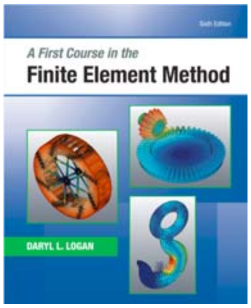
Chapter 7 – Practical Considerations in Modeling



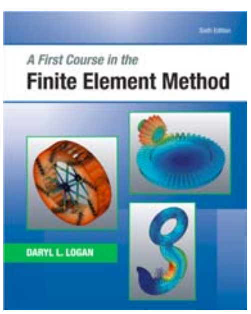
Learning Objectives

•To present concepts that should be considered when modeling for a situation by the finite element method, such as aspect ratio, symmetry, natural subdivisions, sizing of elements and the h, p, and r methods of refinement, concentrated loads and infinite stress, infinite medium, and connecting different kinds of elements

•To describe some of the approximations inherent in finite element solutions

•To illustrate convergence of solution and introduce the patch test for convergence of solution

Chapter 7 – Practical Considerations in Modeling



Learning Objectives

•To discuss the interpretation of stresses in an element, including a common method of averaging the nodal values (also called smoothing)

•To present a flowchart of typical finite element processes used for the analysis of plane stress and plane strain

•To describe a computer assisted step-by-step solution of a bicycle wrench

•To demonstrate various real-world applications where plane stress/strain element models are applicable

Practical Considerations in Modeling

Introduction

In this section we will discuss some modeling considerations and guidelines, including mesh size, natural subdivisions, and the use of symmetry and associated boundary conditions.

We will also introduce the concept of ***static condensation***, which enables us to apply the basis of the CST stiffness matrix to a quadrilateral element.

Practical Considerations in Modeling

Finite Element Modeling

Finite element modeling is *partly an art guided by visualizing physical interactions taking place within a body.*

In modeling the user is confronted with the difficult task of understanding physical behavior taking place and understanding the physical behavior of various elements available for use.

Matching the appropriate finite element to the physical behavior being modeling is one of many decisions that must be made by the modeler.

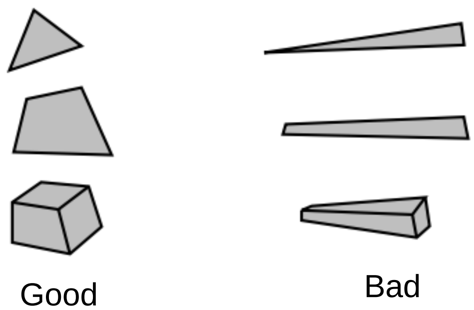
Understanding the ***boundary conditions*** can be one of the most difficult tasks a modeler must face in construction a useable finite element model.

Practical Considerations in Modeling

Aspect Ratio and Element Shape

The ***aspect ratio*** is define as *the ratio of the longest dimension to the shortest dimension of a quadrilateral element.*

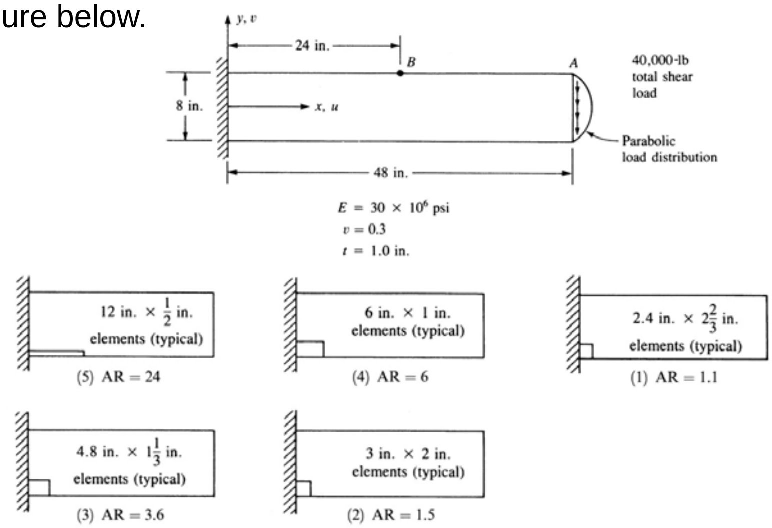
In general, as the aspect ratio increases, the inaccuracy of the finite element solution increases.



Practical Considerations in Modeling

Aspect Ratio and Element Shape

Consider the five different finite element model shown in the figure below.



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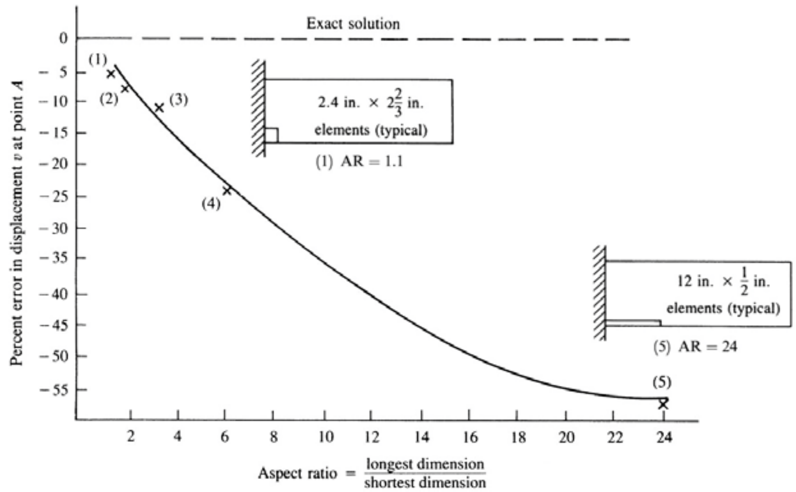
Aspect Ratio and Element Shape

A plot of the resulting error in the displacement at point *A* of the beam verse aspect ratio is given.



