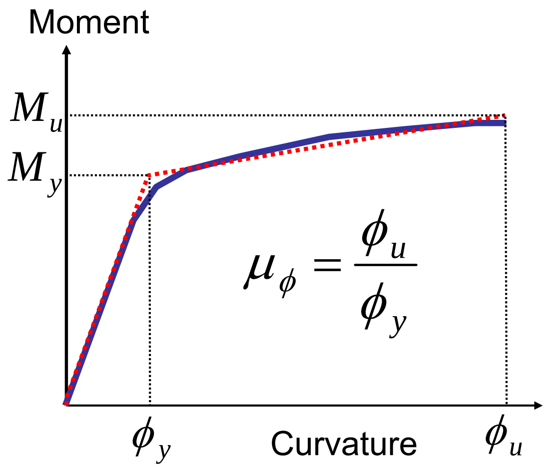
Source of photo unknown.



The next level of the hierarchy is from material to cross section. Definitions of yield moment and curvature are illustrated.

Idealized Inelastic Behavior

To Section…..



NOTE: $\mu_\phi \le \mu_\varepsilon$

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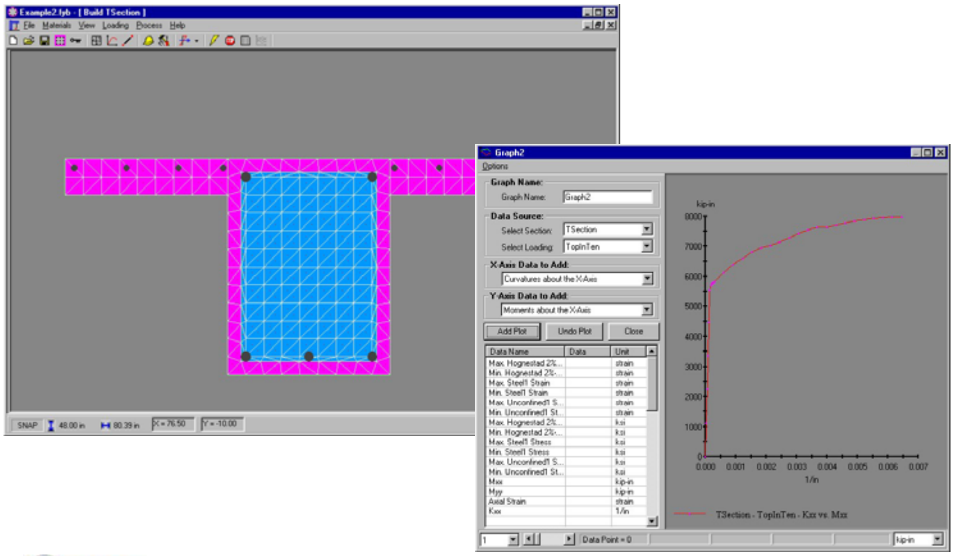
The moment-curvature relation for the section is shown. Again, it is important to note that the ductility indicated is the ductility SUPPLIED by the cross section. The ductility demanded by the earthquake must be less than this.

Software for Moment - Curvature Analysis "XTRACT"



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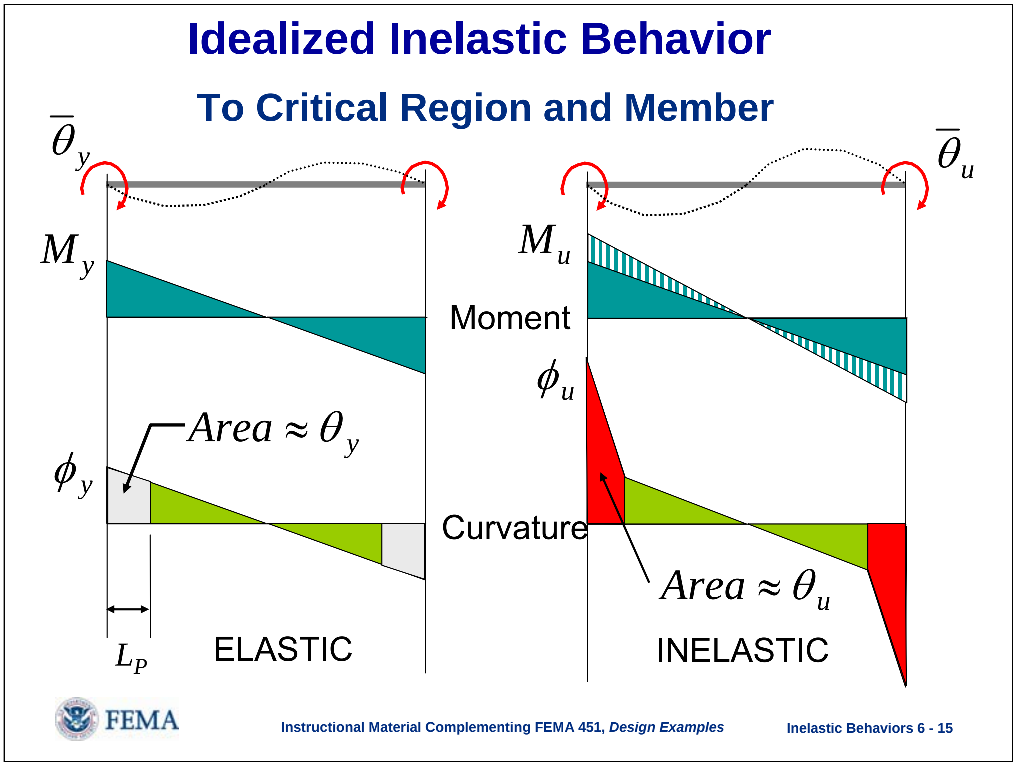
A variety of software exists for performing moment-curvature analysis. The program, called XTRACT, used here may be obtained at the following web site: http://www.imbsen.com/xtract.htm.

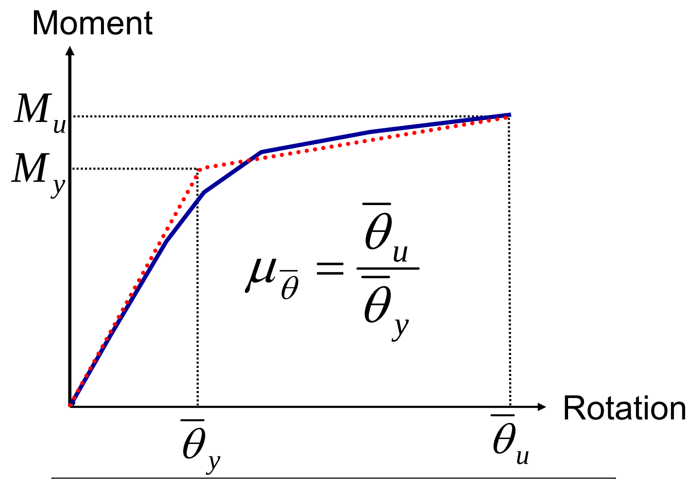


We now move from the cross section to the "critical region." This region is that part of the element over which significant inelastic behavior is expected to occur. In this slide, the critical region coincides with the flexural plastic hinging of a beam. When the applied member end rotations are such that the yield moment is just developed at the ends, the yield rotation in the hinging region is a shown. It is necessary, of course, to assume the length of this hinging region, which is approximately equal to the depth of the section.

When additional end rotations are applied and the ultimate moments are developed at the member ends, inelastic curvature occurs in the hinging region, and the integration of these curvatures over the hinge length provides the inelastic rotation in the hinge.

Idealized Inelastic Behavior

To Critical Region and Member



NOTE: $\mu_{\bar{\theta}} \leq \mu_{\theta} \leq \mu_{\phi}$

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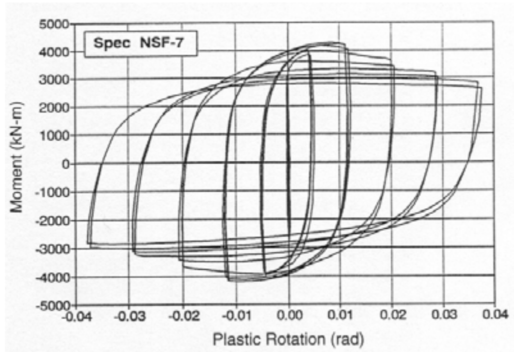
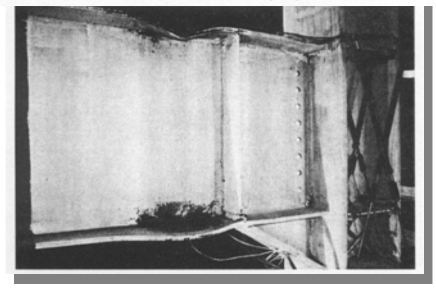
The rotational ductility SUPPLY is determined from the curve. Development of such curves is difficult, and requires static nonlinear analysis.

Note that the rotation with the overbar is the applied rotation at the end of the element, and the rotation without the overbar is the plastic hinge rotation.

Critical Region Behavior of a Steel Girder

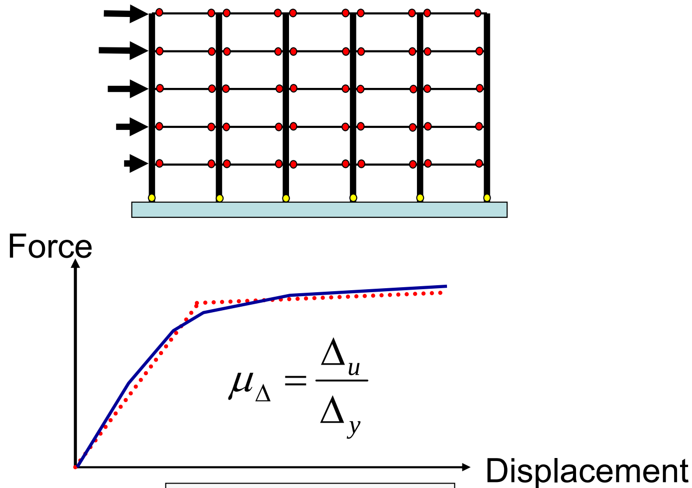




In this slide, the moment rotation relationship for a steel member is determined experimentally. The important thing to note is that inelastic buckling of the flanges has resulted (after a number of cycles) in a loss of moment capacity. However, this member is maintaining more that 80 percent of its capacity at rotations of 0.038 radians, which is approximately twice the limit allowed by the code. If the loss of strength was excessive, it would lead to a reduction in the available rotational ductility supply.

Photo from the FEMA Program to Reduce the Earthquake Hazards in Steel Moment Frame Structures; a brief description and links to reports are available at http://www.sacsteel.org/library/publications.html

Idealized Inelastic Behavior
To Structure…..



$$\mu_\Delta = \frac{\Delta_u}{\Delta_y}$$

Note: $\mu_\Delta \leq \mu_{\bar{\theta}}$

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The next level of the hierarchy is the structure. Here, the overall response is given in terms of force vs displacement. This curve is sometimes called a capacity curve or a pushover curve. The displacement ductility SUPPLY is as indicated. Note that this supply may depend on the applied loading pattern.

Loss of Ductility Through Hierarchy

Strain μ_{ε}= 100

Curvature μ_{ϕ}= 12 to 20

Rotation μ_{θ}= 8 to 14

Displacement μ_{Δ}= 4 to 10

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It is generally true that there is a reduction in ductility over the hierarchy. Hence, even though a material might be highly ductile (like mild steel), the system ductility may only be moderate.

However, system level ductility SUPPLY of 4 to 6 is all that is needed for good seismic performance because the ductility DEMANDS rarely exceed 4 to 6 (for structures designed for the larger *R* factors.)

Ductility and Energy Dissipation Capacity

- System ductility of 4 to 6 is required for acceptable seismic behavior.
- Good hysteretic behavior requires ductile materials. However, ductility in itself is insufficient to provide acceptable seismic behavior.
- Cyclic energy dissipation capacity is a better indicator of performance.

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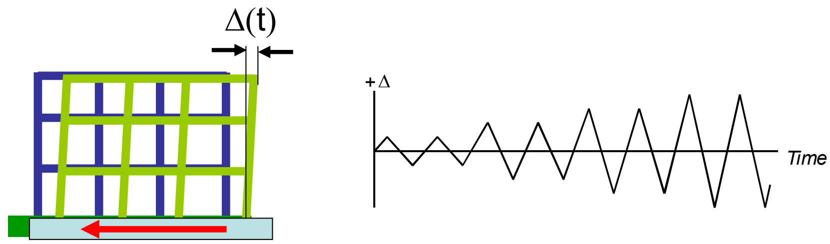
The first bullet summarized the major point form the previous slide. However, ductility in itself is insufficient. Structures must have good energy dissipation capacity, and this capacity must be sustainable over several cycles of deformation.

