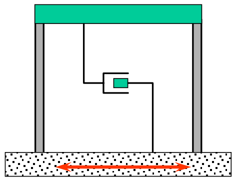
Structural Dynamics of Linear Elastic Single-Degree-of-Freedom (SDOF) Systems



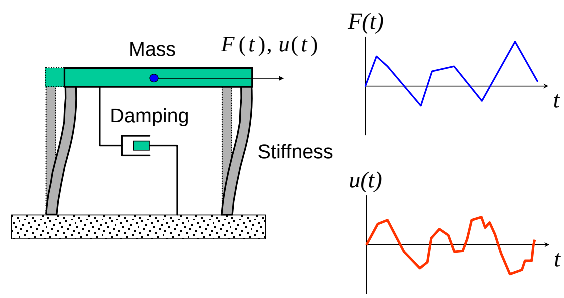
Structural Dynamics

- Equations of motion for SDOF structures
- Structural frequency and period of vibration
- Behavior under dynamic load
- Dynamic magnification and resonance
- Effect of damping on behavior
- Linear elastic response spectra

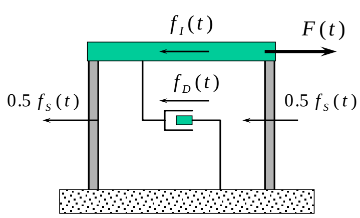
Importance in Relation to ASCE 7-05

- Ground motion maps provide ground accelerations in terms of *response spectrum* coordinates.
- Equivalent lateral force procedure gives base shear in terms of *design spectrum* and *period of vibration*.
- Response spectrum is based on *5% critical damping* in system.
- Modal superposition analysis uses design *response spectrum* as basic ground motion input.

Idealized SDOF Structure



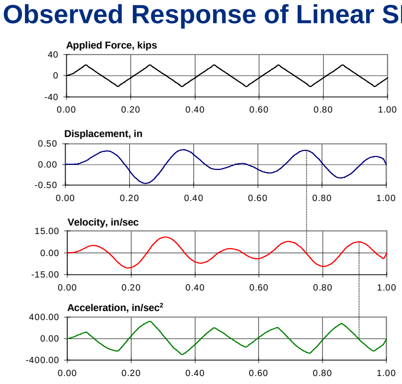
Equation of Dynamic Equilibrium



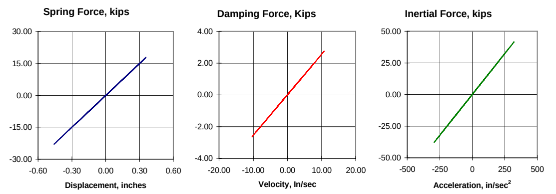
$$F(t) - f_I(t) - f_D(t) - f_S(t) = 0$$

$$f_I(t) + f_D(t) + f_S(t) = F(t)$$

Observed Response of Linear SDOF



Observed Response of Linear SDOF
(Development of Equilibrium Equation)



Slope = *k* = 50 kip/in

$f_S(t) = k\,u(t)$

Slope = *c* = 0.254 kip-sec/in

$f_D(t) = c\,\dot{u}(t)$

Slope = *m* = 0.130 kip-sec²/in

$f_I(t) = m\,\ddot{u}(t)$

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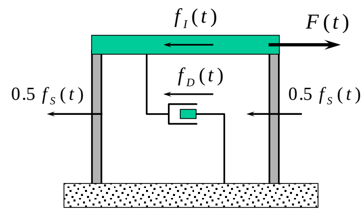
Equation of Dynamic Equilibrium



$$f_I(t) + f_D(t) + f_S(t) = F(t)$$

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = F(t)$$

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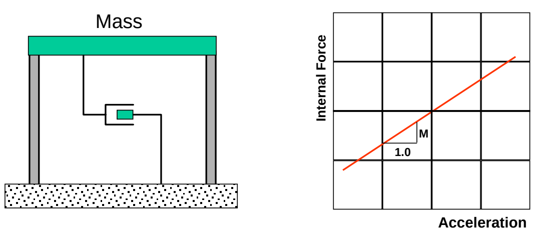
Properties of Structural Mass



- Includes all dead weight of structure
- May include some live load
- Has units of force/acceleration

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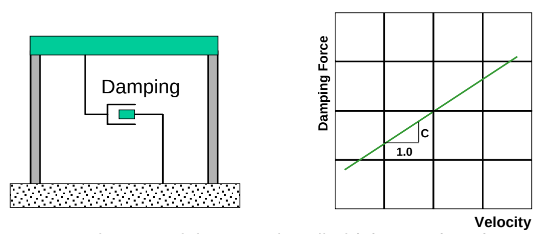
Properties of Structural Damping



- In absence of dampers, is called *inherent damping*
- Usually represented by *linear* viscous dashpot
- Has units of force/velocity

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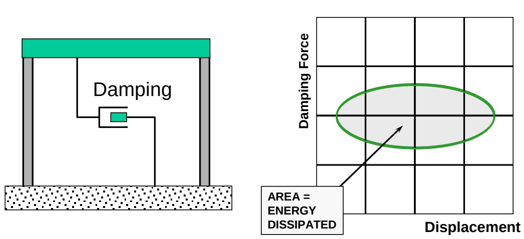
Properties of Structural Damping (2)



Damping vs displacement response is elliptical for linear viscous damper.

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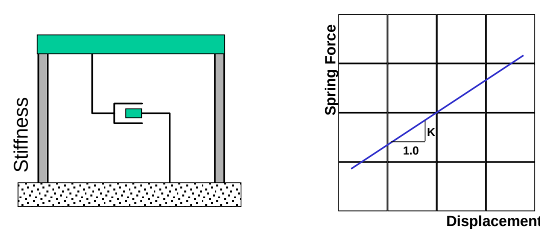
Properties of Structural Stiffness



- Includes all structural members
- May include some “seismically nonstructural” members
- Requires careful mathematical modelling
- Has units of force/displacement

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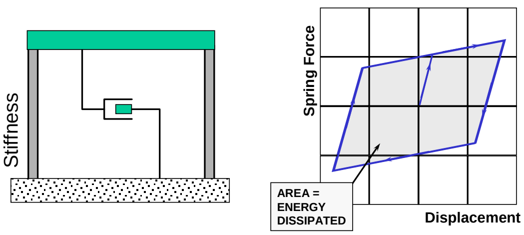
Properties of Structural Stiffness (2)



- Is almost always nonlinear in real seismic response
- Nonlinearity is implicitly handled by codes
- Explicit modelling of nonlinear effects is possible

Undamped Free Vibration

Equation of motion: $m\ddot{u}(t) + k\,u(t) = 0$

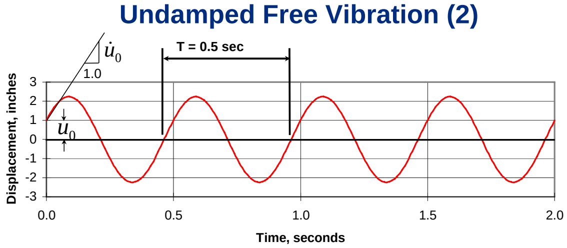
Initial conditions: $\dot{u}_0 \quad u_0$

Assume: $u(t) = A\sin(\omega t) + B\cos(\omega t)$

Solution: $A = \dfrac{\dot{u}_0}{\omega} \qquad B = u_0 \qquad \omega = \sqrt{\dfrac{k}{m}}$

$$u(t) = \frac{\dot{u}_0}{\omega}\sin(\omega t) + u_0\cos(\omega t)$$

Undamped Free Vibration (2)