Practical Considerations in Modeling

Aspect Ratio and Element Shape

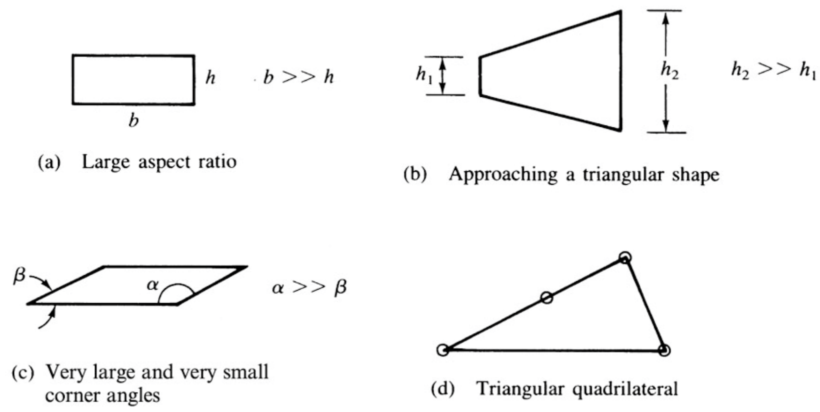
In addition, the numerical answers are given in the following table.

Case	Aspect Ratio	Number of Nodes	Number of Elements	Point A	Point B	% Error
1	1.1	84	60	-1.093	-0.346	5.2
2	1.5	85	64	-1.078	-0.339	6.4
3	3.6	77	60	-1.014	-0.238	11.9
4	6.0	81	64	-0.886	-0.280	23.0
5	24.0	85	64	-0.500	-0.158	56.0
			Exact Solution	-1.152	-0.360	

Practical Considerations in Modeling

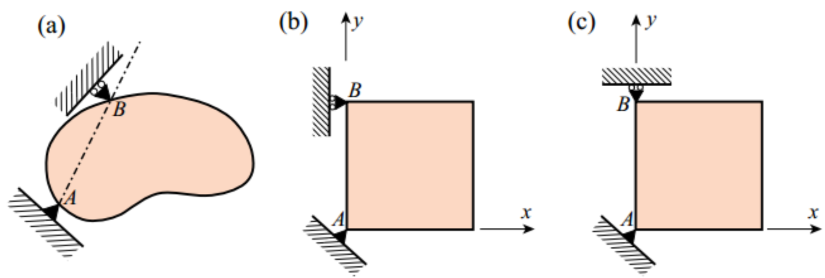
Aspect Ratio and Element Shape

In general, elements that yield the best results are compact and regular in shape will: (1) aspect ratios near one; and (2) corner angles of quadrilaterals near 90°.



Practical Considerations in Modeling

Minimum Support Conditions to Suppress Rigid Body Motions in 2D



stable stable unstable

Practical Considerations in Modeling

Use of Symmetry

The use of ***symmetry*** will often expedite the modeling of a problem.

Symmetry allows us to consider a reduced problem instead of the actual problem.

This will allow us to use a finer discretization of element with less computational cost.

Practical Considerations in Modeling

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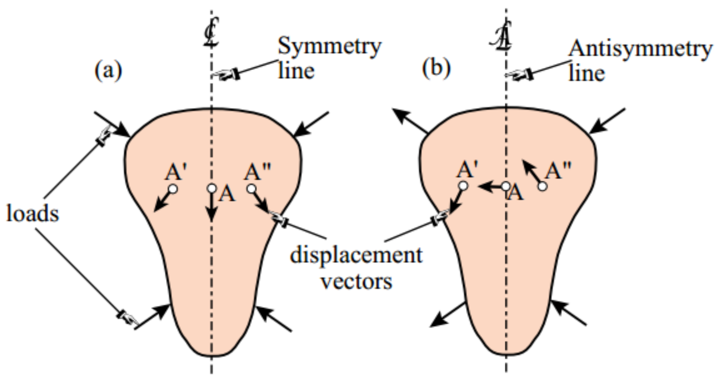

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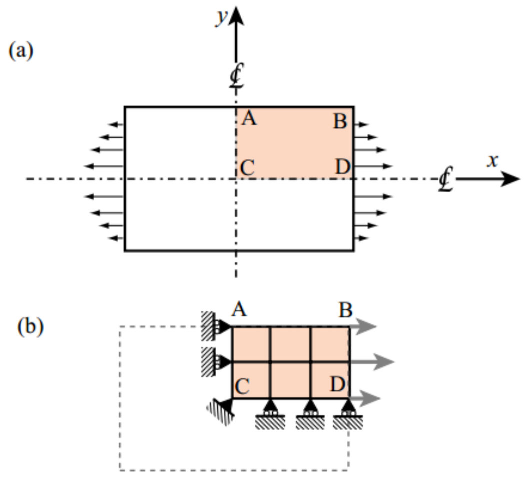

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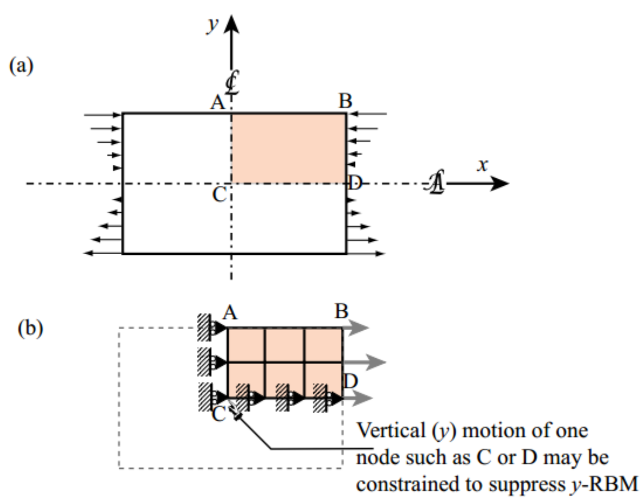