Response Under Reversed Cyclic "Loading"



Earthquakes impose *DEFORMATIONS*. Internal forces develop as a result of the deformations.

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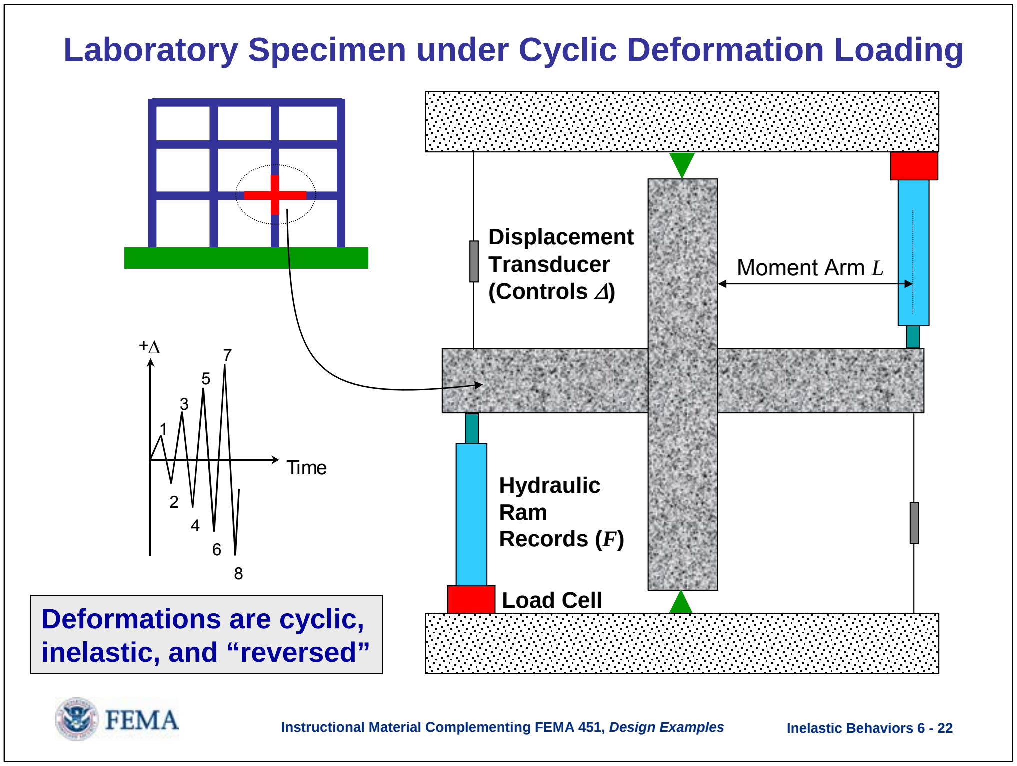
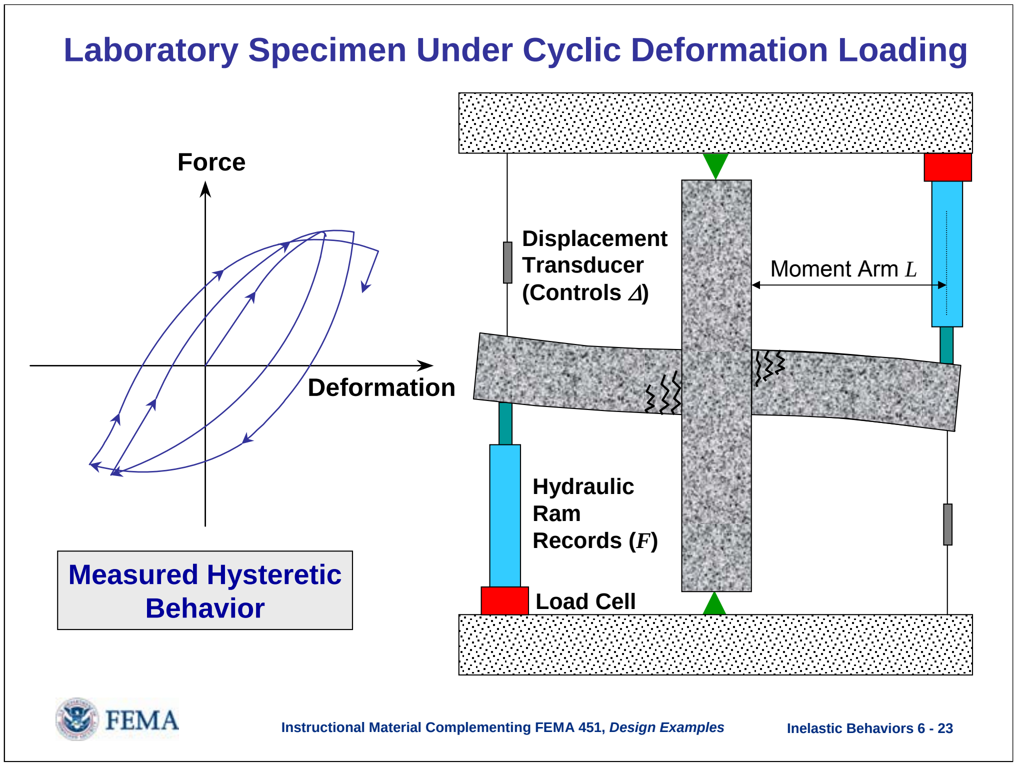
It is important to remember that earthquakes impose deformations, not forces on structures. The forces we apply in the equivalent static method of analysis are entirely fictitious.

In the next series of slides, a hypothetical experiment on a beam-column subassemblage is used to determine the cyclic ductility and the cyclic energy dissipation capacity of the critical regions of the subassemblage.

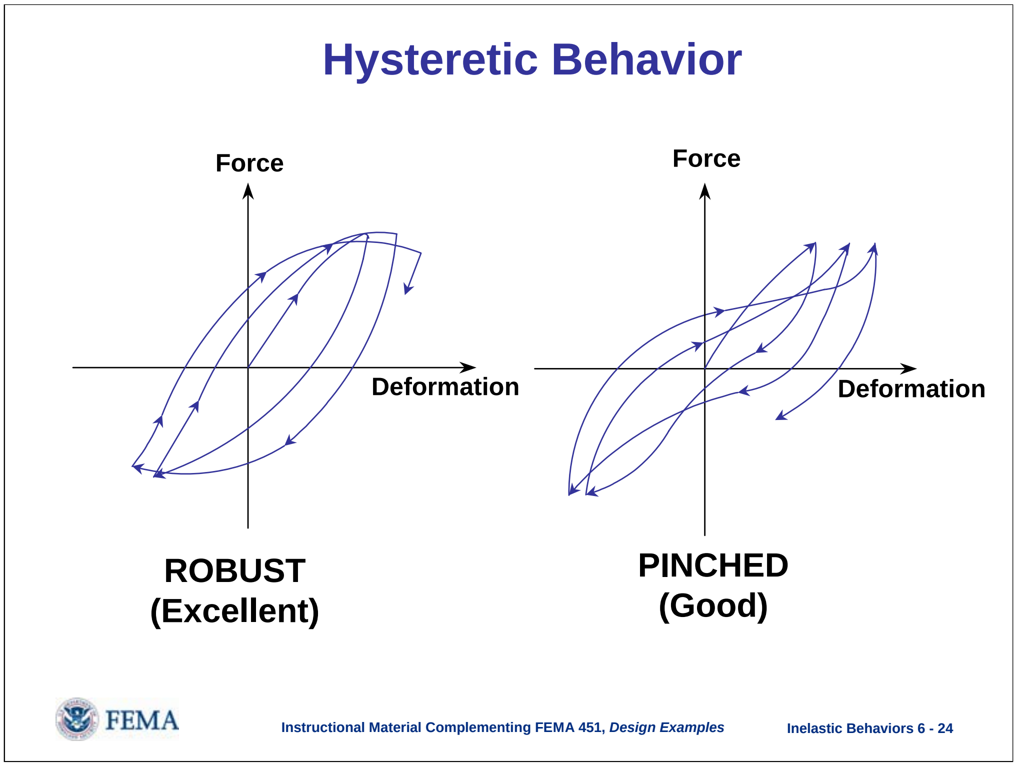
Subassemblage tests are often used in research, and in fact, are required for qualification of connections of special and intermediate moment frames in the AISC Seismic Provisions.



This is the test setup. A predefined cycle of displacements is applied, and the resisting force is monitored. The displacement cycle is often provided in material-specific loading protocols.

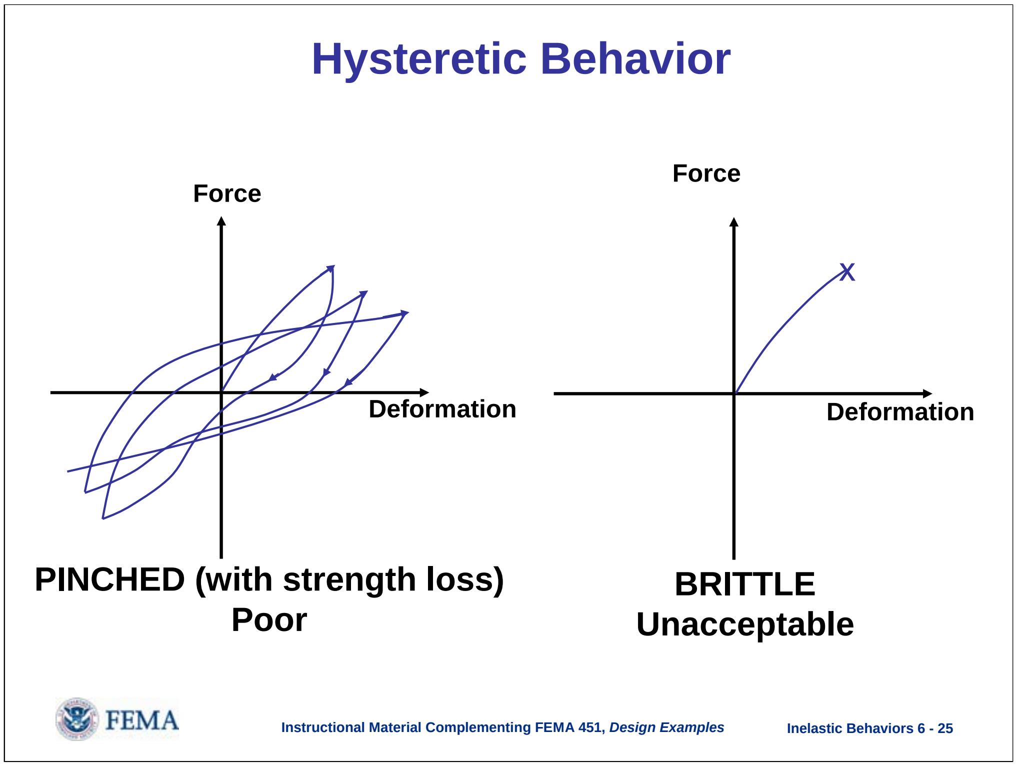


The force-displacement curve shown at the left is obtained from the experiment.