Circular Frequency (radians/sec)

$$\omega = \sqrt{\frac{k}{m}}$$

Cyclic Frequency (cycles/sec, Hertz)

$$f = \frac{\omega}{2\pi}$$

Period of Vibration (sec/cycle)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Approximate Periods of Vibration (ASCE 7-05)

$$T_a = C_t h_n^x$$

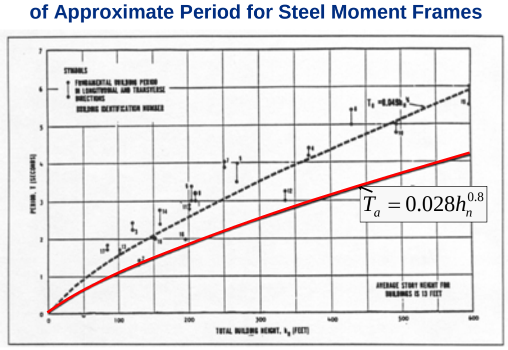
$C_t =$ 0.028, x = 0.8 for steel moment frames
$C_t =$ 0.016, x = 0.9 for concrete moment frames
$C_t =$ 0.030, x = 0.75 for eccentrically braced frames
$C_t =$ 0.020, x = 0.75 for all other systems

Note: This applies ONLY to building structures!

$$T_a = 0.1N$$

For moment frames < 12 stories in height, minimum story height of 10 feet. N = number of stories.

Empirical Data for Determination of Approximate Period for Steel Moment Frames



Periods of Vibration of Common Structures

20-story moment resisting frame	T = 1.9 sec
10-story moment resisting frame	T = 1.1 sec
1-story moment resisting frame	T = 0.15 sec
20-story braced frame	T = 1.3 sec
10-story braced frame	T = 0.8 sec
1-story braced frame	T = 0.1 sec
Gravity dam	T = 0.2 sec
Suspension bridge	T = 20 sec

Adjustment Factor on Approximate Period
(Table 12.8-1 of ASCE 7-05)

$$T = T_a C_u \le T_{computed}$$

S_{D1}	C_u
> 0.40g	1.4
0.30g	1.4
0.20g	1.5
0.15g	1.6
< 0.1g	1.7

Applicable **ONLY** if $T_{computed}$ comes from a "properly substantiated analysis."

Which Period of Vibration to Use in ELF Analysis?

If you do not have a "more accurate" period (from a computer analysis), you must use $T = T_a$.

If you have a more accurate period from a computer analysis (call this T_c), then:

if $T_c > C_u T_a$ use $T = C_u T_a$

if $T_a < T_c < T_u C_a$ use $T = T_c$

if $T_c < T_a$ use $T = T_a$

Damped Free Vibration

Equation of motion: $m\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = 0$

Initial conditions: $u_0 \quad \dot{u}_0$

Assume: $u(t) = e^{st}$

Solution:

$$u(t) = e^{-\xi\omega t}\left[u_0 \cos(\omega_D t) + \frac{\dot{u}_0 + \xi\omega u_0}{\omega_D}\sin(\omega_D t)\right]$$

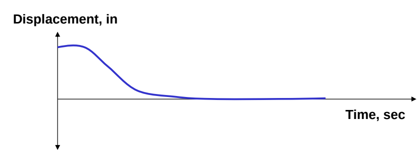
$$\xi = \frac{c}{2m\omega} = \frac{c}{c_c} \qquad \omega_D = \omega\sqrt{1-\xi^2}$$

Damping in Structures

$\xi = \dfrac{c}{2m\omega} = \dfrac{c}{c_c}$ c_c is the c*ritical damping constant.*

ξ is expressed as a ratio (0.0 < ξ < 1.0) in computations.

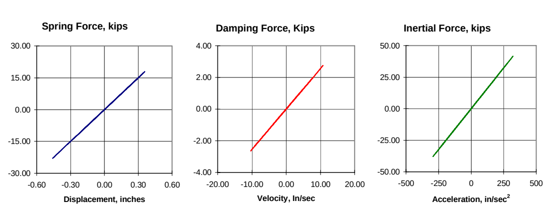
Sometimes ξ is expressed as a% (0 < ξ < 100%).



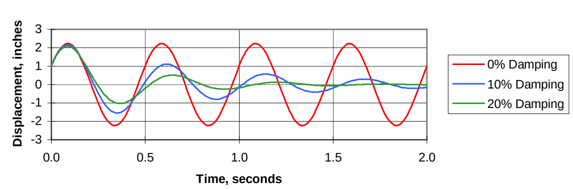
Response of Critically Damped System, ξ=*1.0* or 100% critical

Damping in Structures

True damping in structures is NOT viscous. However, for low damping values, viscous damping allows for linear equations and vastly simplifies the solution.



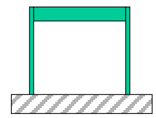
Damped Free Vibration (2)



Damping in Structures (2)

Welded steel frame	ξ = 0.010
Bolted steel frame	ξ = 0.020
Uncracked prestressed concrete	ξ = 0.015
Uncracked reinforced concrete	ξ = 0.020
Cracked reinforced concrete	ξ = 0.035
Glued plywood shear wall	ξ = 0.100
Nailed plywood shear wall	ξ = 0.150
Damaged steel structure	ξ = 0.050
Damaged concrete structure	ξ = 0.075
Structure with added damping	ξ = 0.250

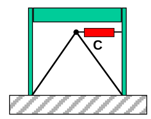
Damping in Structures (3)



Inherent damping

ξ is a structural (material) property independent of mass and stiffness

$$\xi_{Inherent} = 0.5 \text{ to } 7.0\% \text{ critical}$$

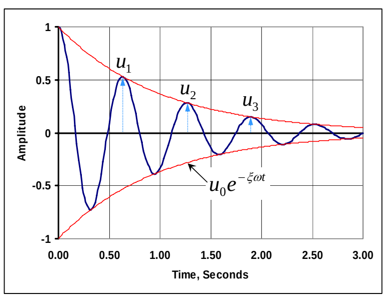


Added damping

ξ is a structural property dependent on mass and stiffness and damping constant C of device

$$\xi_{Added} = 10 \text{ to } 30\% \text{ critical}$$

Measuring Damping from Free Vibration Test



For all damping values

$$\ln \frac{u_1}{u_2} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

For very low damping values

$$\xi \cong \frac{u_1 - u_2}{2\pi u_2}$$

Undamped Harmonic Loading

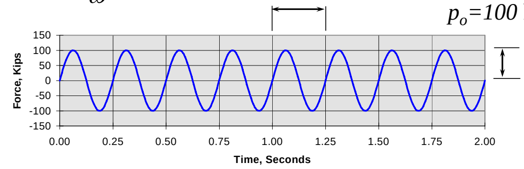
Equation of motion: $m\ddot{u}(t) + ku(t) = p_0 \sin(\bar{\omega} t)$

$\bar{\omega}$ **= frequency of the forcing function**

$$\bar{T} = \frac{2\pi}{\bar{\omega}}$$

$\bar{T}$ = *0.25 sec*

p_o=*100* kips



Undamped Harmonic Loading (2)