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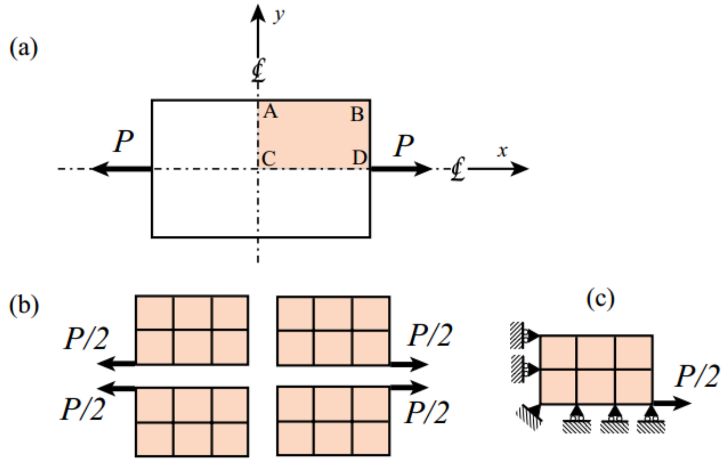

Practical Considerations in Modeling

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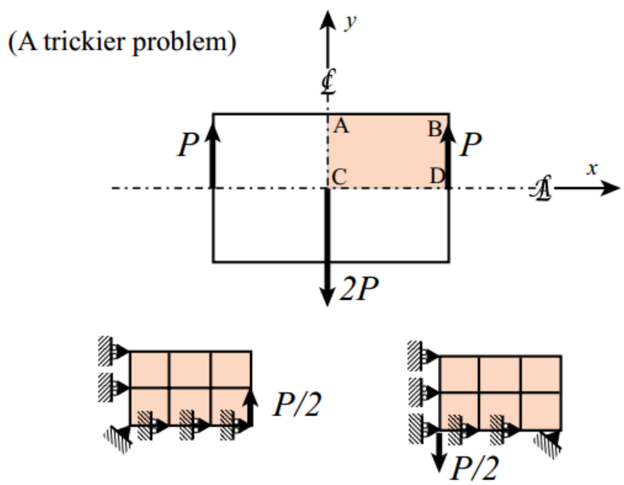


Practical Considerations in Modeling

Natural Subdivisions at Discontinuities

There are a variety of natural subdivisions for finite element discretizations.

For example, natural locations of nodes occur at concentrated loads or discontinuities in loading, other types of boundary conditions, and abrupt changes in geometry of materials.

Practical Considerations in Modeling

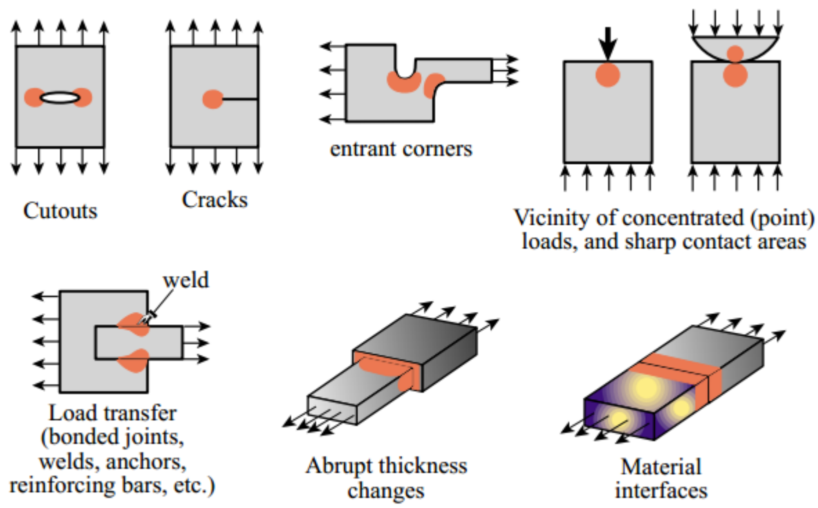
Sizing of Elements and Mesh Refinement

A discretization depends on the geometry of the structure, the loading, and the boundary conditions.

For example, areas of high, rapidity changing stresses require a finer mesh than regions where the stress is constant.

Practical Considerations in Modeling

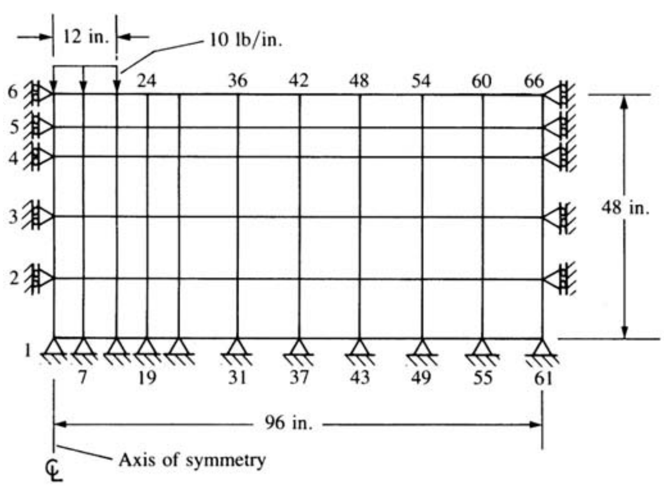
Where Finer Meshes Should Be Used



Practical Considerations in Modeling

Use of Symmetry

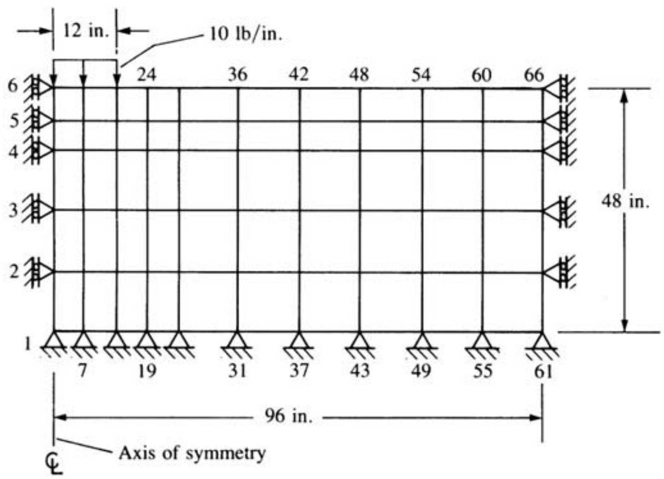
Here the use of symmetry is applied to a soil mass subjected to a foundation loading (66 nodes and 50 elements).