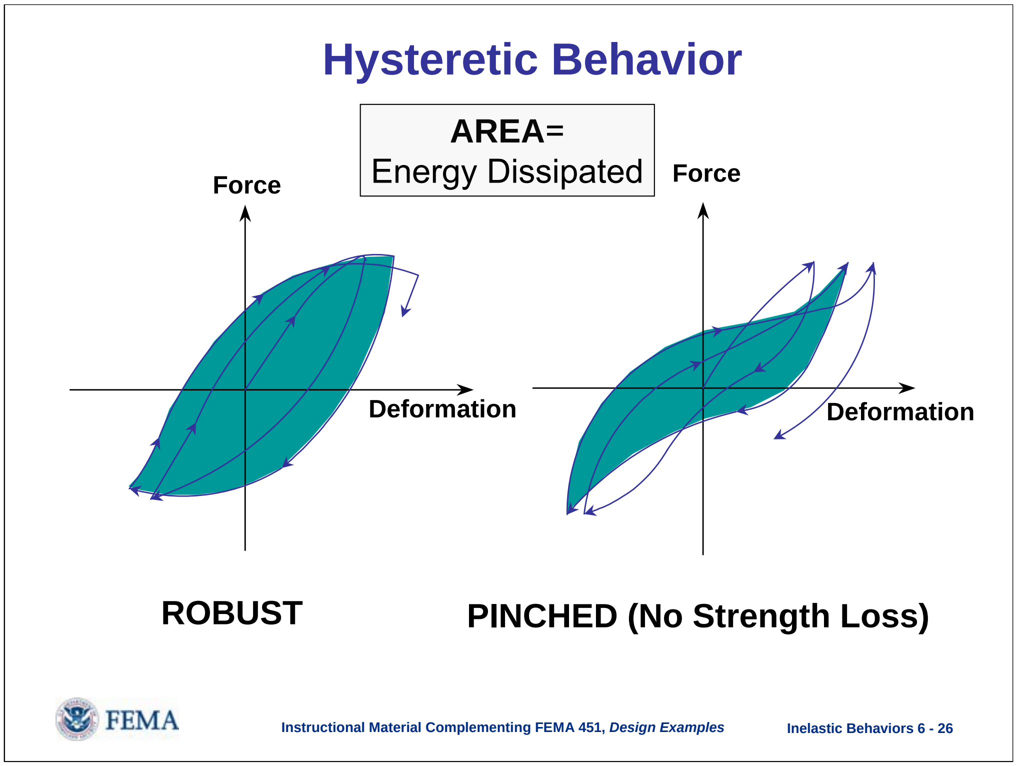
Here are two different typical behaviors. The one shown at the left is excellent because there is virtually no loss of strength over the several cycles of loading. Also, the hysteresis loops are “fat,” indicating a good deal of energy dissipation capacity per cycle.

The loops shown at the right are pinched somewhat. Even though there is no apparent strength loss, the pinching has reduced the area of the hysteresis loops and has thereby reduced the energy dissipated per cycle. Sometimes pinched hysteresis cannot be avoided (as in reinforced concrete T-beams).



The response shown at the left has pinching with strength loss. This behavior is quite poor and is likely to be unacceptable in high seismic risk areas.

The response at the right is brittle and is completely unacceptable in any seismic area except, perhaps, for Seismic Design Category A regions.



This slide reinforces a point made earlier. The area contained within a single hysteresis loop is often a better indicator of performance than is ductility. For example, each of the structures tested have about the same overall ductility and neither suffers appreciable strength loss. However, the energy dissipated per cycle for is greater the robust system and, hence, this system is preferable.

Ductility and Energy Dissipation Capacity

- The structure should be able to sustain several cycles of inelastic deformation without significant loss of strength.
- Some loss of stiffness is inevitable, but excessive stiffness loss can lead to collapse.
- The more energy dissipated per cycle without excessive deformation, the better the behavior of the structure.

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Summary of previous material. Self explanatory.

Ductility and Energy Dissipation Capacity

- The art of seismic-resistant design is in the details.
- With good detailing, structures can be designed for force levels significantly lower than would be required for elastic response.

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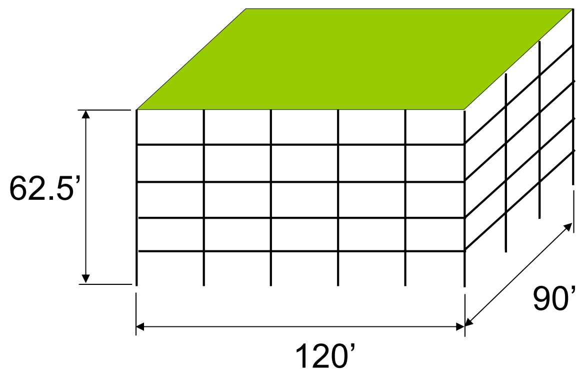
It has been said that in earthquake engineering “strength is essential but otherwise unimportant”. While this statement has some truth, a more “profound” statement is that the are of seismic-resistant design is in the details. Much more will be said about this in the topics on steel, concrete, masonry, and timber systems.

If (and only if) the good detailing is provided, the structure can be designed for forces that are substantially lower than would be required if the structure remained elastic. The basis for this argument is provided in the next several slides.

Why Is Inelastic Response Necessary?

Compare the Wind and Seismic Design of a Simple Building



Building properties:
Moment resisting frames
Density ρ = 8 pcf
Period T = 1.0 sec
Damping ξ = 5%
Soil Site Class “B”

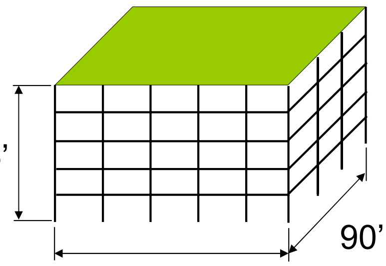
Wind:
100 mph Exposure C

Earthquake:
Assume S_{D1} = 0.48g

This simple example is used to show how seismic forces can significantly exceed wind forces, leading to unacceptable economy in seismic-resistant design. The properties given are typical of commercial office building construction. The lateral load resisting system is assumed to be a moment resisting frame. Wind analysis is from an older version ASCE 7. Note that few locations exist within the United States where the design would be based on 100 mph wind and an earthquake with 0.48g peak acceleration. The southern coast of Alaska is one place where this does occur.

Wind:

100 mph fastest
Exposure C



Velocity pressure q_s= 25.6 psf
Gust/exposure factor C_e = 1.25
Pressure coefficient C_q = 1.3
Load factor for wind = 1.3

Total wind force on 120-foot face:
V_{W120}= 62.5*120*25.6*1.25*1.3*1.3/1000 = **406 kips**

Total wind force on 90-foot face:
V_{W90} = 62.5*90*25.6*1.25*1.3*1.3/1000 = **304 kips**

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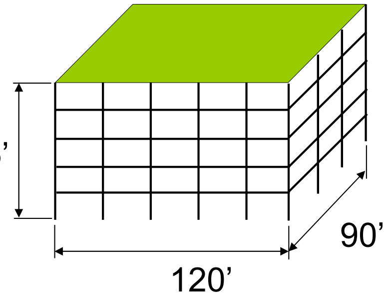
Note that because the "sail area" of the building is different for each direction, the wind force is different in the different directions. For this building, it has been assumed that the wind force is independent of building dynamic properties (frequency of vibration, damping).

Note also that a load factor of 1.3 has been applied to make the working stress wind force consistent with earthquake, which is based on an ultimate strength approach in ASCE 7.

Earthquake:



Building weight, W = 120*90*62.5*8/1000 = 5400 kips

$$V_{EQ} = C_S W$$

$$C_S = \frac{S_{D1}}{T(R/I)} = \frac{0.48}{1.0(1.0/1.0)} = 0.480$$

Total ***ELASTIC*** earthquake force (in each direction):
V_{EQ} = 0.480*5400 = **2592 kips**

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Using the simple ASCE 7 spectrum (with R = 1), this slide shows how much earthquake shear the building would be designed for if it were to remain elastic. A building period of 1.0 second has been assumed for each direction and is within the likely values for a building of this type of construction and height.

Comparison: Earthquake vs. Wind

$$\frac{V_{EQ}}{V_{W120}} = \frac{2592}{406} = 6.4 \qquad \frac{V_{EQ}}{V_{W90}} = \frac{2592}{304} = 8.5$$

- ELASTIC earthquake forces 6 to 9 times wind!
- Virtually impossible to obtain economical design

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Now, comparing the earthquake and wind forces, it is clear that the design for earthquake will be very expensive (relative to wind) if elastic response is required.

How to Deal with Huge Earthquake Force?

- Isolate structure from ground (base isolation)
- Increase damping (passive energy dissipation)
- Allow controlled inelastic response

Historically, building codes use **inelastic response procedure**.
Inelastic response occurs through structural **damage** (yielding).
We must control the damage for the method to be successful.

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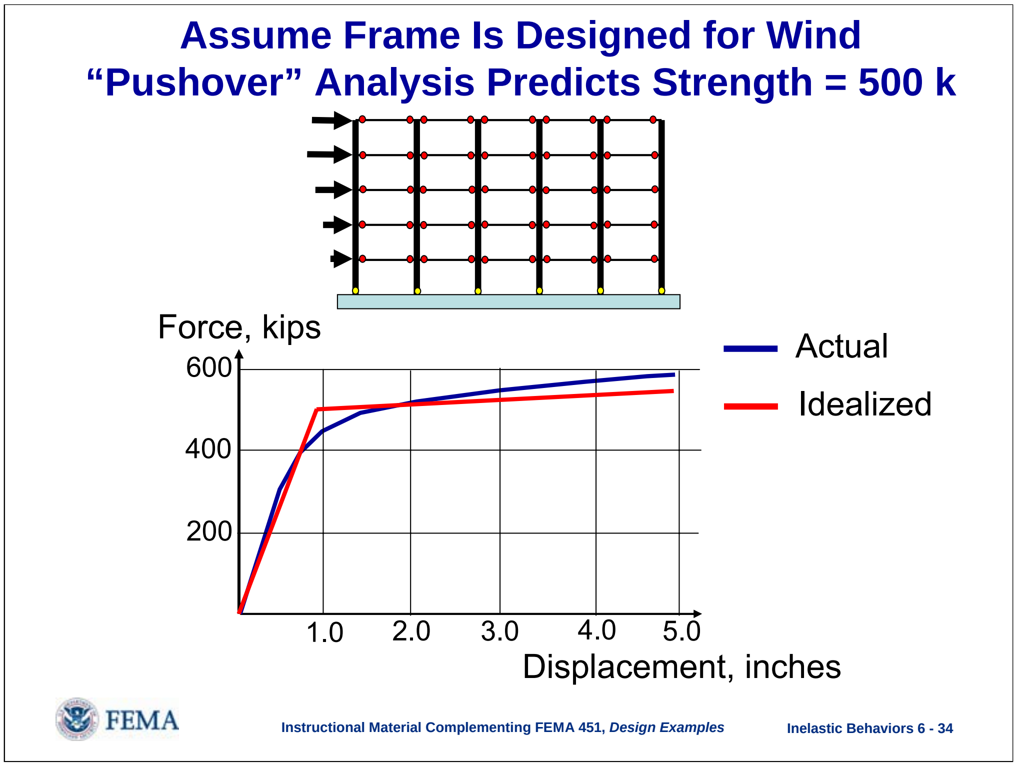
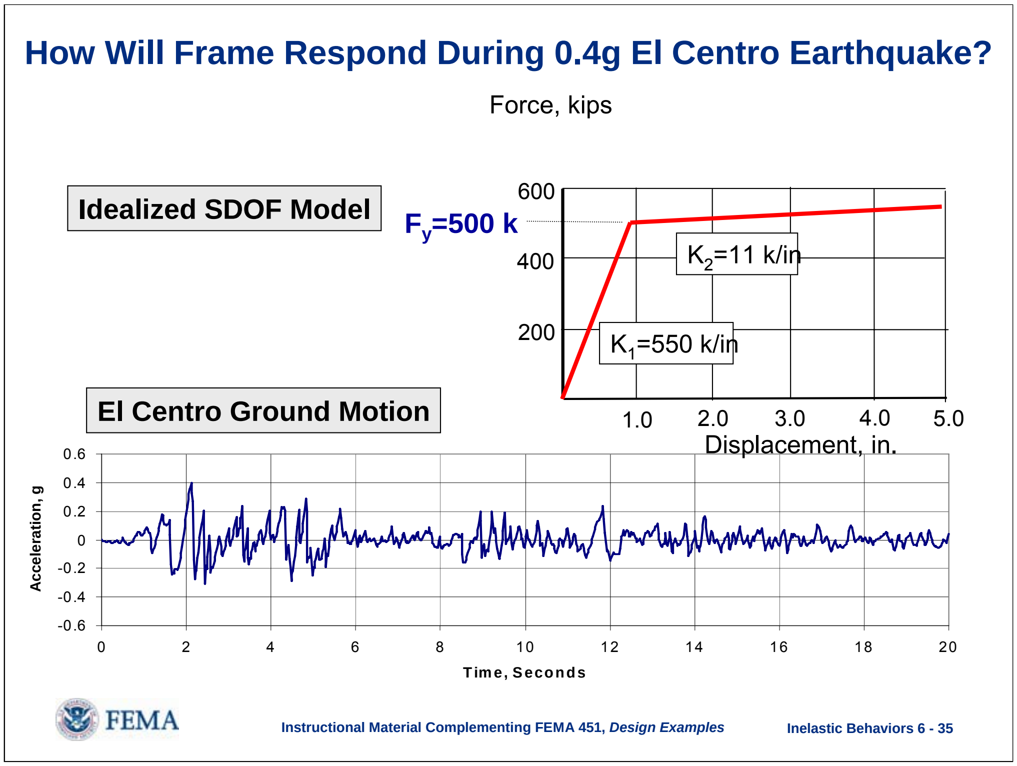
Base Isolation and added damping are typically reserved for very special structures. The cost of these devices is still significantly greater than the cost of traditional construction based on controlled inelastic response.