Equation of motion: $m\,\ddot{u}(t) + k\,u(t) = p_0 \sin(\bar{\omega} t)$

Assume system is initially at rest:

Particular solution: $u(t) = C \sin(\bar{\omega} t)$

Complimentary solution: $u(t) = A\sin(\omega t) + B\cos(\omega t)$

Solution:

$$u(t) = \frac{p_0}{k} \frac{1}{1-(\bar{\omega}/\omega)^2} \left(\sin(\bar{\omega} t) - \frac{\bar{\omega}}{\omega} \sin(\omega t) \right)$$

Undamped Harmonic Loading

Define $\beta = \dfrac{\bar{\omega}}{\omega}$ **Loading frequency** / **Structure's natural frequency**

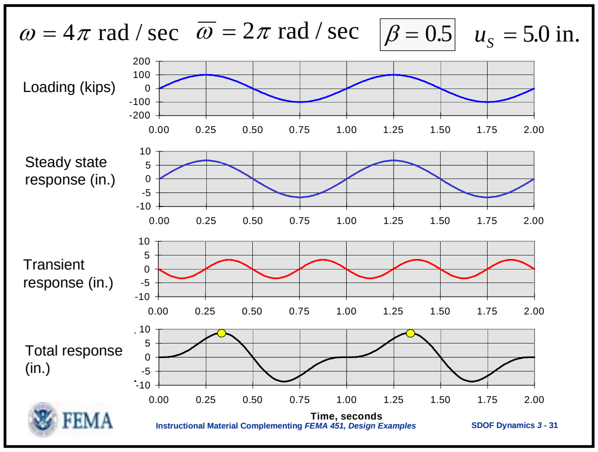
Dynamic magnifier

Transient response (at structure's frequency)

$$u(t) = \frac{p_0}{k} \frac{1}{1-\beta^2} \left(\sin(\bar{\omega} t) - \beta \sin(\omega t) \right)$$

Static displacement

Steady state response (at loading frequency)


$\omega = 4\pi$ rad / sec $\overline{\omega} = 2\pi$ rad / sec $\beta = 0.5$ $u_S = 5.0$ in.
Loading (kips)
Steady state response (in.)
Transient response (in.)
Total response (in.)
Time, seconds
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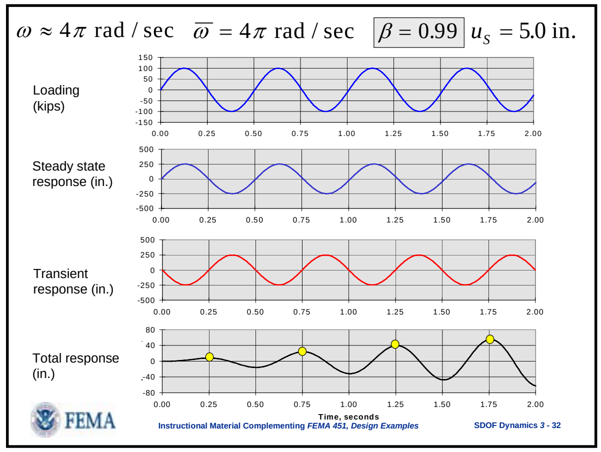
$\omega \approx 4\pi$ rad / sec $\overline{\omega} = 4\pi$ rad / sec $\beta = 0.99$ $u_S = 5.0$ in.
Loading (kips)
Steady state response (in.)
Transient response (in.)
Total response (in.)
Time, seconds
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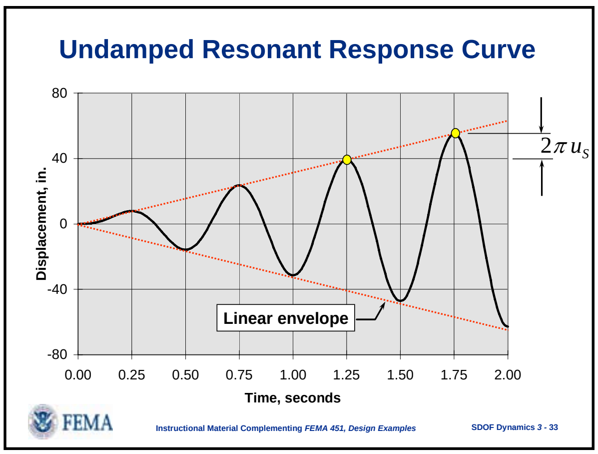
Undamped Resonant Response Curve
Displacement, in.
Time, seconds
$2\pi u_S$
Linear envelope
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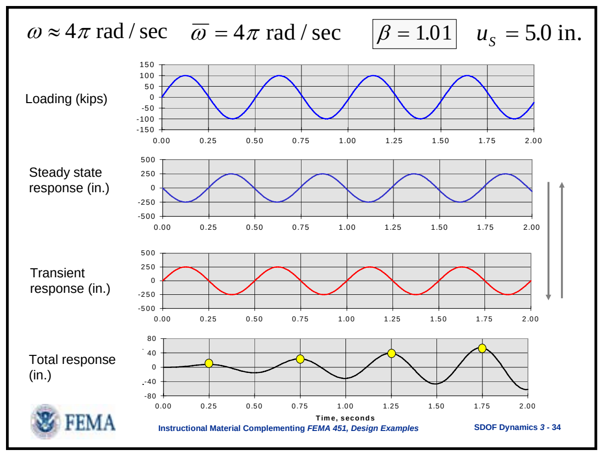
SDOF Dynamics 3 - 33


$\omega \approx 4\pi$ rad / sec $\overline{\omega} = 4\pi$ rad / sec $\beta = 1.01$ $u_S = 5.0$ in.
Loading (kips)
Steady state response (in.)
Transient response (in.)
Total response (in.)
Time, seconds
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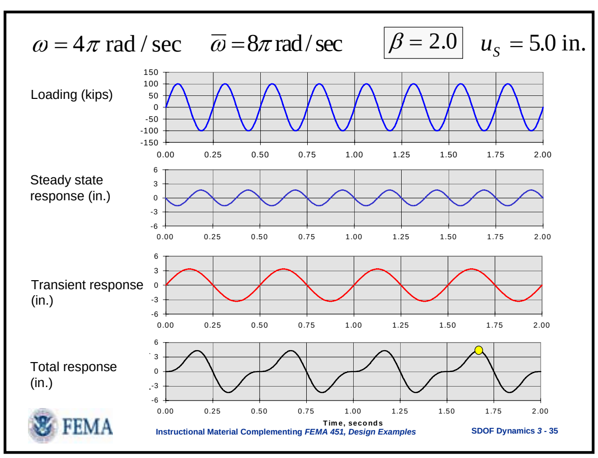
$\omega = 4\pi$ rad / sec $\overline{\omega} = 8\pi$ rad / sec $\beta = 2.0$ $u_S = 5.0$ in.
Loading (kips)
Steady state response (in.)
Transient response (in.)
Total response (in.)
Time, seconds
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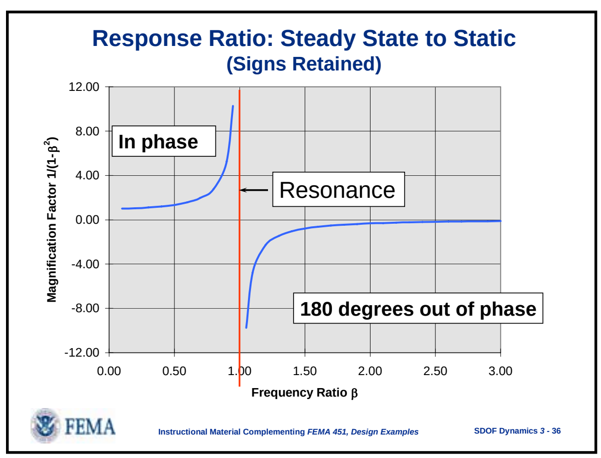
Response Ratio: Steady State to Static
(Signs Retained)
Magnification Factor $1/(1-\beta^2)$
Frequency Ratio β
In phase
Resonance
180 degrees out of phase
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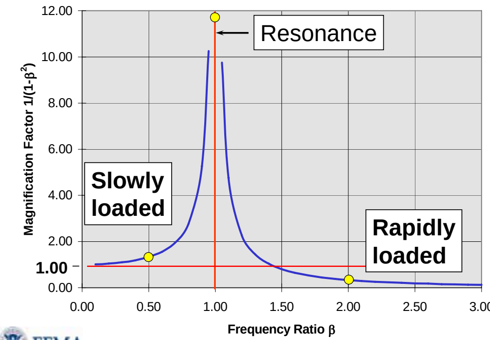
Response Ratio: Steady State to Static
(Absolute Values)