Practical Considerations in Modeling

Use of Symmetry

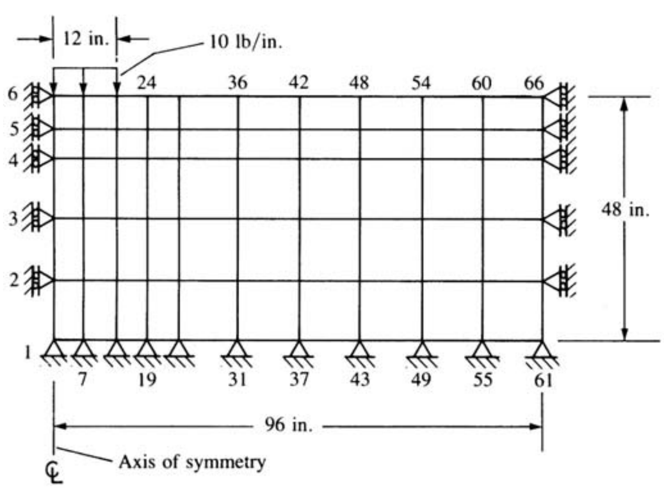
Note that at the place of symmetry the displacements in the direction perpendicular to the plane must be zero.



Practical Considerations in Modeling

Use of Symmetry

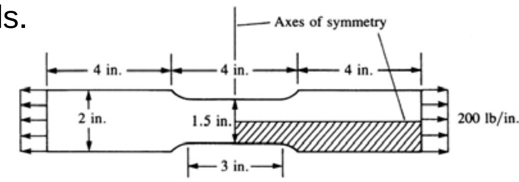
This is modeled by rollers at nodes 2 - 6.



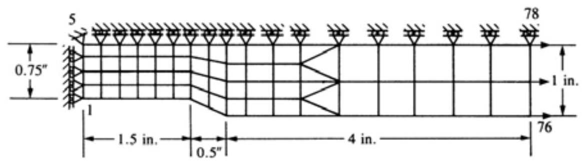
Practical Considerations in Modeling

Use of Symmetry

The figure below illustrates the use of triangular elements for transitions from smaller quadrilaterals to larger quadrilaterals.



(a) Plane stress uniaxially loaded member with fillet

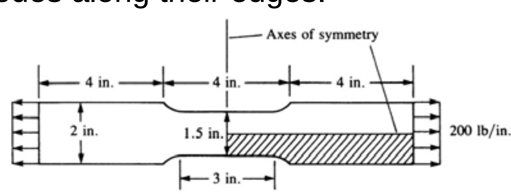


(b) Enlarged finite element model of the cross-hatched quarter of the member (number of nodes = 78, number of elements = 60) (2.54 cm = 1 in.)

Practical Considerations in Modeling

Use of Symmetry

The transitions are required since CST elements do not have immediate nodes along their edges.



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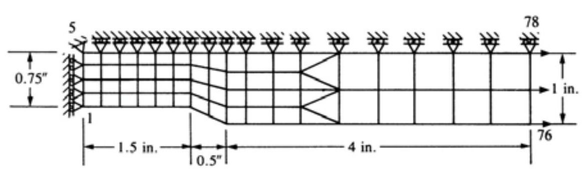
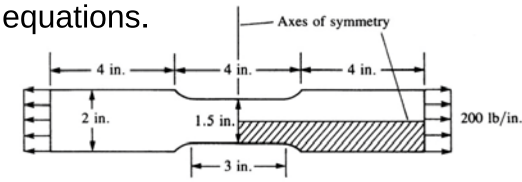


(b) Enlarged finite element model of the cross-hatched quarter of the member (number of nodes = 78, number of elements = 60) (2.54 cm = 1 in.)

Practical Considerations in Modeling

Use of Symmetry

If an element had an intermediate node, the resulting equations would be inconsistent with the energy formulation for the CST equations.



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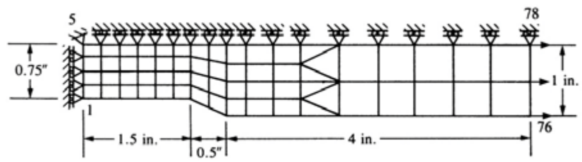

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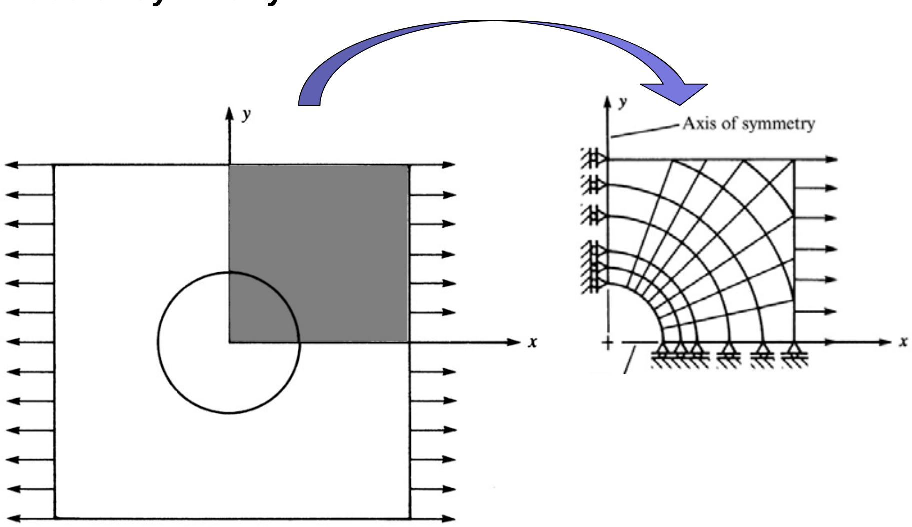
Practical Considerations in Modeling