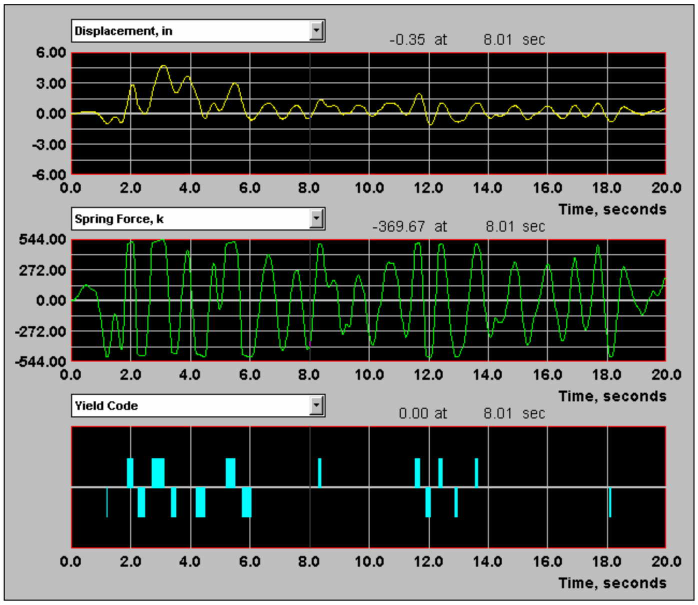
In the next several slides, we will develop the basis for the code-based inelastic design response spectrum.

The "actual" force-displacement curve came from a static pushover analysis of the frame, using Drain-2Dx. Using a bilinear idealization of the response, a simplified single-degree-of-freedom (SDOF) time-history analysis of the frame will be used.



This is the simplified SDOF model of the real multiple-degree-of-freedom (MDOF) system. Analysis will be carried out using NONLIN.

Response Computed by NONLIN



Maximum displacement:

4.79”

Maximum shear force:

542 k

Number of yield events:

15

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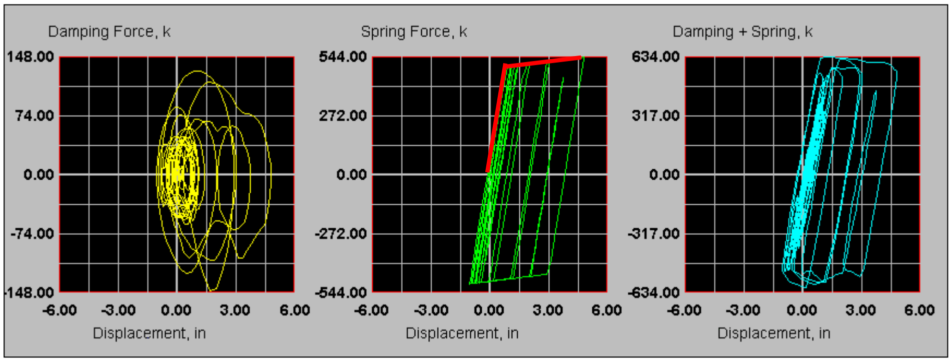
These screens are from the NONLIN program. This model has a 500 kip strength, with 2% strain hardening.

Note that, because of strain hardening, the base shear is somewhat larger than the yield strength.

Response Computed by NONLIN



Yield displacement = 500/550 = 0.91 inch

$$\textbf{\textit{Ductility Demand}} \equiv \frac{\text{Maximum Displacement}}{\text{Yield Displacement}} = \frac{4.79}{0.91} = 5.26$$

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This slide gives the basic definition for ductility *demand*. If this much ductility is not actually *supplied* by the structure, collapse may occur.

Note the difference in the second and third force-displacement panels of the slide. The second panel is member shear only and the third panel is member shear plus damping, or total base shear.

Write basic design equation:

Ductility Demand < Ductility Supply

Interim Conclusion (The Good News)

The frame, designed for a wind force <u>that is 15% of the ELASTIC earthquake force</u>, can survive the earthquake if:

- It has the capability to undergo ***numerous cycles of INELASTIC deformation.***
- It has the capability to ***deform at least 5 to 6 times the yield deformation.***
- It suffers ***no appreciable loss of strength.***

REQUIRES ADEQUATE DETAILING

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Note that detailing requirements are covered in the various material specifications (e.g., ACI 318).

Interim Conclusion (The Bad News)

As a result of the large displacements associated with the inelastic deformations, the structure will suffer considerable structural and nonstructural damage.

- This damage must be controlled by adequate detailing and by limiting structural deformations (drift).

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Note that damage from the Northridge and Kobe earthquakes has caused many engineers, building owners, and building officials to rethink the current philosophy.

The Performance-Based Design concept has been forwarded to give the engineer-owner team more control in designing the structure for damage control and limited or full functionality during and after an earthquake.

Development of "Equal Displacement" Concept of Seismic Resistant Design

Concept used by:

IBC NEHRP ASCE-7	In association with "force based" design concept. Used to predict design forces and displacements
FEMA 273	In association with static pushover analysis. Used to predict displacements at various performance points.

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The equal displacement concept is the basis for dividing the "elastic" force demands by the factor *R*. This is one of the most important concepts in earthquake engineering. The basis for the equal displacement concept is illustrated in the following slides.

The Equal Displacement Concept

"The displacement of an inelastic system, with stiffness K and strength F_y, subjected to a particular ground motion, is approximately equal to the displacement of the same system responding elastically."

(The displacement of a system is independent of the yield strength of the system.)

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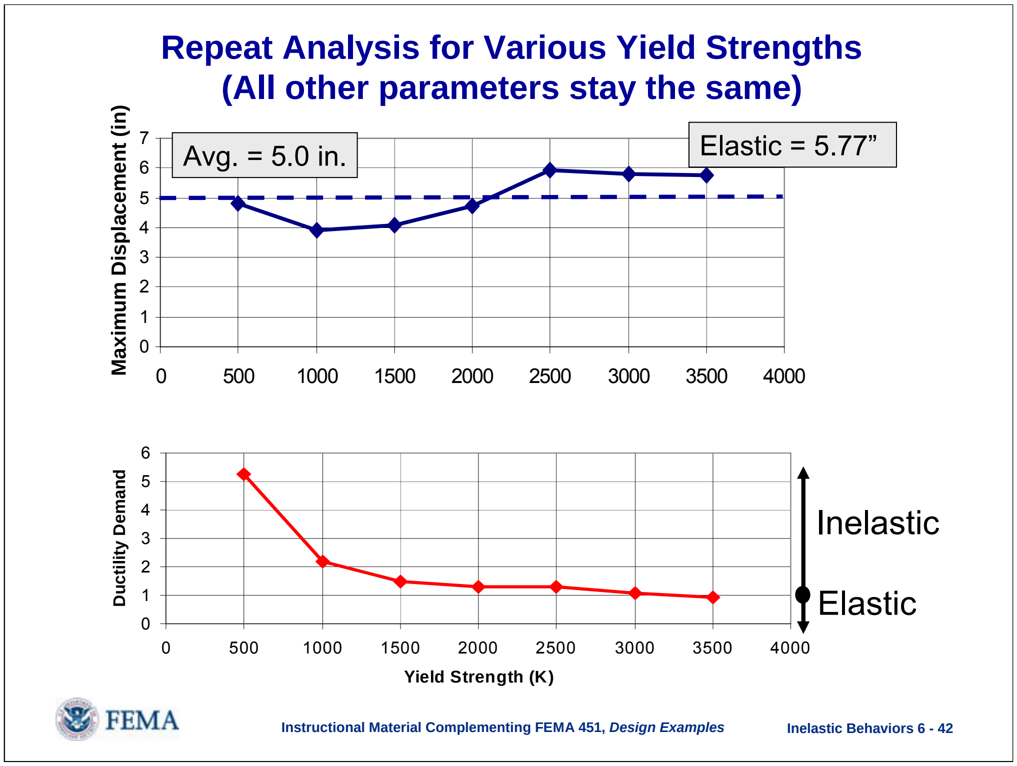
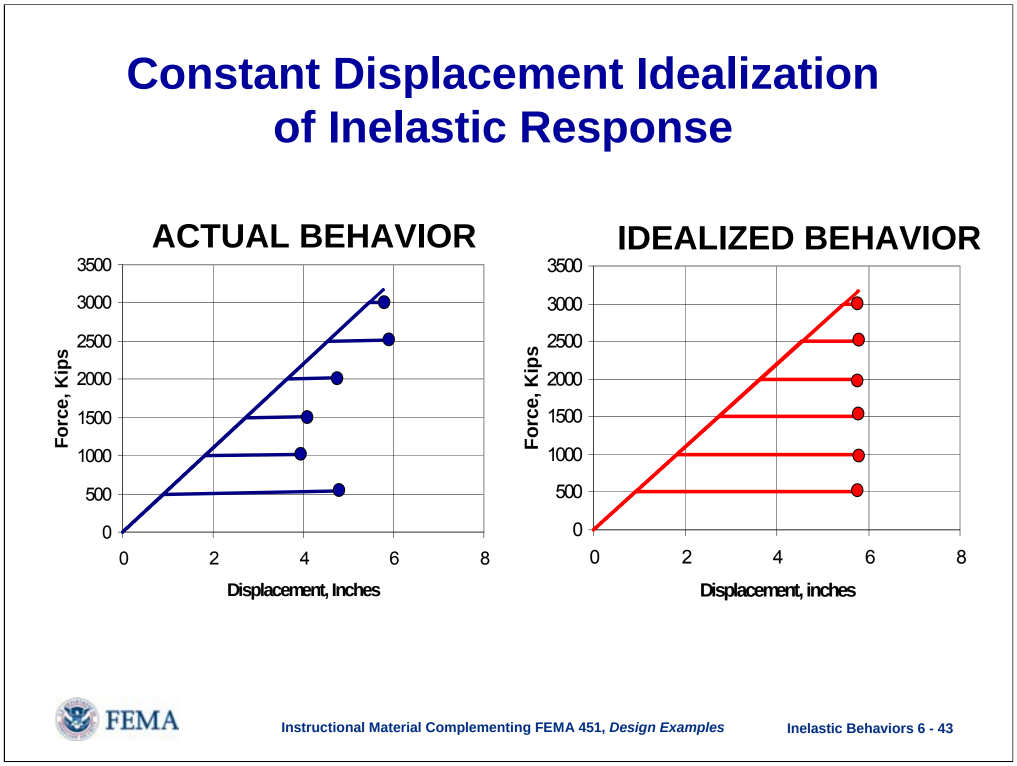
The equal displacement concept is stated in words.



This slide is based on a series of NONLIN analyses wherein all parameters were kept the same as in the original model except for the yield strength, which was systematically increased in 500 kip increments to a maximum of 3,500 kips. The structure with a 3,500 kip strength remains elastic during the earthquake.

Note that the displacement appears to be somewhat independent of yield strength, but the ductility demand is much higher for relatively lower strengths.



This slide shows simplified force-displacement envelopes from the different analyses. An apparently conservative assumption (with regard to displacements) is shown on the right. The basic assumption is that the displacement demand is relatively insensitive to system yield strength. This is often referred to as the “equal displacement” concept of seismic-resistant design.

Equal Displacement Idealization of Inelastic Response

- For design purposes, it may be assumed that inelastic displacements are equal to the displacements that would occur during an elastic response.
- The required force levels under inelastic response are much less than the force levels required for elastic response.