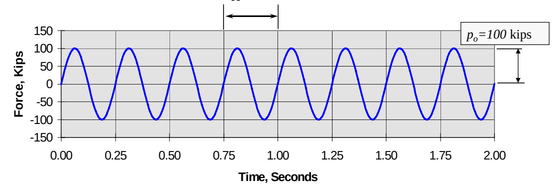
FEMA

Damped Harmonic Loading

Equation of motion:

$$m\,\ddot{u}(t) + \boxed{c\dot{u}(t)} + k\,u(t) = p_0 \sin(\overline{\omega} t)$$

$$\overline{T} = \frac{2\pi}{\overline{\omega}} = 0.25\,\text{sec}$$



FEMA

Damped Harmonic Loading

Equation of motion:

$$m\,\ddot{u}(t) + c\dot{u}(t) + k\,u(t) = p_0 \sin(\overline{\omega} t)$$

Assume system is initially at rest

Particular solution: $u(t) = C\sin(\overline{\omega}t) + D\cos(\overline{\omega}t)$

Complimentary solution:

$$u(t) = e^{-\xi\omega t}\left[A\sin(\omega_D t) + B\cos(\omega_D t)\right]$$

$$\xi = \frac{c}{2m\omega}$$

Solution:

$$u(t) = e^{-\xi\omega t}\left[A\sin(\omega_D t) + B\cos(\omega_D t)\right] + C\sin(\overline{\omega}t) + D\cos(\overline{\omega}t)$$

$$\omega_D = \omega\sqrt{1-\xi^2}$$

FEMA

Damped Harmonic Loading

Transient response at structure's frequency (eventually damps out)

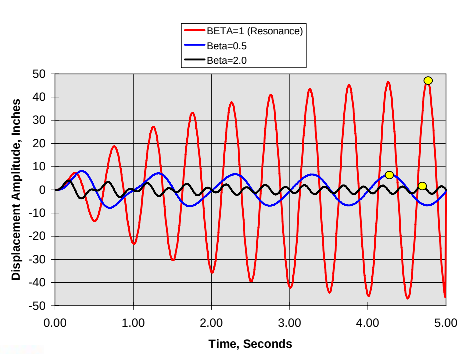
$$u(t) = \boxed{e^{-\xi\omega t}\left[A\sin(\omega_D t) + B\cos(\omega_D t)\right]} + \boxed{C\sin(\overline{\omega}t) + D\cos(\overline{\omega}t)}$$

Steady state response, at loading frequency

$$C = \frac{p_o}{k}\frac{1-\beta^2}{(1-\beta^2)^2 + (2\xi\beta)^2} \qquad D = \frac{p_o}{k}\frac{-2\xi\beta}{(1-\beta^2)^2 + (2\xi\beta)^2}$$

FEMA

Damped Harmonic Loading (5% Damping)



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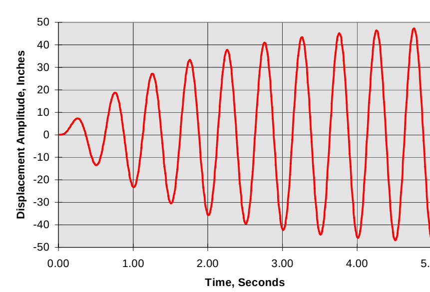
Damped Harmonic Loading (5% Damping)

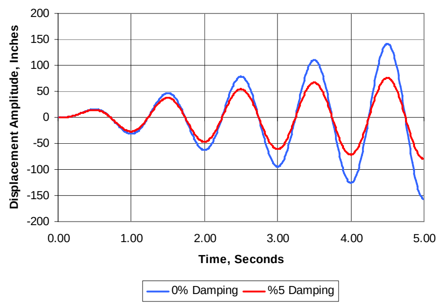


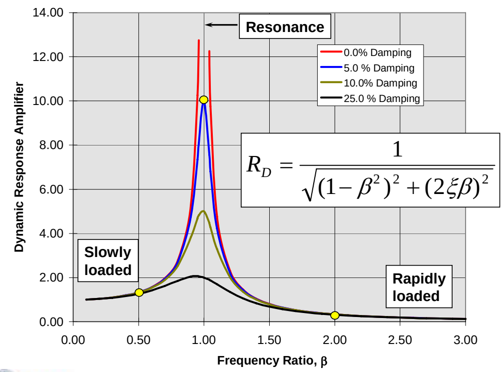
$$\frac{1}{2\xi}\delta_{Static}$$

FEMA

Harmonic Loading at Resonance

Effects of Damping





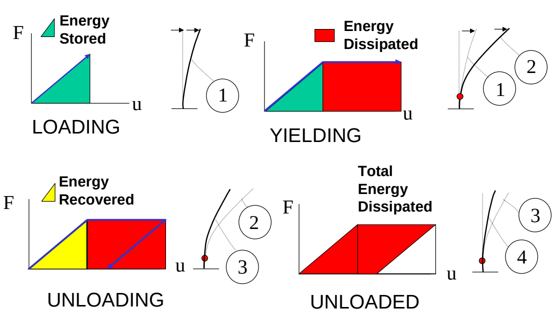
Summary Regarding Viscous Damping in Harmonically Loaded Systems

- For systems loaded at a frequency near their natural frequency, the dynamic response exceeds the static response. This is referred to as *dynamic amplification*.
- An undamped system, loaded at resonance, will have an unbounded increase in displacement over time.

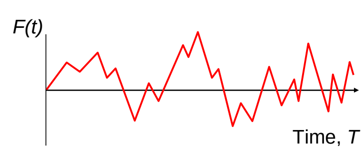
Summary Regarding Viscous Damping in Harmonically Loaded Systems

- Damping is an effective means for *dissipating energy* in the system. Unlike strain energy, which is recoverable, dissipated energy is not recoverable.
- A damped system, loaded at resonance, will have a limited displacement over time with the limit being ($1/2\xi$) times the static displacement.
- Damping is most effective for systems loaded at or near resonance.