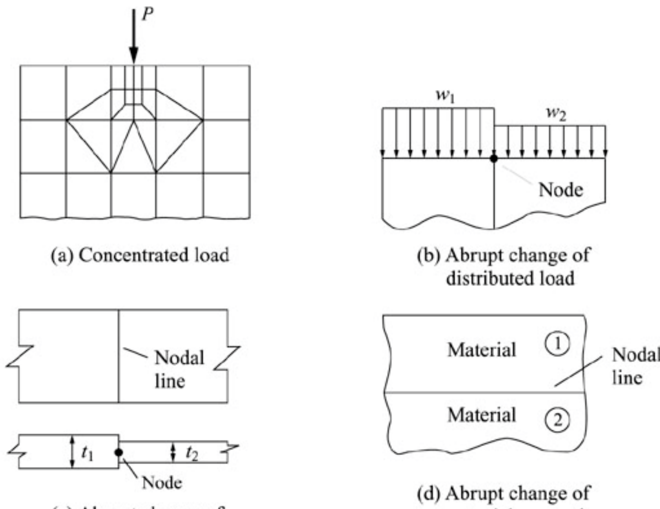
Use of Symmetry



(a) Plate with hole under plane stress

Practical Considerations in Modeling

Natural Subdivisions at Discontinuities



(a) Concentrated load

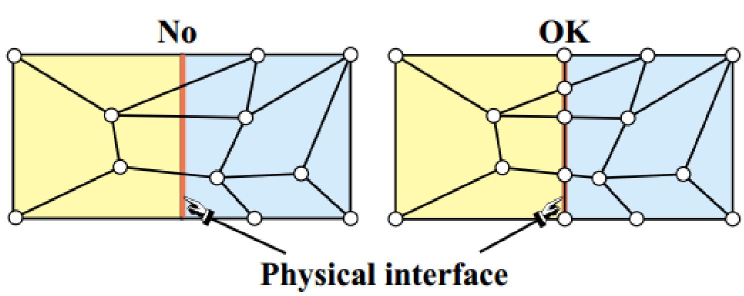
(b) Abrupt change of distributed load

(c) Abrupt change of plate thickness

(d) Abrupt change of material properties

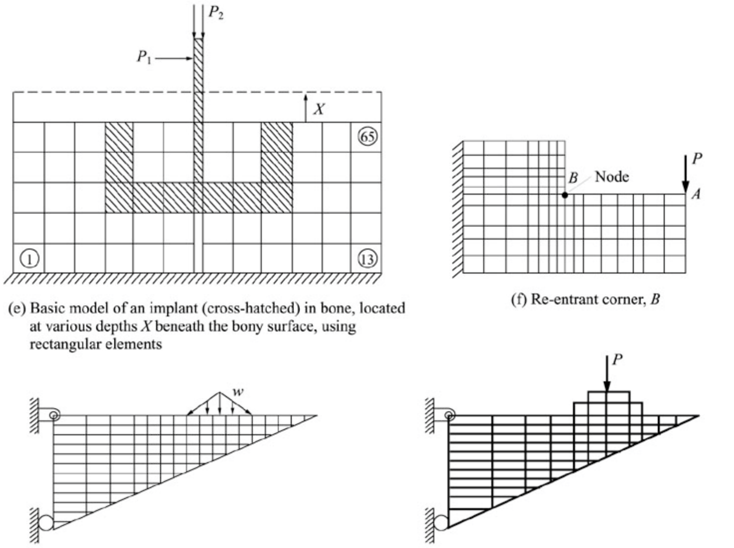
Practical Considerations in Modeling

Elements Must Not Cross Interfaces



Practical Considerations in Modeling

Natural Subdivisions at Discontinuities



(e) Basic model of an implant (cross-hatched) in bone, located at various depths X beneath the bony surface, using rectangular elements

(f) Re-entrant corner, B

(g) Structure with a distributed load

(h) Using elements to distribute the loading and spread the concentrated load

Practical Considerations in Modeling

Mesh Revision (Refinement)

Mesh revision or refinement is used to increase the accuracy of the results produced by a finite element model.

First, start with a basic mesh using the fewest, reasonable number of elements, and obtain a benchmark result.

Then refine the model by doing actions like increasing the mesh density and compare the results.

This process continues until the results converge to some user-defined criteria.

Consider this original mesh for the discussion of types of methods:

Practical Considerations in Modeling

Mesh Revision (Refinement)

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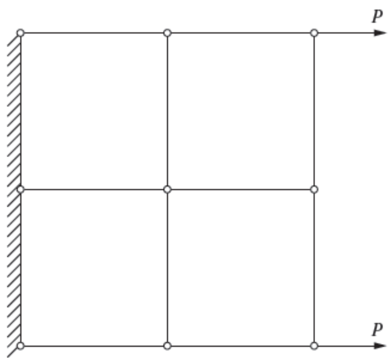


(a) Original mesh

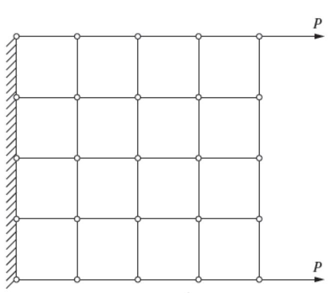
Practical Considerations in Modeling

h Method of Refinement

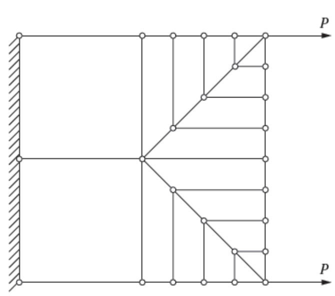
In the *h* method, we use the particular element based on the shape functions for that element.