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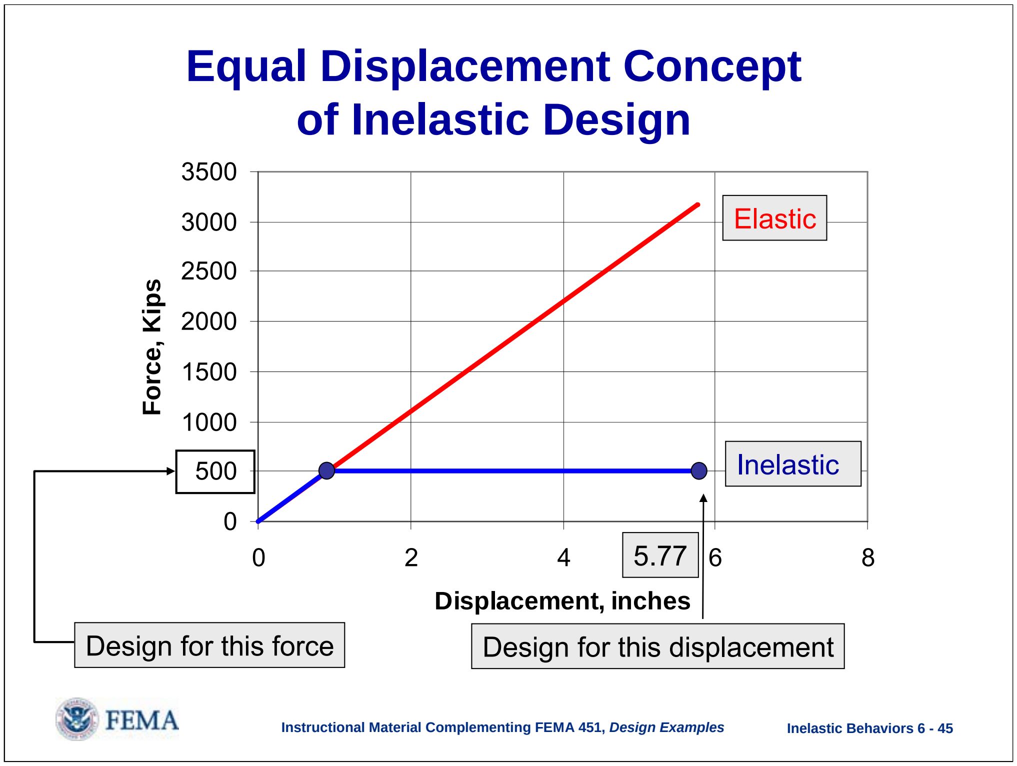
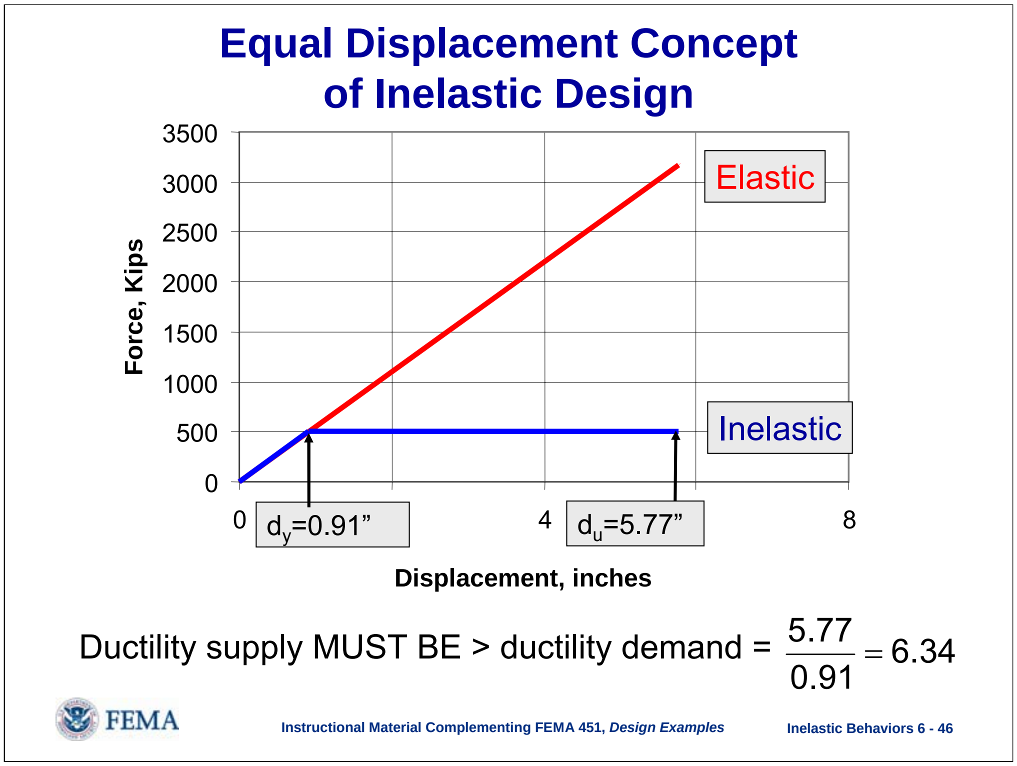
This slide summarizes the previous points.

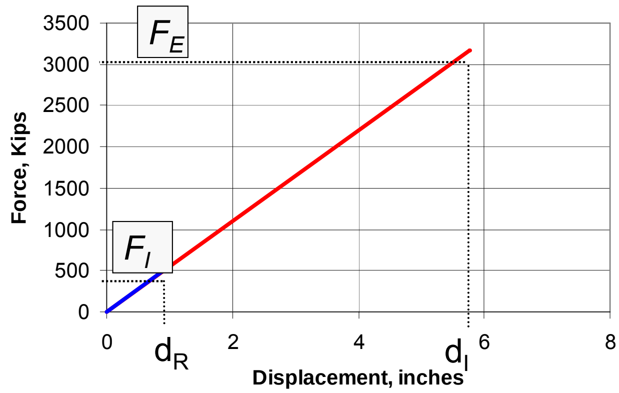


The equal displacement concept allows us to use elastic analysis to predict inelastic displacements. For the example system, the predicted elastic displacement (red line) is 5.77 inches, and it is assumed that the inelastic response (blue line) displacement is the same.



For this simple bilinear system, the ductility demand can now be computed. The system must be detailed to have this level of ductility.

For design purposes, we typically reverse the process. We assume some ductility supply (based on the level of detailing provided) and, using this, we can estimate the strength requirements.



Using response spectra, estimate elastic force demand $\boldsymbol{F_E}$

Estimate ductility supply, μ, and determine **inelastic** force demand $\boldsymbol{F_I = F_E/\mu}$.. **Design structure for** $\boldsymbol{F_{I.}}$

Compute reduced displacement. $\boldsymbol{d_R}$, and multiply by μ to obtain true inelastic eisplacement, $\boldsymbol{d_{I.}}$ **Check drift using** $\boldsymbol{d_{i.}}$

The procedure mentioned in the previous slide is explained in more detail here.

Building codes allow for an elastic structural analysis based on applied forces reduced to account for the presumed ductility supplied by the structure. For elastic analysis, use of the reduced forces will result in a significant underestimate of displacement demands. Therefore, the displacements from the reduced-force elastic analysis must be multiplied by the ductility factor to produce the true “inelastic” displacements.

ASCE 7 Approach

Use basic elastic spectrum but, for strength, divide all pseudoacceleration values by ***R***, a response modification factor that accounts for:

- Anticipated ductility supply
- Overstrength
- Damping (if different than 5% critical)
- Past performance of similar systems
- Redundancy

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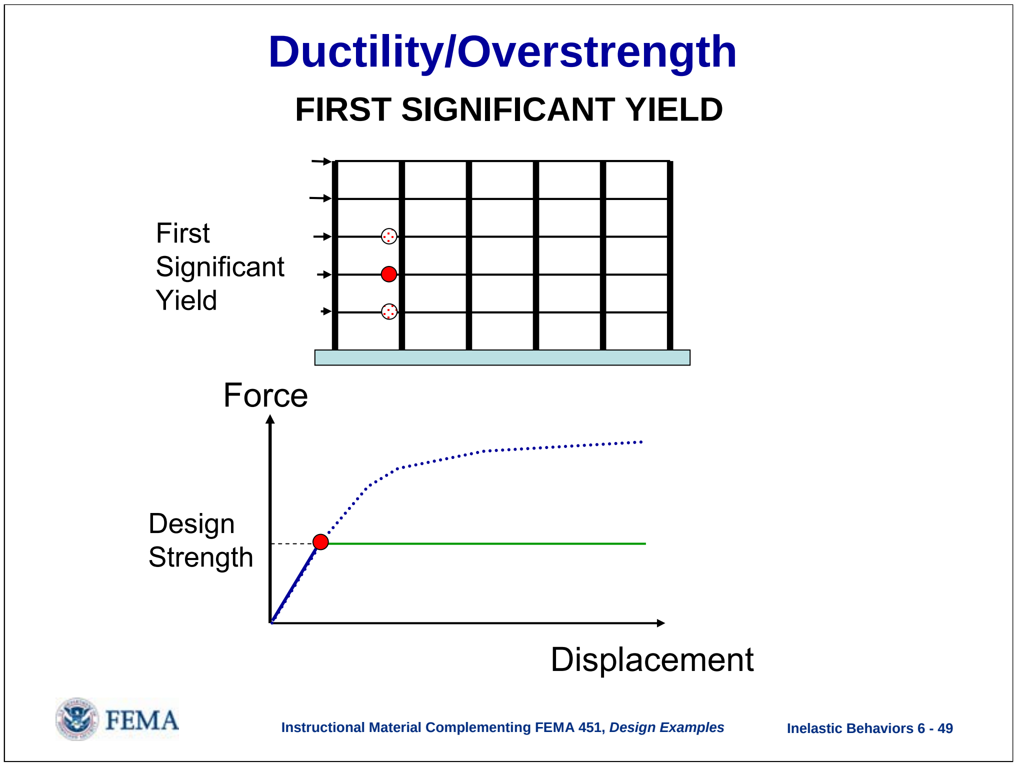
The ASCE 7 approach to using the equal displacement concept is discussed in the next several slides. One of the key aspects of the method is the use of the response modification factor, *R*. This term includes a variety of “ingredients,” the most important of which are ductility and overstrength.

Note that overstrength did not enter into the previous discussion because we were working with idealized systems. Real structures are usually much stronger than required by design. This extra strength, when recognized, can be used to reduce the ductility demands. (If the overstrength was so large that the response was elastic, the ductility demand would be less than 1.0.)



In this slide and several that follow as structure is being subjected to a pushover analysis. The structure remains essentially elastic until the first full plastic hinge forms. The formation of this “first significant yield” occurs at a level of load referred to as the design strength of the system.

If the hinging region has adequate ductility, it can sustain increased plastic rotations without loss of strength. At the same time, the other potential hinging regions of the structure will attract additional moment until they begin to yield.

First Significant Yield is the level of force that causes complete plastification of at least the most critical region of the structure (e.g., formation of the first plastic hinge).

The **design strength** of a structure is equal to the resistance at first significant yield.