CONCEPT of ENERGY STORED and Energy DISSIPATED



General Dynamic Loading

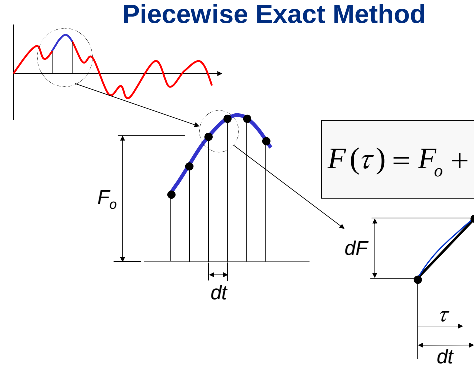


General Dynamic Loading Solution Techniques

- Fourier transform
- Duhamel integration
- Piecewise exact
- Newmark techniques

All techniques are carried out numerically.

Piecewise Exact Method



$$F(\tau) = F_o + \tau \frac{dF}{dt}$$

Piecewise Exact Method

Initial conditions $u_{o,0} = 0 \quad \dot{u}_{o,0} = 0$

Determine "exact" solution for 1st time step

$$u_1 = u(\tau) \qquad \dot{u}_1 = \dot{u}(\tau) \qquad \ddot{u}_1 = \ddot{u}(\tau)$$

Establish new initial conditions

$$u_{o,1} = u(\tau) \qquad \dot{u}_{0,1} = \dot{u}(\tau)$$

LOOP

Obtain exact solution for next time step

$$u_2 = u(\tau) \qquad \dot{u}_2 = \dot{u}(\tau) \qquad \ddot{u}_2 = \ddot{u}(\tau)$$

Piecewise Exact Method

Advantages:

- Exact if load increment is linear
- Very computationally efficient

Disadvantages:

- Not generally applicable for inelastic behavior

Note: NONLIN uses the piecewise exact method for response spectrum calculations.

Newmark Techniques

- Proposed by Nathan Newmark
- General method that encompasses a family of different integration schemes
- Derived by:
  - Development of incremental equations of motion
  - Assuming acceleration response over short time step

Newmark Method

Advantages:

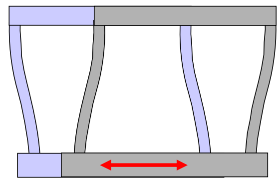
- Works for inelastic response

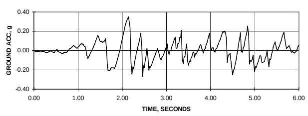
Disadvantages:

- Potential numerical error

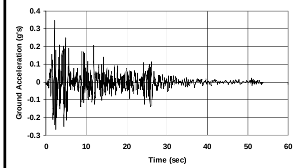
Note: NONLIN uses the Newmark method for general response history calculations

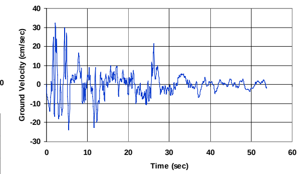
Development of Effective Earthquake Force

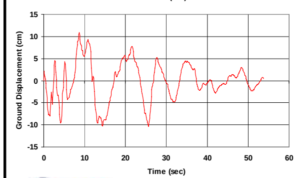




Earthquake Ground Motion, 1940 El Centro

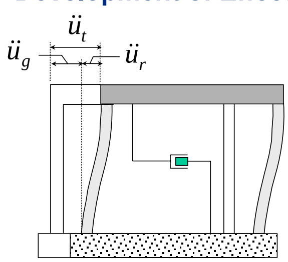




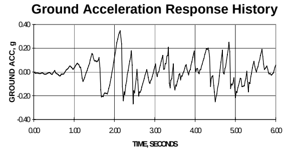


Many ground motions now are available via the Internet.

Development of Effective Earthquake Force



Ground Acceleration Response History



$$m[\ddot{u}_g(t)+\ddot{u}_r(t)]+c\,\dot{u}_r(t)+k\,u_r(t)=0$$

$$\boxed{m\ddot{u}_r(t)+c\,\dot{u}_r(t)+k\,u_r(t)=-m\ddot{u}_g(t)}$$

"Simplified" form of Equation of Motion:

$$m\ddot{u}_r(t)+c\dot{u}_r(t)+ku_r(t)=-m\ddot{u}_g(t)$$

Divide through by ***m***:

$$\ddot{u}_r(t)+\frac{c}{m}\dot{u}_r(t)+\frac{k}{m}u_r(t)=-\ddot{u}_g(t)$$

Make substitutions:

$$\frac{c}{m}=2\xi\omega \qquad \frac{k}{m}=\omega^2$$

Simplified form:

$$\boxed{\ddot{u}_r(t)+2\xi\omega\dot{u}_r(t)+\omega^2u_r(t)=-\ddot{u}_g(t)}$$

For a given ground motion, the response history $u_r(t)$ is function of the structure's frequency ω and damping ratio ξ.

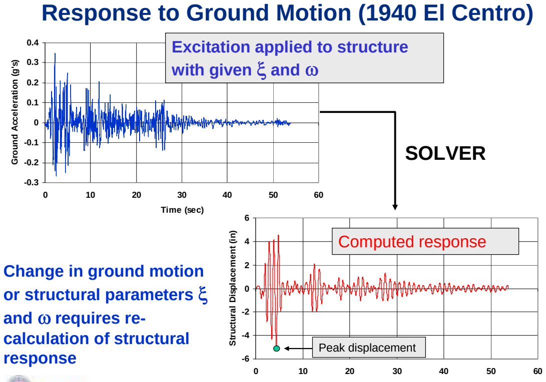
Structural frequency

$$\boxed{\ddot{u}_r(t)+2\xi\omega\dot{u}_r(t)+\omega^2u_r(t)=-\ddot{u}_g(t)}$$

Damping ratio

Ground motion acceleration history

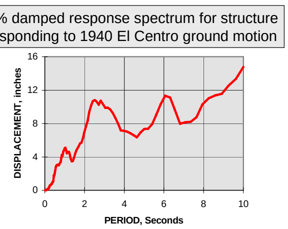
Response to Ground Motion (1940 El Centro)



Change in ground motion or structural parameters ξ and ω requires re-calculation of structural response

The Elastic Displacement Response Spectrum

An ***elastic displacement response spectrum*** is a plot of the peak computed relative displacement, u_r, for an elastic structure with a constant damping ξ, a varying fundamental frequency ω (or period $T = 2\pi/\omega$), responding to a given ground motion.



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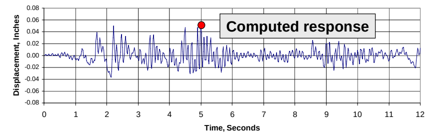
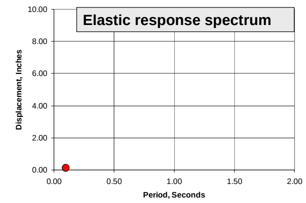
Computation of Response Spectrum for El Centro Ground Motion



$\xi = 0.05$

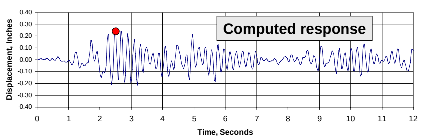
$T = 0.10$ sec

$U_{max} = 0.0543$ in.



