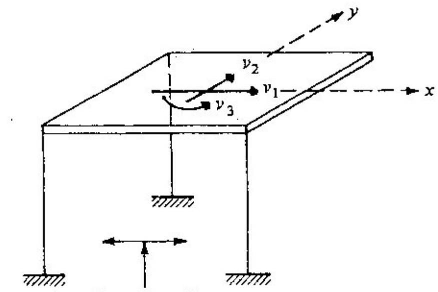

FIGURE 27-7
Rigid slab subjected to base translation.

it is assumed that the earthquake motions act in the direction of the x axis, the influence vector for this case is given by $\mathbf{r}^T = [1 \quad 0 \quad 0]$. The modal earthquake-excitation factors for this structure are then obtained by substituting this vector into Eq. (27-52), and the response is given finally by Eqs. (27-33) to (27-39).

EXAMPLE E27-6 Because the earthquake-response analysis of a rigid slab structure of this type involves several features of special interest, the example structure of Fig. E27-5 will be discussed in some detail. It is assumed that the three columns supporting the slab are rigidly attached to the foundation and to the slab, so that the resistance at the top of each column to lateral displacement in any direction is $12EI/L^3 = 5$ kips/ft. The torsional stiffness of the columns is negligible.

For the purpose of this example, the three degrees of freedom of the

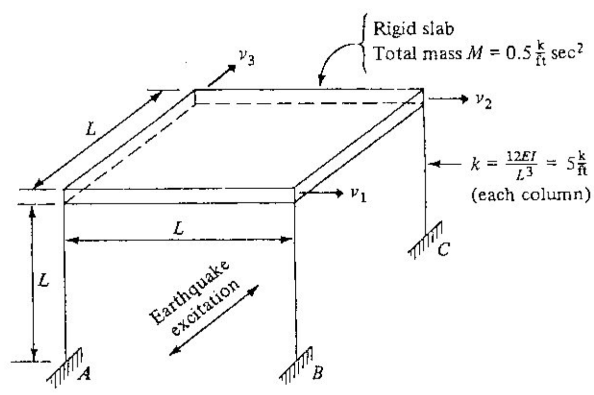

FIGURE E27-5
Slab supported by three columns.

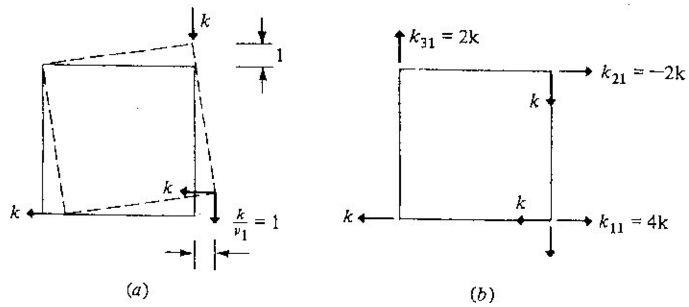

FIGURE E27-6
Evaluation of stiffness coefficients for $v_1 = 1$: (a) displacement $v_1 = 1$ and resisting column forces; (b) column forces and equilibrating stiffness coefficients.

slab are represented by the displacement components of the corners as shown. The total mass of the slab is $m = 0.5\ \text{kip}\cdot\text{s}^2/\text{ft}$ and is distributed uniformly over the area. The structure is subjected to an earthquake having the response spectrum of Fig. E27-1 and acting in the direction parallel with coordinate v_3. It is desired to determine the maximum displacements of the slab due to this earthquake.

The mass and stiffness matrices of this system can be evaluated by direct application of the definitions of the influence coefficients. Considering first the stiffness matrix, a unit displacement $v_1 = 1$ is applied while the other coordinates are constrained, as shown in Fig. E27-6*a*. The forces exerted by the columns in resisting this displacement are shown in this sketch, and the equilibrating forces corresponding to the degrees of freedom are shown in Fig. 27-6*b*. By applying unit displacements of the other two coordinates, the remaining stiffness coefficients can be determined similarly.

The mass matrix is evaluated by applying a unit acceleration separately to each degree of freedom and determining the resulting inertia forces in the slab. For example, Fig. E27-7*a* shows the unit acceleration $\ddot{v}_2 = 1$ and the slab inertia forces resisting this acceleration, while Fig. E27-7*b* shows the mass influence coefficients which equilibrate these inertial forces. The other mass coefficients can be found by unit accelerations of the other two coordinates. The complete stiffness and mass matrices for the system are

$$\mathbf{k} = \frac{12EI}{L^3}\begin{bmatrix} 4 & -2 & 2 \\ -2 & 3 & -2 \\ 2 & -3 & 3 \end{bmatrix} \qquad \mathbf{m} = \frac{m}{6}\begin{bmatrix} 4 & -1 & 3 \\ -1 & 4 & -3 \\ 3 & -3 & 6 \end{bmatrix}$$

−2

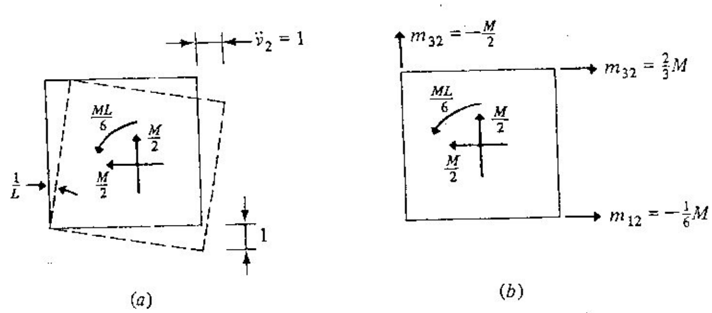

FIGURE E27-7
Evaluation of mass coefficients for $\ddot{v}_2 = 1$. (a) Acceleration $\ddot{v}_2 = 1$ and resisting inertia forces; (b) slab inertia forces and equilibrating mass coefficients.