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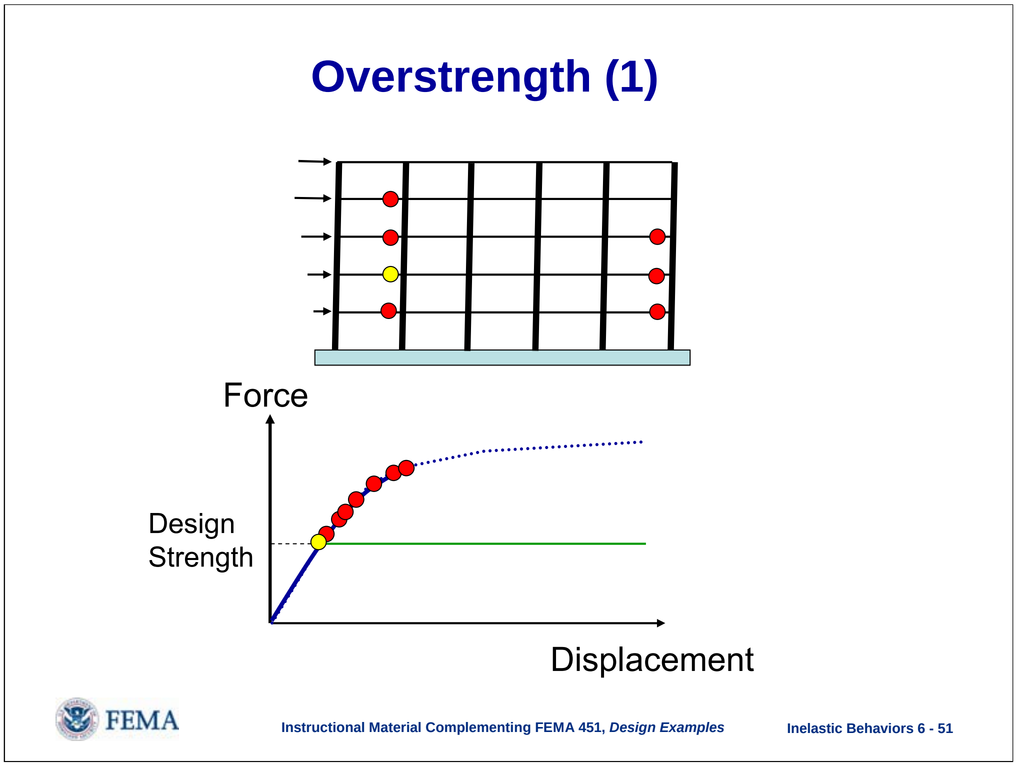
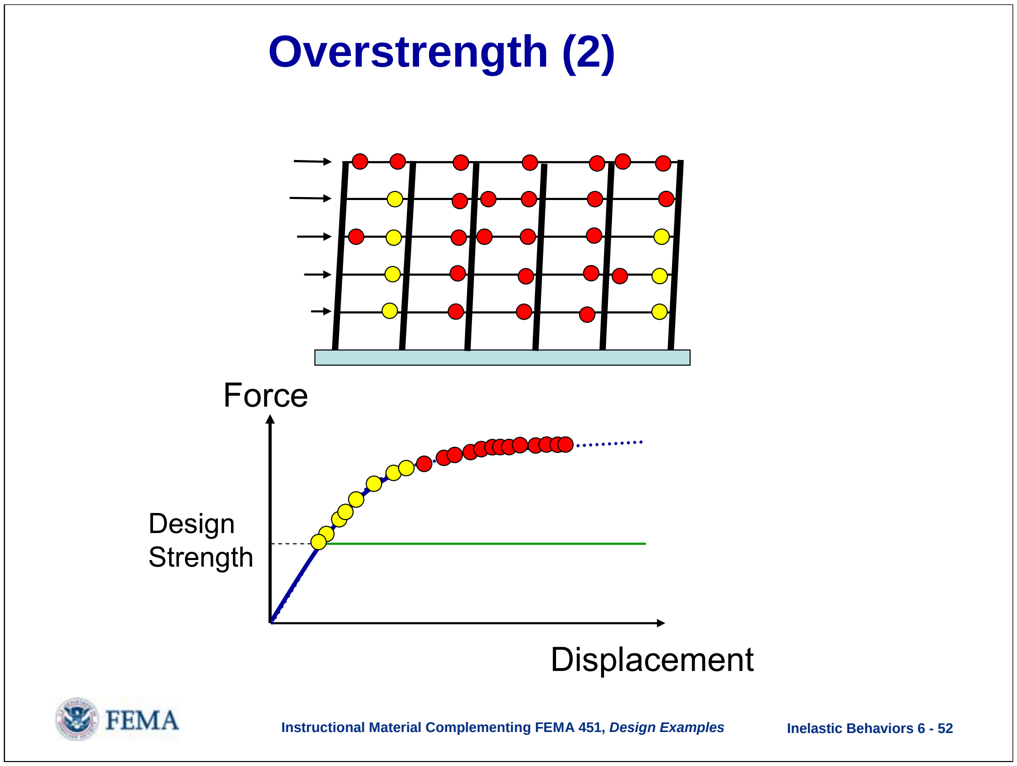
These definitions come form the commentary to the *NEHRP Recommended Provisions* and the commentary to ASCE 7.

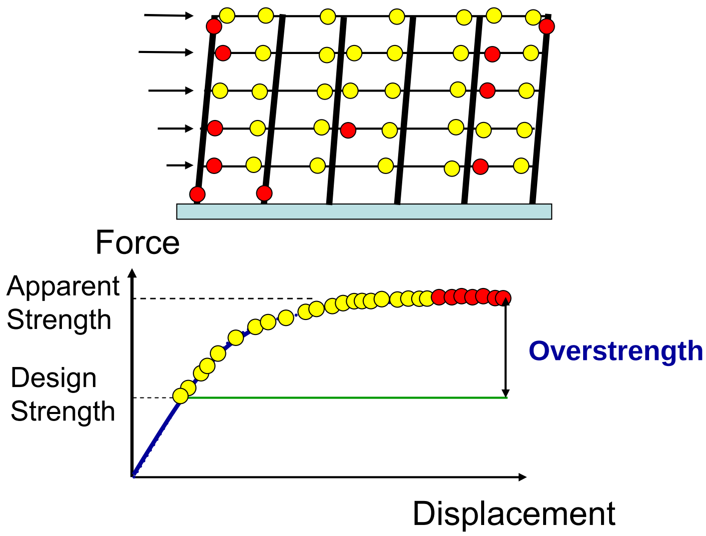


As additional load is applied to the structure, more hinges begin to form. The first hinge to form is continuing to rotate inelastically but has not reached its rotational capacity.



Even more load can be applied as additional hinges form. However, the first hinges to form are near their rotational capacity and may begin to loose strength. Hence, the pushover curve begins to flatten out.

Overstrength (3)



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The structure has finally reached its strength and deformation capacity. The additional strength beyond the design strength is called the overstrength. Most structures display considerable overstrength.

The total strength of the system is referred to as the apparent strength.

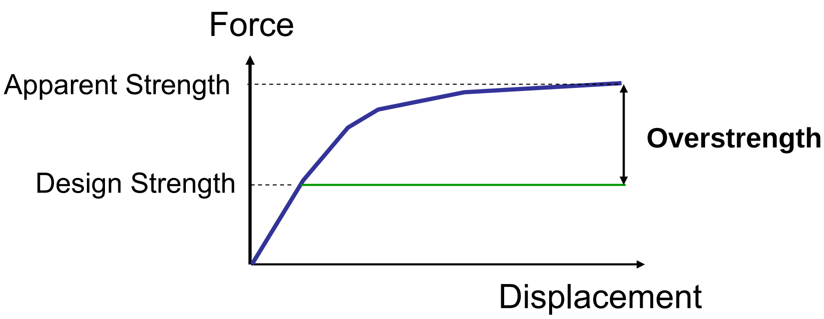
Sources of Overstrength

- Sequential yielding of critical regions
- Material overstrength (actual vs specified yield)
- Strain hardening
- Capacity reduction (ϕ) factors
- Member selection

This slide lists most of the sources of overstrength. It is not uncommon for the true strength of a structure, including overstrength, to be two to three times the design strength.

Definition of Overstrength Factor Ω

$$\text{Overstrength Factor } \Omega = \frac{\text{Apparent Strength}}{\text{Design Strength}}$$



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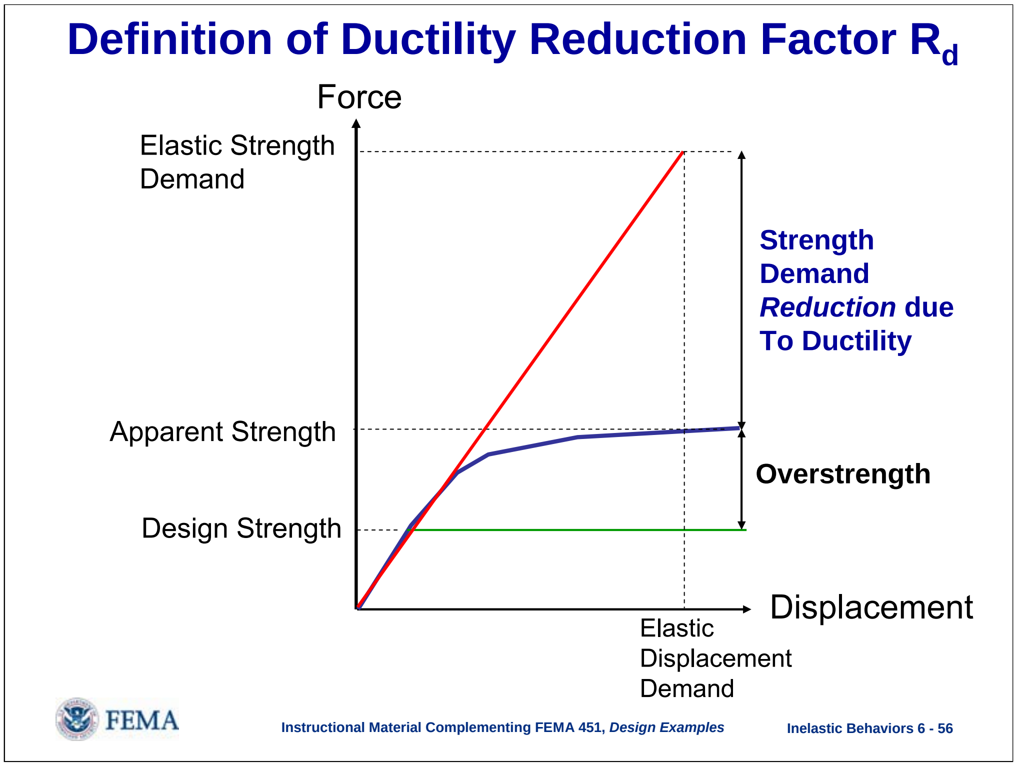
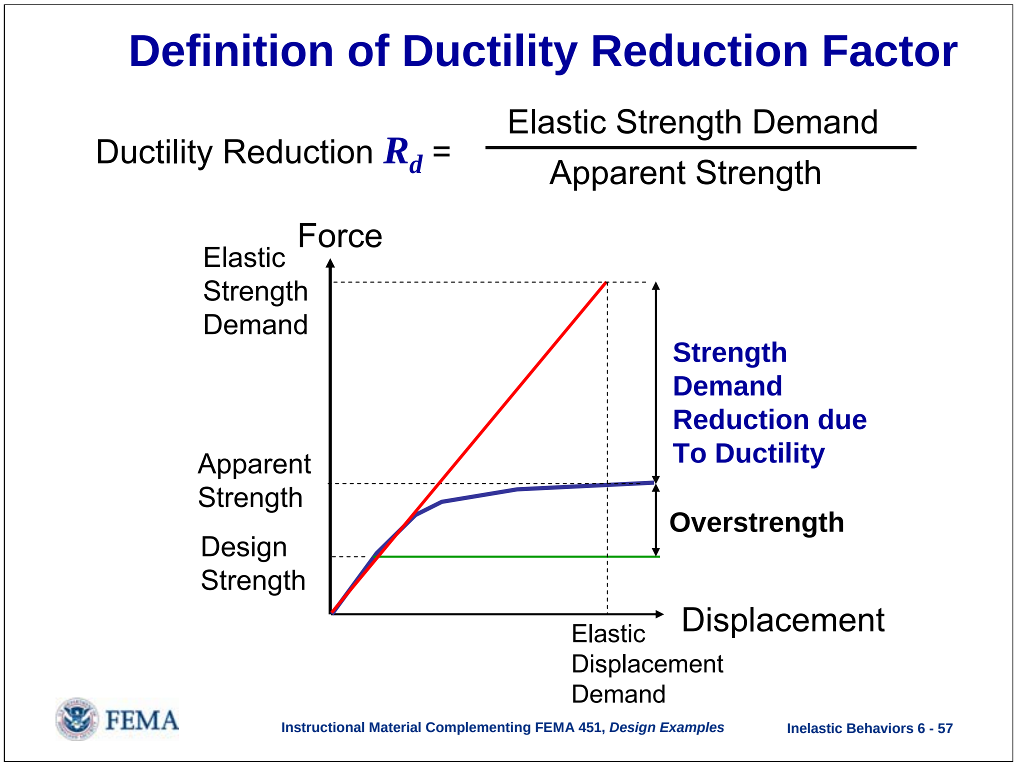
The apparent strength divided by the design strength is called the "overstrength factor." Note that the symbol Ω used for the overstrength factor is similar to the term Ω_0 in ASCE 7.

As implemented in ASCE 7, Ω_0 is an upper bound estimate of true overstrength. More will be said about this in the topic on seismic load analysis.



In the first part of this topic, it was shown that the displacement demand for a system of a given initial stiffness is independent of the yield strength. It was also shown that for a system of given minimum ductility, the maximum required strength demand was equal to the elastic strength demand divided by the ductility supply. This observation was made with regards to the apparent strength, not the design strength.

Hence, the apparent strength is 1/(ductility supply) times the elastic strength demand. The ductility supply is referred to as the ductility reduction factor and is represented by the symbol Rd.



This is simply a rearrangement of the information presented on the previous slide.