Instructional Material Complementing *FEMA 451, Design Examples* SDOF Dynamics 3 - 62

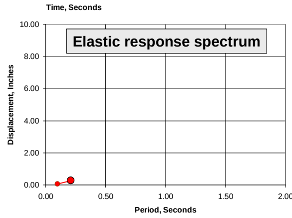
Computation of Response Spectrum for El Centro Ground Motion



$\xi = 0.05$

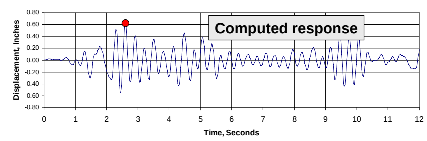
$T = 0.20$ sec

$U_{max} = 0.254$ in.



Instructional Material Complementing *FEMA 451, Design Examples* SDOF Dynamics 3 - 63

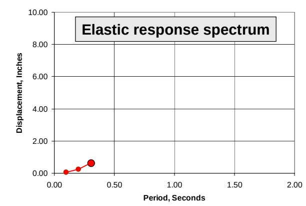
Computation of Response Spectrum for El Centro Ground Motion



$\xi = 0.05$

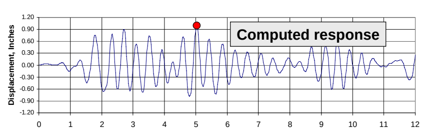
$T = 0.30$ sec

$U_{max} = 0.622$ in.



Instructional Material Complementing *FEMA 451, Design Examples* SDOF Dynamics 3 - 64

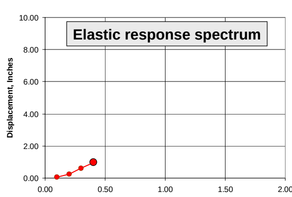
Computation of Response Spectrum for El Centro Ground Motion



$\xi = 0.05$

$T = 0.40$ sec

$U_{max} = 0.956$ in.



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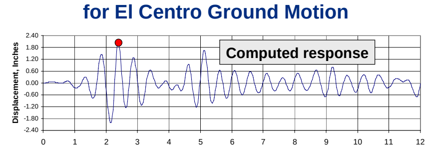
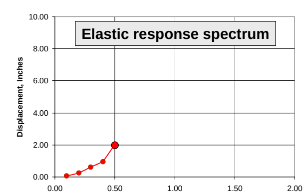
Computation of Response Spectrum for El Centro Ground Motion



$\xi = 0.05$

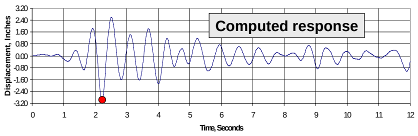
$T = 0.50$ sec

$U_{max} = 2.02$ in.



Instructional Material Complementing *FEMA 451, Design Examples* SDOF Dynamics 3 - 66

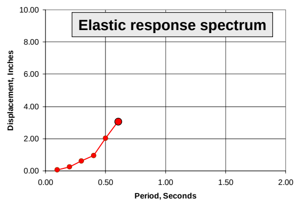
Computation of Response Spectrum for El Centro Ground Motion



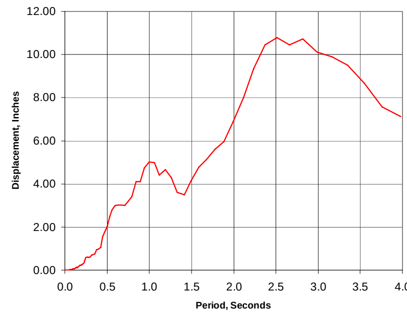
$\xi = 0.05$

$T = 0.60$ sec

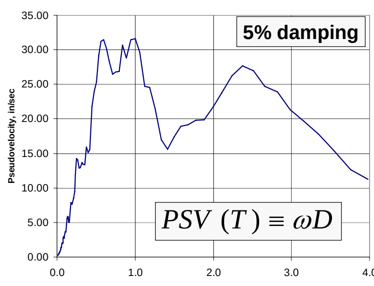
$U_{max} = -3.00$ in.



Complete 5% Damped Elastic Displacement Response Spectrum for El Centro Ground Motion

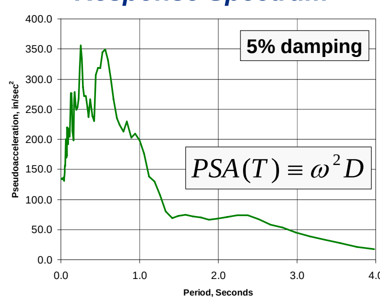


Development of *Pseudovelocity Response Spectrum*



$$PSV(T) \equiv \omega D$$

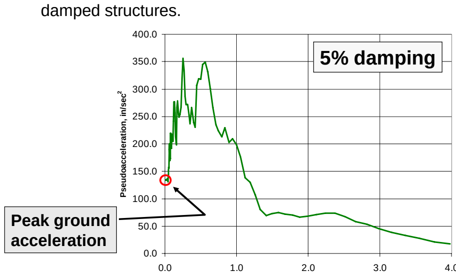
Development of *Pseudoacceleration Response Spectrum*



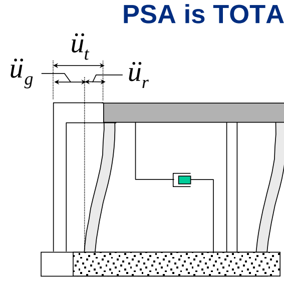
$$PSA(T) \equiv \omega^2 D$$

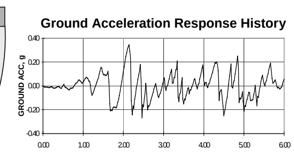
Note About the Pseudoacceleration Response Spectrum

The pseudoacceleration response spectrum represents the **total acceleration** of the system, not the relative acceleration. It is nearly identical to the true total acceleration response spectrum for lightly damped structures.



PSA is TOTAL Acceleration!



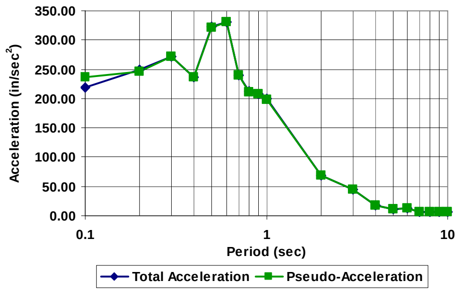


$$m[\ddot{u}_g(t) + \ddot{u}_r(t)] + c\dot{u}_r(t) + k u_r(t) = 0$$

$$m\ddot{u}_r(t) + c\dot{u}_r(t) + k u_r(t) = -m\ddot{u}_g(t)$$

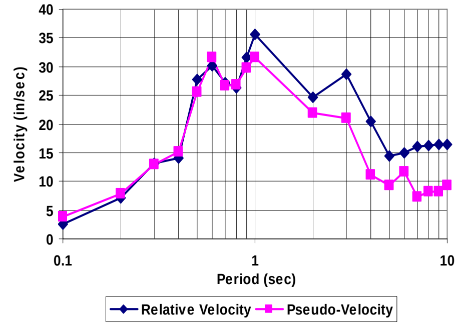
Difference Between Pseudo-Acceleration and Total Acceleration

(System with 5% Damping)