When the eigenproblem $(\mathbf{k} - \omega^2\mathbf{m})\hat{\mathbf{v}} = \mathbf{0}$ is solved, the mode shapes and frequencies of the system are found to be

$$\mathbf{\Phi} = \begin{bmatrix} 0.366 & 1.000 & -1.366 \\ 1.000 & 1.000 & 1.000 \\ 1.000 & -1.000 & 1.000 \end{bmatrix} \qquad \omega^2 = \begin{bmatrix} 25.36 \\ 30.00 \\ 94.64 \end{bmatrix} (\text{rad/s})^2$$

Study of these mode shapes reveals that the first and third represent rotations about points on the symmetry diagonal while the second is simple translation along this diagonal. Obviously these motions could have been identified more easily by a more appropriate coordinate system; translation of the center of mass in the direction of the two diagonals plus rotation about the center of mass would have been a better choice of coordinates.

The frequencies, periods of vibration, and the spectral velocities given by Fig. E27-1 (assuming 5 percent damping) for the three modes of this structure are

$$\omega = \begin{bmatrix} 5.036 \\ 5.477 \\ 9.464 \end{bmatrix} \text{rad/s} \qquad \mathbf{T} = \begin{bmatrix} 1.25 \\ 1.15 \\ 0.65 \end{bmatrix} \text{s} \qquad \mathbf{S}_v = \begin{bmatrix} 0.55 \\ 0.54 \\ 0.48 \end{bmatrix} \text{ft/s}$$

Also the generalized masses M_n and modal earthquake-excitation factors $\mathscr{L}_n = \phi_n^T\mathbf{m}\mathbf{r}$ where $\mathbf{r}^T = [0 \quad 0 \quad 1]$ are

$$\mathbf{M} = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.5 \end{bmatrix} \text{kips}\cdot\text{s}^2/\text{ft} \qquad \mathscr{L} = \begin{bmatrix} 0.3415 \\ -0.5000 \\ -0.0915 \end{bmatrix} \text{kips}\cdot\text{s}^2/\text{ft}$$

.07459
-.0493
-.00928

Hence the maximum modal displacements are found, from

$$\mathbf{v}_{n,\,\max} = \phi_n \frac{\mathscr{L}_n}{M_n} \frac{S_{vn}}{n}$$

to be

$$\mathbf{v}_{1,\,\max} = \begin{bmatrix} 0.0272 \\ 0.0745 \\ 0.0745 \end{bmatrix} \text{ft} \qquad \mathbf{v}_{2,\,\max} = \begin{bmatrix} 0.0493 \\ 0.0493 \\ -0.0493 \end{bmatrix} \text{ft} \qquad \mathbf{v}_{3,\,\max} = \begin{bmatrix} -0.0124 \\ 0.0091 \\ 0.0091 \end{bmatrix} \text{ft}$$

An approximation of the maximum displacement in each coordinate could be determined from these results by the root-sum-square method. ////

Comparison with Uniform Building Code Requirements

It is of interest to compare the foregoing formulation of expressions for the forces developed in a building due to seismic excitation with the seismic design requirements of a typical building code. For example, in the Uniform Building Code (UBC)[1] the principal seismic provision defines the effective intensity of the design earthquake in terms of the maximum shear force which it produces at the base of the building. The expression for this code base shear force $\mathcal{V}_0$ is of the form

$$\boxed{\mathcal{V}_{0,\,\max} = k\hat{C}W} \tag{a}$$

where W is the weight of the building, $\hat{C}$ is the base-shear coefficient, and k is a factor which depends on the type of structural framing system. This factor is intended to account for the relative energy-absorbing capacity of the framing type and varies from $^2/_3$ for a rigid jointed frame which resists lateral forces by flexure of the columns and girders to $^4/_3$ for a box-type structure assembled from shear panels in the horizontal and vertical planes. The base-shear coefficient is expressed as a function of the fundamental period of vibration T of the structure as

$$\boxed{\hat{C} = \frac{0.05}{\sqrt[3]{T}}} \tag{b}$$

Also contained in the UBC provisions is a zone factor which reduces the design forces for zones of less seismicity; the zone factor of unity implied in Eq. (a) is intended for the regions of highest seismicity.

An analytical expression corresponding to the code formula of Eq. (a) above can easily be derived from Eq. (27-40) by considering only the fundamental mode and

[1] Published by the International Conference of Building Officials, Pasadena, California.