Definition of Response Modification Coefficient R

$$\text{Overstrength Factor } \Omega = \frac{\text{Apparent Strength}}{\text{Design Strength}}$$

$$\text{Ductility Reduction } R_d = \frac{\text{Elastic Strength Demand}}{\text{Apparent Strength}}$$

$$R = \frac{\text{Elastic Strength Demand}}{\text{Design Strength}} = R_d \, \Omega$$

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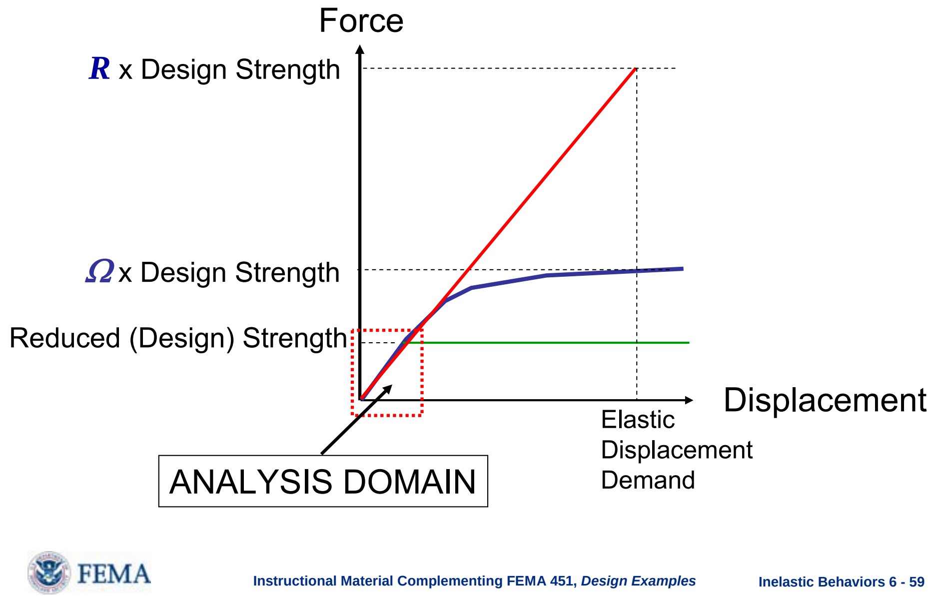
The ASCE 7 response modification factor, R, is equal to the ductility reduction factor times the overstrength factor.

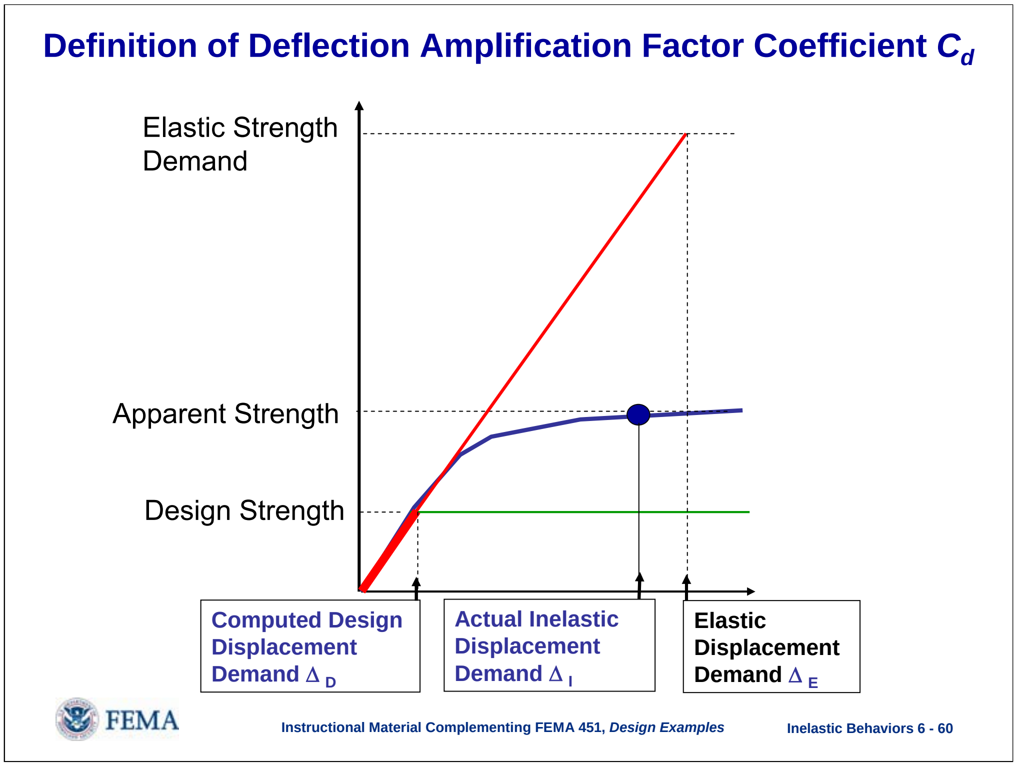
Definition of Response Modification Coefficient *R*



This figure is a graphical version of the information presented in the previous slide. The ASCE 7 response modification factor, *R*, is used to reduce the expected elastic strength demand to the DESIGN level strength demand.

On the basis of the equal displacement theory the inelastic displacement demand is the same as the elastic displacement demand. For design purposes, however, the reduced design strength is applied to the structure to determine the member forces.

The analysis domain represents the response of the linear elastic system as analyzed with the reduced forces. Clearly the displacement predicted by this analysis is too low. ASCE 7 compensates through the use of the C_d factor.



To correct for the too-low displacement predicted by the reduced force elastic analysis, the "computed design displacement" is multiplied by the factor C_d . This factor is always less than the *R* factor because *R* contains ingredients other than pure ductility.

ASCE 7 Approach for Displacements

Determine design forces: $V = C_s W$, where C_s includes ductility/overstrength reduction factor R.

Distribute forces vertically and horizontally and compute displacements using linear elastic analysis.

Multiply computed displacements by C_d to obtain estimate of true inelastic response.

FEMA Instructional Material Complementing FEMA 451, *Design Examples* Inelastic Behaviors 6 - 61

This slide summarizes the ASCE 7 approach with regards to computing system displacements.

Examples of Design Factors for Steel Structures ASCE 7-05

	R	Ω_o	R_d	C_d
Special Moment Frame	8	3	2.67	5.5
Intermediate Moment Frame	4.5	3	1.50	4.0
Ordinary Moment Frame	3.5	3	1.17	3.0
Eccentric Braced Frame	8	2	4.00	4.0
Eccentric Braced Frame (Pinned)	7	2	3.50	4.0
Special Concentric Braced Frame	6	2	3.00	5.0
Ordinary Concentric Braced Frame	3.25	2	1.25	3.25
Not Detailed	3	3	1.00	3.0

Note: R_d is ductility demand ONLY IF Ω_o is achieved.

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Table 12.2.1 of ASCE 7 provides the R, Ω_0, and C_d factors for a large number of structural systems. A small sample of these systems is shown here for steel structures.

It is very important to note that because Ω_0 as listed in the table is the "maximum expected overstrength," the "ductility demands," R_d, are minimums. Ductility demands of 1 or less indicate that the "expected" response for these systems is essentially elastic.

The "Not Detailed" systems are allowed only in SDC A through C.

Examples of Design Factors for Reinforced Concrete Structures ASCE 7-05

	R	Ω_o	R_d	C_d
Special Moment Frame	8	3	2.67	5.5
Intermediate Moment Frame	5	3	1.67	4.5
Ordinary Moment Frame	3	3	1.00	2.5
Special Reinforced Shear Wall	5	2.5	2.00	5.0
Ordinary Reinforced Shear Wall	4	2.5	1.60	4.0
Detailed Plain Concrete Wall	2	2.5	0.80	2.0
Ordinary Plain Concrete Wall	1.5	2.5	0.60	1.5

Note: R_d is Ductility Demand ONLY IF Ω_o is Achieved.

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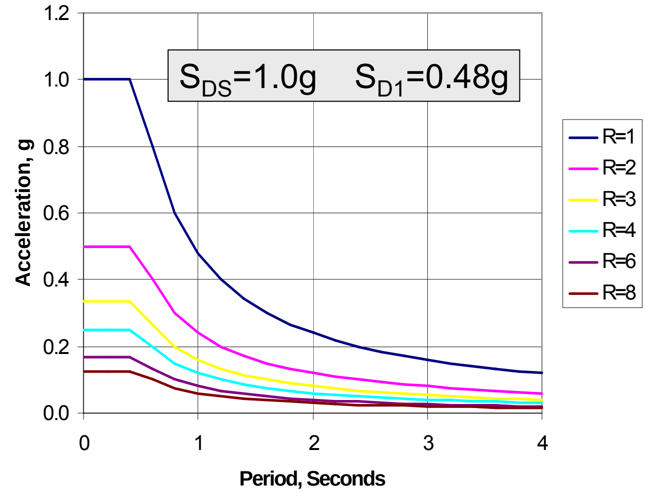
These are the design coefficients for a few selected concrete systems. Note the very low R_d values for the plain walls. These plain wall systems are allowed only in SDC A and B buildings.

ASCE 7 Elastic Spectra as Adjusted for Ductility and Overstrength

$$C_S = \frac{S_{DS}}{R/I}$$

$$C_S = \frac{S_{D1}}{T(R/I)}$$



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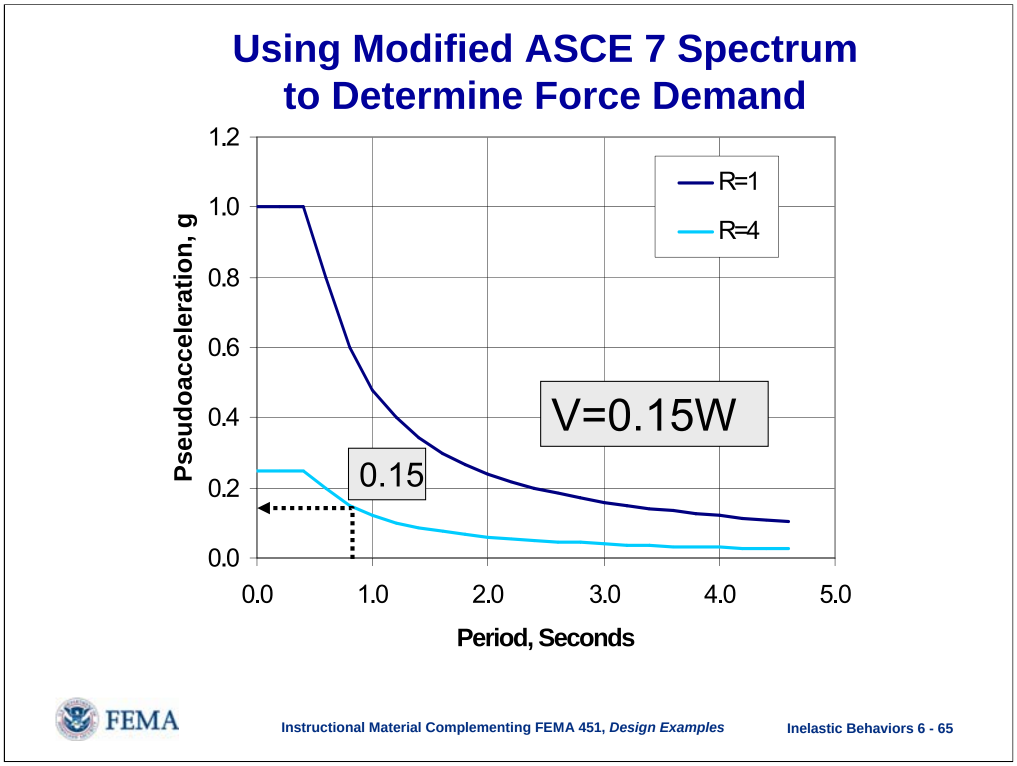
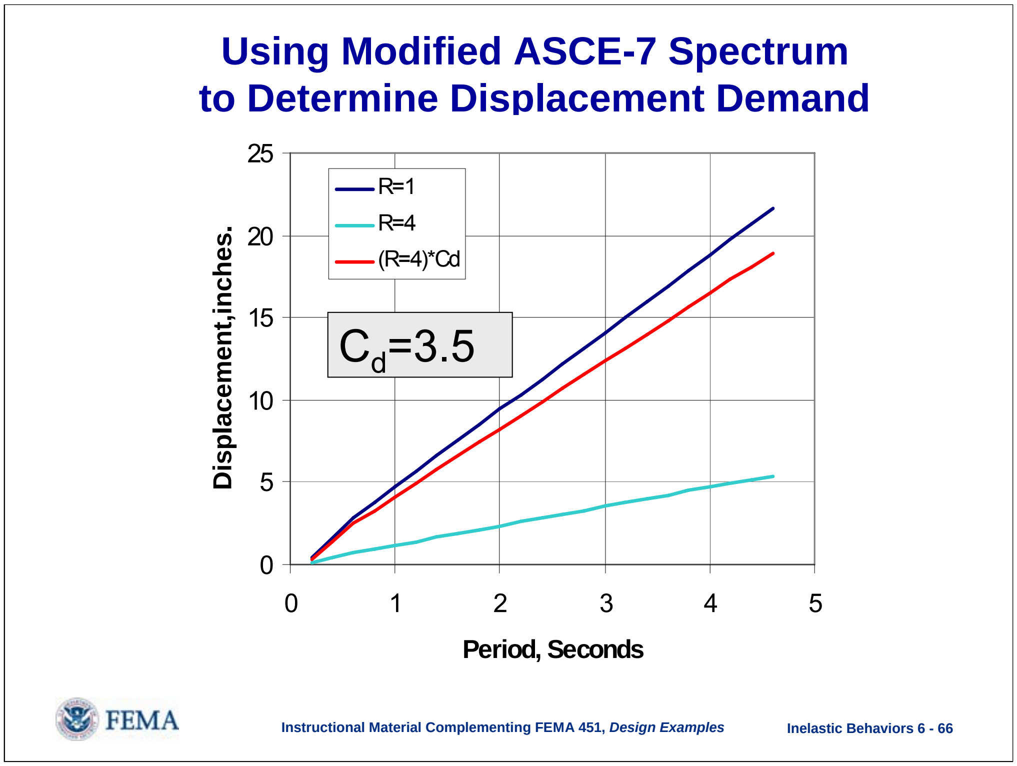
In the previous slides, the concept of the reduction factor, *R*, was presented, and several values were illustrated. Here, the effect of the R value of the design response spectrum is illustrated for *R* = 1 (elastic) through 6. The value for *R* = 1 has been normalized to produce a peak short period acceleration of 1.0g.