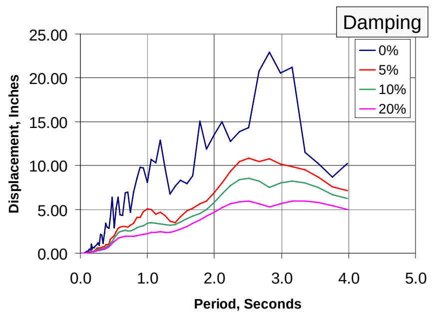


Difference Between Pseudovelocity and Relative Velocity

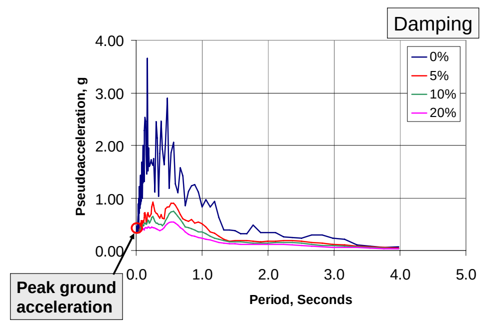
(System with 5% Damping)



Displacement Response Spectra for Different Damping Values



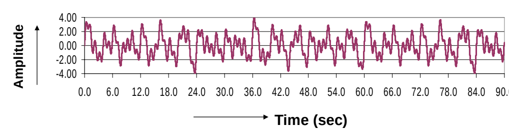
Pseudoacceleration Response Spectra for Different Damping Values



Damping Is Effective in Reducing the Response for (Almost) Any Given Period of Vibration

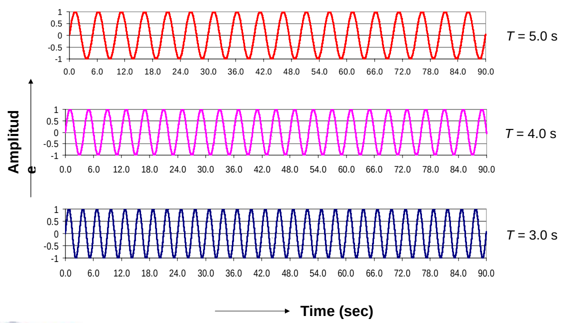
- An earthquake record can be considered to be the combination of a large number of harmonic components.
- Any SDOF structure will be in near resonance with one of these harmonic components.
- Damping is most effective at or near resonance.
- Hence, a response spectrum will show reductions due to damping at all period ranges (except $T = 0$).

Damping Is Effective in Reducing the Response for Any Given Period of Vibration

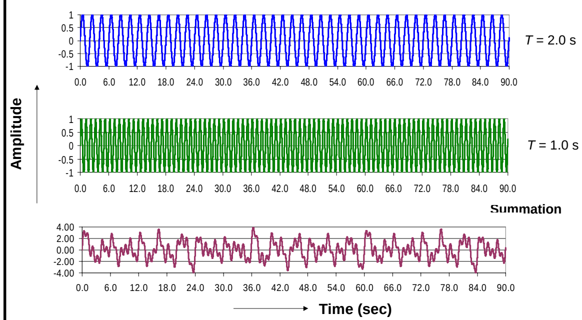


- Example of an artificially generated wave to resemble a real time ground motion accelerogram.
- Generated wave obtained by combining five different harmonic signals, each having equal amplitude of 1.0.

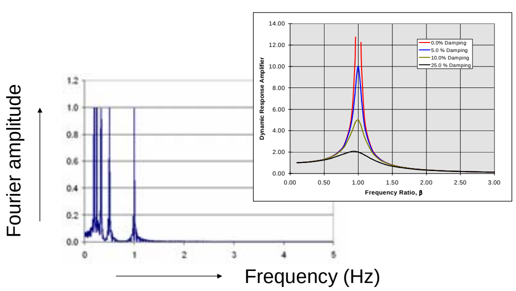
The Artificial Wave Is the Sum of Five Harmonics



The Artificial Wave Is the Sum of Five Harmonics



Damping Reduces the Response at Each Resonant Frequency

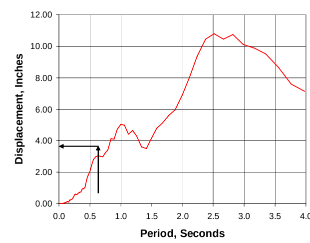


FFT curve for the combined wave

Use of an Elastic Response Spectrum

Example Structure

K = 500 k/in
W = 2,000 k
M = 2000/386.4 = 5.18 k-sec^2/in
$\omega = (K/M)^{0.5}$ =9.82 rad/sec
T = $2\pi/\omega$ = 0.64 sec
5% critical damping

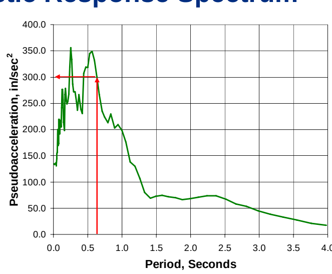


At *T* = 0.64 sec, displacement = 3.03 in.

Use of an Elastic Response Spectrum

Example Structure

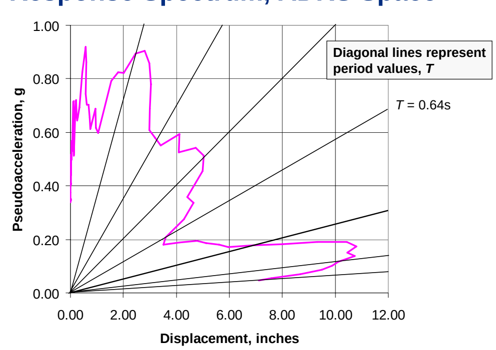
K = 500 k/in
W = 2,000 k
M = 2000/386.4 = 5.18 k-sec^2/in
$\omega = (K/M)^{0.5}$ =9.82 rad/sec
T = $2\pi/\omega$ = 0.64 sec
5% critical damping



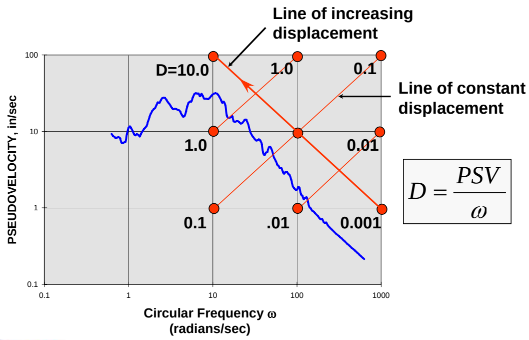
At *T* = 0.64 sec, pseudoacceleration = 301 in./sec^2

Base shear = *M* x *PSA* = 5.18(301) = 1559 kips

Response Spectrum, ADRS Space



Four-Way Log Plot of Response Spectrum



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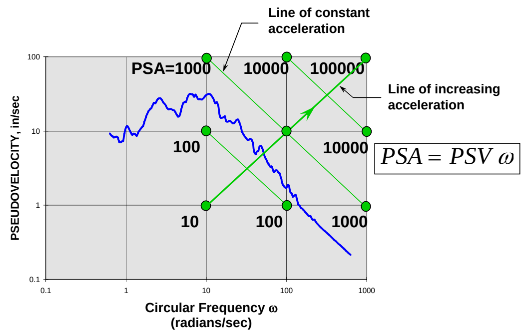
Four-Way Log Plot of Response Spectrum



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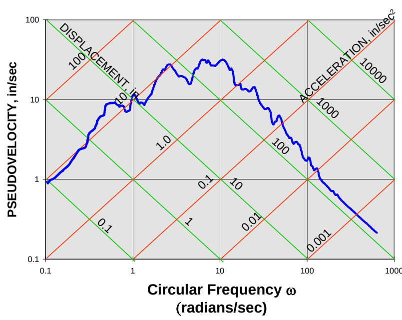
Four-Way Log Plot of Response Spectrum



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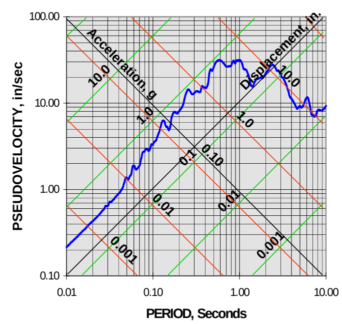
Four-Way Log Plot of Response Spectrum

Plotted vs Period



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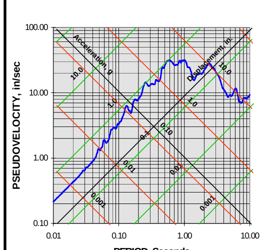
Development of an Elastic Response Spectrum



Problems with Current Spectrum:

For a given earthquake, small variations in structural frequency (period) can produce significantly different results.