Elements of the same kind are then added or made smaller.

Uniform or non-uniform refinements are possible as shown below:



(b) A uniformly refined *h* mesh



(c) A possible nonuniform *h* refinement

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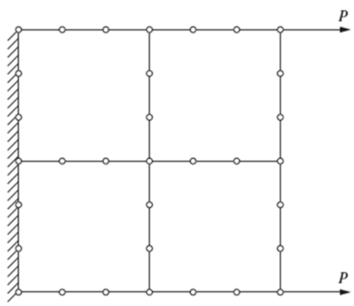
p Method of Refinement

In the *p* method, the polynomial p is increased to higher-order polynomials based on the degree of accuracy specified.

These iterations are repeated until a convergence is reached.



(d) A possible uniform *p* refinement

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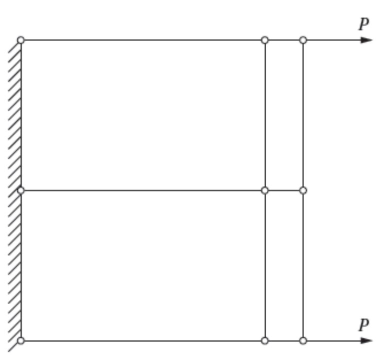
r Method of Refinement

In the *r* method, the nodes are rearranged or relocated without changing the number of elements of the polynomial degree of the field quantities.



(e) A possible *r* refinement

Practical Considerations in Modeling

Infinite Medium

A typical example of infinite medium is a soil foundation problem.

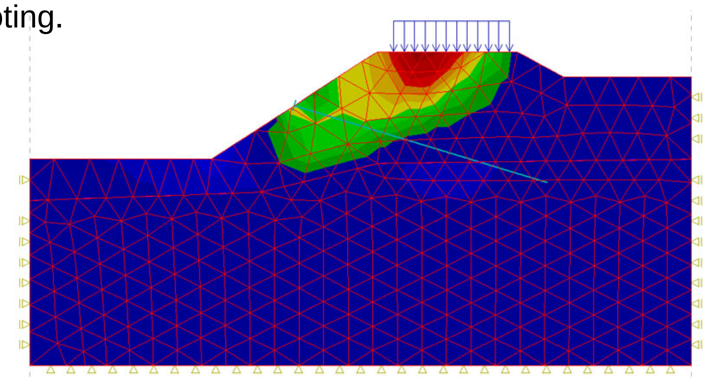
The guideline for the finite element model is that enough material must be included such that the displacements at nodes and stresses within the elements become negligibly small at locations far from the foundation load.

The level of discretization can be determined by a trail-and-error procedure in which the horizontal and vertical distances from the load are varied and the resulting effects on the displacements and stresses are observed.

Practical Considerations in Modeling

Infinite Medium

For a homogeneous soil mass, experience has shown the influence of a footing becomes insignificant if the horizontal distance of the model is taken as approximately four and six times the width of the footing and the vertical distance is taken as approximately four to ten times the width of the footing.



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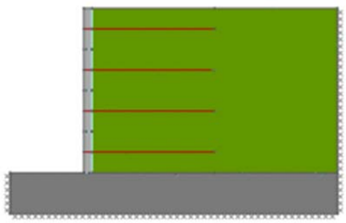

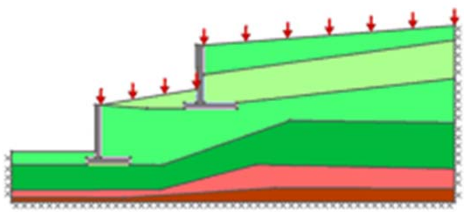

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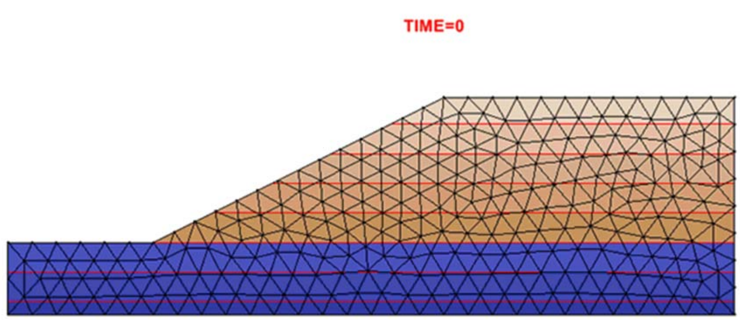

Practical Considerations in Modeling

Checking the Model

The discretized finite element model should be checked carefully before results are computed.

Ideally, a model should be checked by an analyst not involved in the preparation of the model, who is then more likely to be objective.

Preprocessors with their detailed graphical display capabilities now make it comparatively easy to find errors, particularly with a misplaced node or missing element or a misplaced load or boundary condition.

Preprocessors include the ability to color, shrink, rotate, and section a model mesh.

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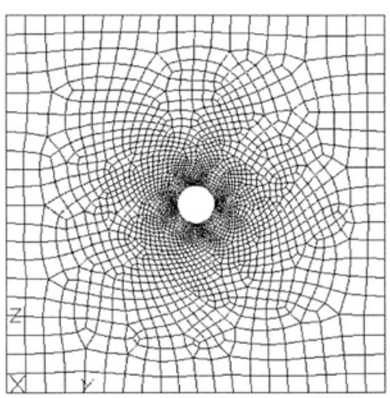
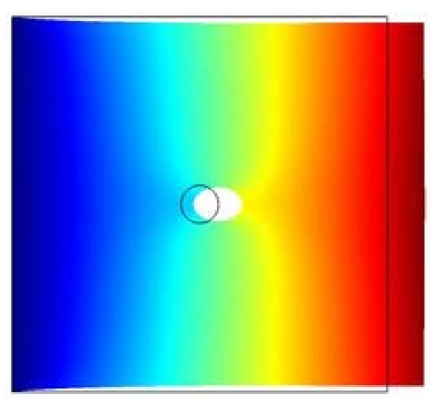
Checking the Results and Typical Postprocessor Results

An analyst should probability spend as much time processing, checking, and analyzing results as spent in data preparation.