This slide simply shows how the design base shear is determined for a system with T = 0.8 seconds and R = 4.   The pseudoacceleration spectrum is used.

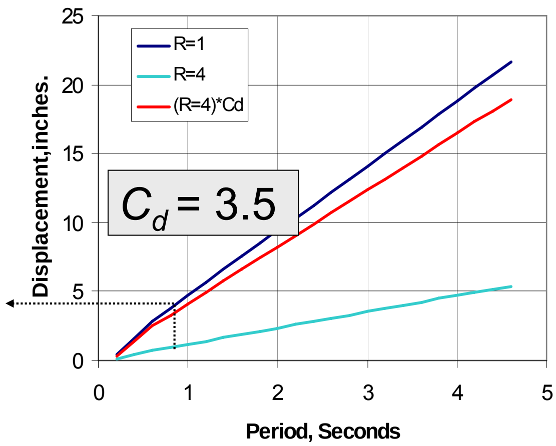


Here, the displacements are computed. First, the elastic displacement spectrum (determined by dividing the pseudoaccelration spectrum by ω^2) is obtained. Next, this is divided by *R*, producing the light blue line. This line cannot be used for determining displacements until is amplified by C_d at all period values, resulting in the red line.

Displacements must be multiplied by factor C_d because displacements based on reduced force would be too low

$$\Delta_{INELASTIC} = C_d \times \Delta_{REDUCEDELASTIC}$$

$$\Delta_{INELASTIC} = 3.65\ in.$$



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This slide further illustrates the point made in the previous slide. At the period of 0.8 the displacement is read of the red line, which includes C_d.

In practice, one would not generally use a displacement spectrum Instead, the displacements would be determined from the elastic model with the reduced (1/R) loads would be obtained, and these would be multiplied by C_d.

“Equal displacement” approach may not be applicable at very low period values.

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It has been shown that the equal displacement approach does not work very well for systems with low periods. The primary reason for this is that the low period systems tend to display significant residual deformations.

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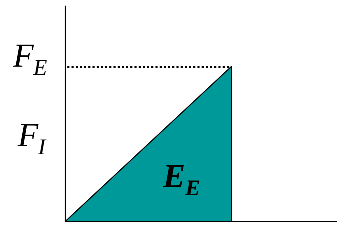
Equal Energy Concept
(Applicable at Low Periods)

ELASTIC ENERGY



$$\frac{F_E}{F_I}\delta_y$$

$$\boxed{E_E = 0.5F_E \frac{F_E}{F_I}\delta_y = 0.5\delta_y \frac{F_E^2}{F_I}}$$

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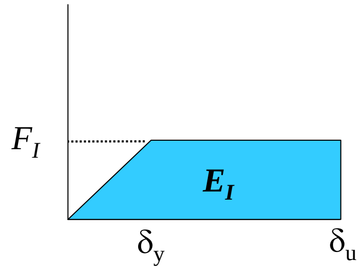
In this slide, the elastic energy demand is computed on the basis of the maximum elastic force, F_E, being attained.

Equal Energy Concept
(Applicable at Low Periods)

INELASTIC ENERGY



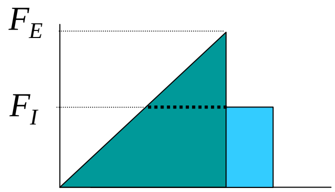
$$\boxed{E_I = F_I\delta_u - 0.5F_I\delta_y = F_I\delta_y\left(\mu - 0.5\right)}$$

FEMA   Instructional Material Complementing FEMA 451, *Design Examples*   Inelastic Behaviors 6 - 70

Now, the inelastic energy demand is computed.

Equal Energy Concept
(Applicable at Low Periods)

Assuming $E_E = E_I$:



$$\frac{F_E}{F_I} = \sqrt{2\mu - 1}$$

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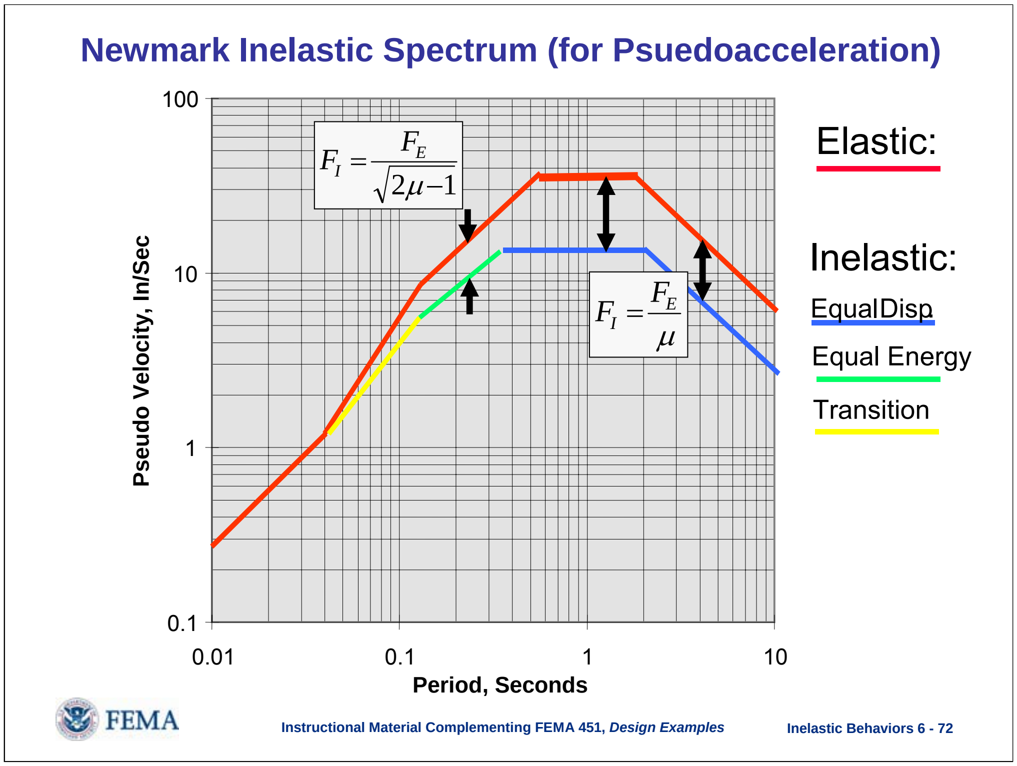
In the equal energy demand concept, the reduction from elastic force to design force is as shown. For example, if the ductility is 5, the ratio of elastic to inelastic force demand would be only 3. In the equal displacement realm, the ratio of elastic force to inelastic force demand would be 5.



The Newmark-Hall spectrum (presented in the topic on SDOF dynamics) may be converted into an "inelastic design response spectrum" by making the appropriate adjustments. To determine strength demands, the spectrum is divided by ductility in the higher period (equal displacement) realm but is divided by (2μ - 1) in the short period (equal energy) realm. Note that there is no modification at zero period because the spectral acceleration there must be equal to the peak ground acceleration.

Newmark's
Inelastic Design Response Spectrum

To obtain inelastic displacement spectrum, multiply the spectrum shown in previous slide by μ (for all periods).

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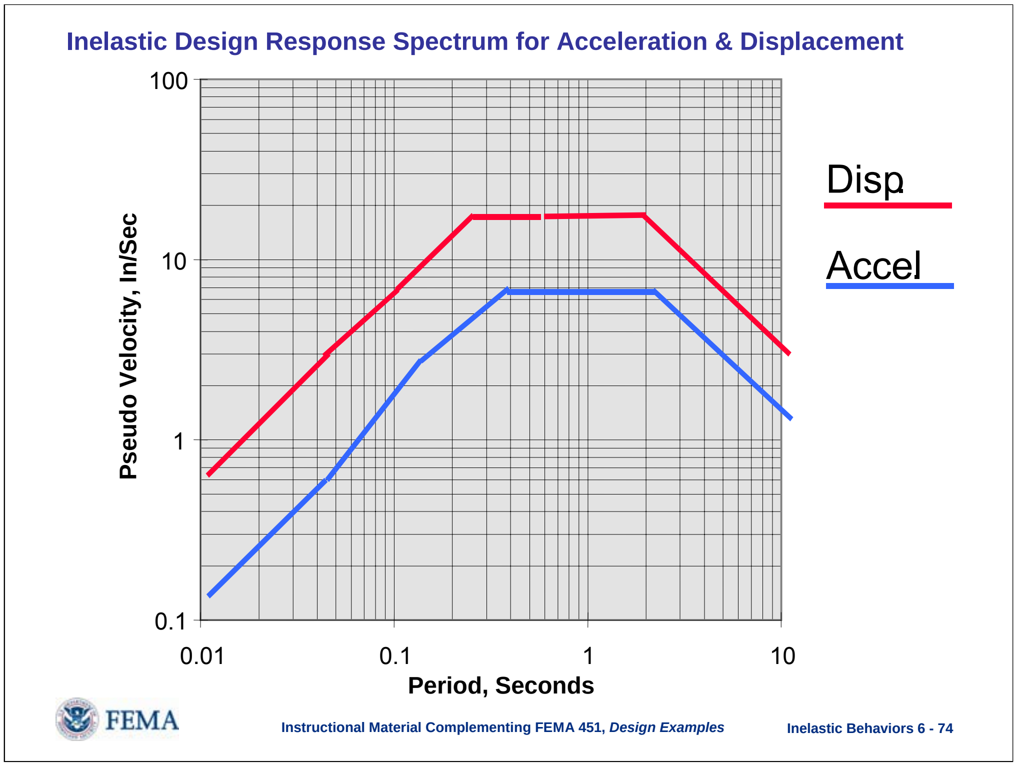
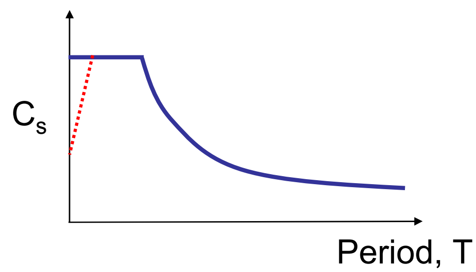
Self explanatory. See next slide for illustration.



To compute displacement, Newmark and Hall recommend that the inelastic spectrum be multiplied by the ductility across the ENTIRE period range. Note that his “amplifies” the displacements at the short period region.

At very low periods, the ASCE 7 spectrum does not reduce to ground acceleration so this partially compensates for "error" in equal displacement assumption at low period values.



Note: FEMA 273 has explicit modifications for computing "target at low periods."

ASCE 7 effectively divides the acceleration spectrum by ductility (the *R* factor) at all period ranges. However, there is some conservatism in this approach because the code allows no reduction to the peak ground acceleration in the very short period region.