It is for a single earthquake; other earthquakes will have different Characteristics.

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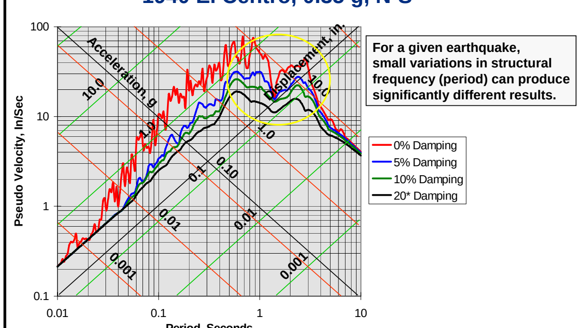
1940 El Centro, 0.35 g, N-S



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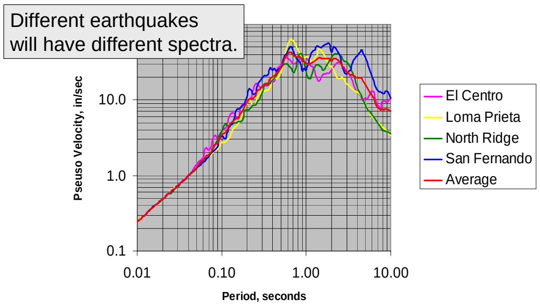
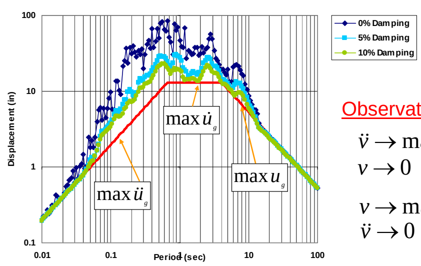
5% Damped Spectra for Four California Earthquakes
Scaled to 0.40 g (PGA)



Smoothed Elastic Response Spectra (Elastic DESIGN Response Spectra)

- Newmark-Hall spectrum
- ASCE 7 spectrum

Newmark-Hall Elastic Spectrum

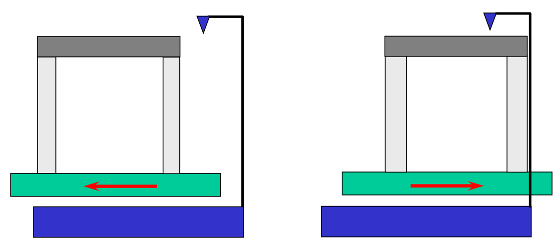


Observations

$\ddot{v} \rightarrow \max \ddot{v}_g$
$v \rightarrow 0$ at short *T*

$v \rightarrow \max v_g$
$\ddot{v} \rightarrow 0$ at long *T*

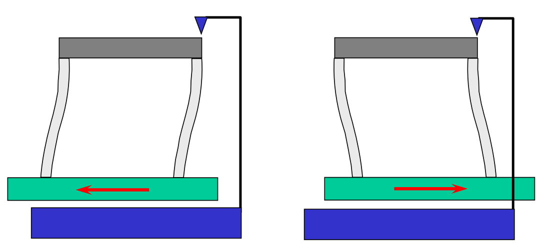
Very Stiff Structure (T < 0.01 sec)



Relative displacement ⟹ Zero

Total acceleration ⟹ Ground acceleration

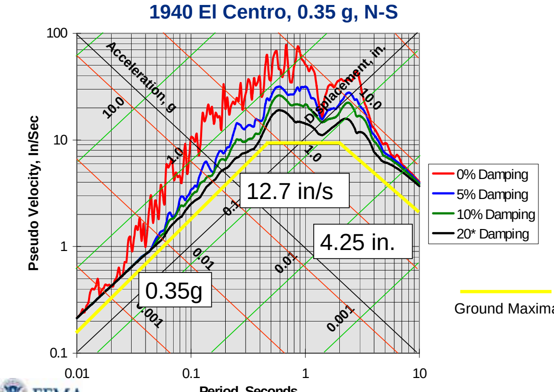
Very Flexible Structure (T > 10 sec)



Relative displacement ⟹ Ground displacement

Total acceleration ⟹ Zero

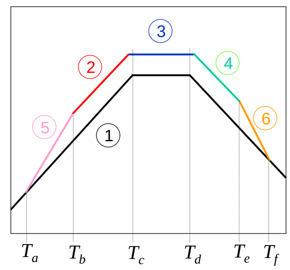
1940 El Centro, 0.35 g, N-S



Newmark's Spectrum Amplification Factors for Horizontal Elastic Response

Damping	One Sigma (84.1%)			Median (50%)		
% Critical	a_a	a_v	a_d	a_a	a_v	a_d
.05	5.10	3.84	3.04	3.68	2.59	2.01
1	4.38	3.38	2.73	3.21	2.31	1.82
2	3.66	2.92	2.42	2.74	2.03	1.63
3	3.24	2.64	2.24	2.46	1.86	1.52
5	**2.71**	**2.30**	**2.01**	2.12	1.65	1.39
7	2.36	2.08	1.85	1.89	1.51	1.29
10	1.99	1.84	1.69	1.64	1.37	1.20
20	1.26	1.37	1.38	1.17	1.08	1.01

Newmark-Hall Elastic Spectrum



1) Draw the lines corresponding to max $\ddot{v}_g, \dot{v}_g, v_g$

2) Draw line $\alpha_A \max \ddot{v}_g$ from T_b to T_c

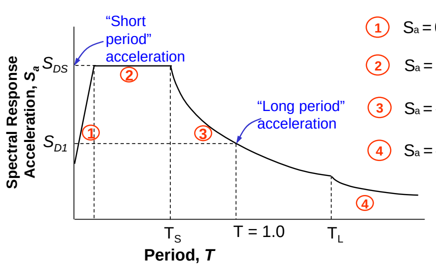
3) Draw line $\alpha_V \max \dot{v}_g$ from T_c to T_d

4) Draw line $\alpha_D \max v_g$ from T_d to T_e

5) Draw connecting line from T_a to T_b

6) Draw connecting line from T_e to T_f

ASCE 7 Uses a Smoothed Design Acceleration Spectrum



1. $S_a = 0.6 \frac{S_{DS}}{T_0} T + 0.4\, S_{DS}$
2. $S_a = S_{DS}$
3. $S_a = \frac{S_{D1}}{T}$
4. $S_a = \frac{T_L S_{D1}}{T^2}$

Note exceptions at larger periods

The ASCE 7 Response Spectrum

is a uniform hazard spectrum based on probabilistic and deterministic seismic hazard analysis.