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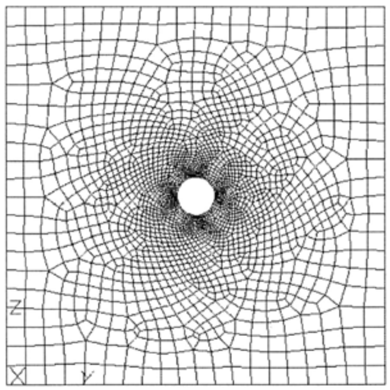
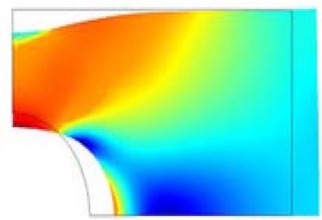
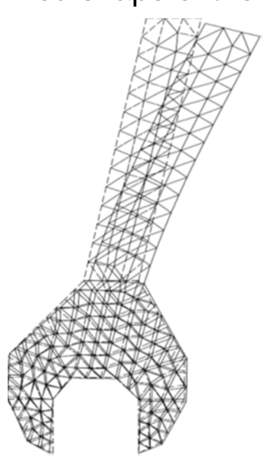




Practical Considerations in Modeling

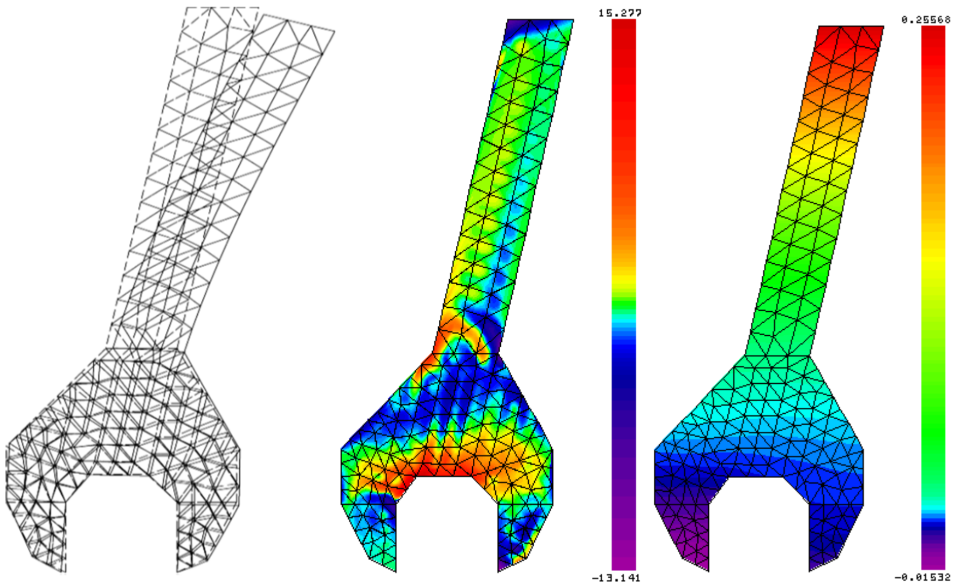
Checking the Results and Typical Postprocessor Results

The wrench in this example is modeled by 307 constraint strain triangular elements (plane stress assumption). Below is a plot of the deformed shape of the wrench over the original mesh.



Practical Considerations in Modeling

Checking the Results and Typical Postprocessor Results



Practical Considerations in Modeling

Equilibrium and Compatibility of Finite Element Results

An approximate solution for a stress analysis problem using the finite element method based on assumed displacement fields does not generally satisfy all the requirements for equilibrium and compatibility that an exact theory-of-elasticity solution satisfies.

However, remember that relatively few exact solutions exist.

Hence, the finite element method is a very practical one for obtaining reasonable, but approximate, numerical solutions.

Practical Considerations in Modeling

Equilibrium and Compatibility of Finite Element Results

We now describe some of the approximations generally inherent with finite element solutions.

1. Equilibrium of nodal forces and moments is satisfied.

   This is true because the global equation $\boldsymbol{F} = \boldsymbol{K}\boldsymbol{d}$ is a nodal equilibrium equation whose solution for $\boldsymbol{d}$ is such that the sums of all forces and moments applied to each node are zero.

   Equilibrium of the whole structure is also satisfied because the structure reactions are included in the global forces, and hence, in the nodal equilibrium equations.

Practical Considerations in Modeling

Equilibrium and Compatibility of Finite Element Results

We now describe some of the approximations generally inherent with finite element solutions.

2. Equilibrium within an element is not always satisfied.

   However, for the constant-strain bar and the constant-strain triangle, element equilibrium is satisfied.

   Also the cubic displacement function is shown to satisfy the basic beam equilibrium differential equation, and hence, to satisfy element force and moment equilibrium.

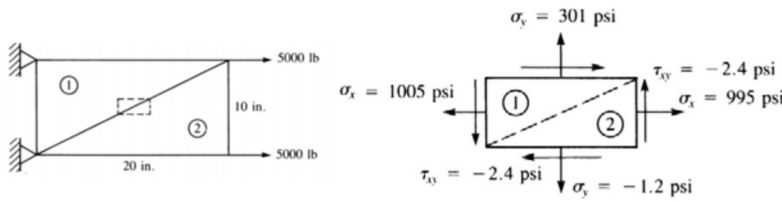
Practical Considerations in Modeling

Equilibrium and Compatibility of Finite Element Results

We now describe some of the approximations generally inherent with finite element solutions.

3. Equilibrium is not usually satisfied between elements.

   A differential element including parts of two adjacent finite elements is usually not in equilibrium (see the figure below).



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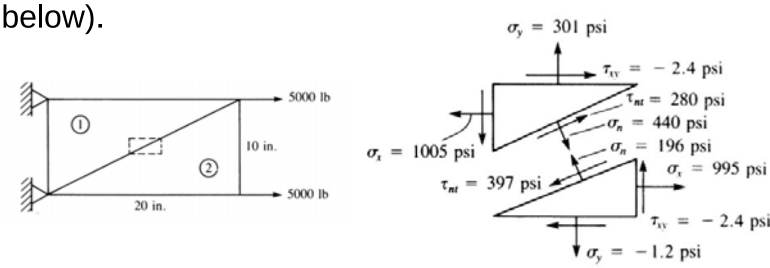


Practical Considerations in Modeling

Equilibrium and Compatibility of Finite Element Results

We now describe some of the approximations generally inherent with finite element solutions.

3. Equilibrium is not usually satisfied between elements.

   For line elements, such as used for truss and frame analysis, interelement equilibrium is satisfied.

   However, for two- and three-dimensional elements, interelement equilibrium is not usually satisfied.

   Also, the coarseness of the mesh causes this lack of interelement equilibrium to be even more pronounced.

Practical Considerations in Modeling

Equilibrium and Compatibility of Finite Element Results

We now describe some of the approximations generally inherent with finite element solutions.

4. Compatibility is satisfied within an element as long as the element displacement field is continuous; hence, individual elements do not tear apart.

5. In the formulation of the element equations, compatibility is invoked at the nodes.

   Hence, elements remain connected at their common nodes. Similarly, the structure remains connected to its support nodes because boundary conditions are invoked at these nodes.

Practical Considerations in Modeling

Equilibrium and Compatibility of Finite Element Results

We now describe some of the approximations generally inherent with finite element solutions.

6. Compatibility may or may not be satisfied along interelement boundaries.

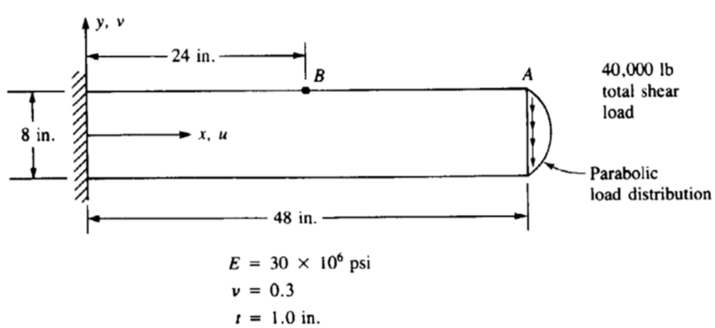
   For line elements such as bars and beams, interelement boundaries are merely nodes.

   The constant-strain triangle remain straight sided when deformed and therefore, interelement compatibility exists for these elements. Incompatible elements, those that allow gaps or overlaps between elements, can be acceptable and even desirable.

Practical Considerations in Modeling

Convergence of Solution

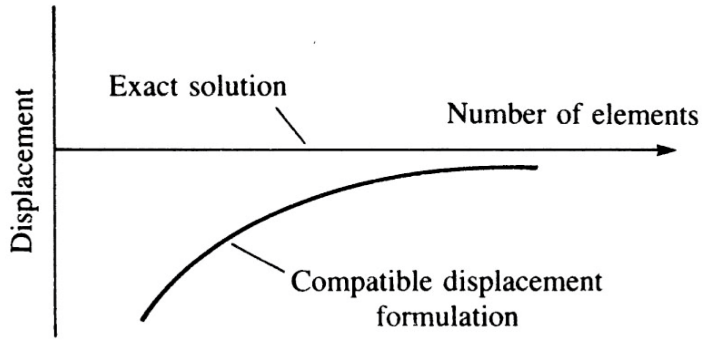
When the mesh size is reduced - that is the number of elements is increased - we are ensured of monotonic convergence of the solution when compatible and complete displacement functions are used.



Practical Considerations in Modeling

Convergence of Solution

When the mesh size is reduced - that is the number of elements is increased - we are ensured of monotonic convergence of the solution when compatible and complete displacement functions are used.



Practical Considerations in Modeling

Convergence of Solution

When the mesh size is reduced - that is the number of elements is increased - we are ensured of monotonic convergence of the solution when compatible and complete displacement functions are used.

Case	Number of Elements	Number of Nodes	Aspect Ratio	Point A
1	12	21	2	-0.740
2	24	39	1	-0.980
3	32	45	3	-0.875
4	64	85	1.5	-1.078
5	80	105	1.2	-1.100
		Exact Solution		-1.152

Practical Considerations in Modeling

Patch Test

The Patch test is used to test the convergence of a solution of an element being used in the model.

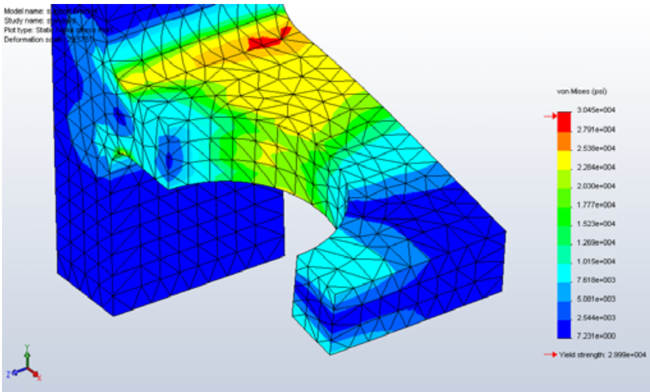
The test requires that the element must be able to accommodate both rigid-body motion and a constant state of strain, as both are possible within a structure.

The test can also be applied to determine if sufficient Gauss points have been used in the numerical integration process to evaluate the stiffness matrix for isoparametric formulations.

Practical Considerations in Modeling

Convergence of Solution

When the mesh size is reduced - that is the number of elements is increased - we are ensured of monotonic convergence of the solution when compatible and complete displacement functions are used.



Practical Considerations in Modeling

Convergence of Solution

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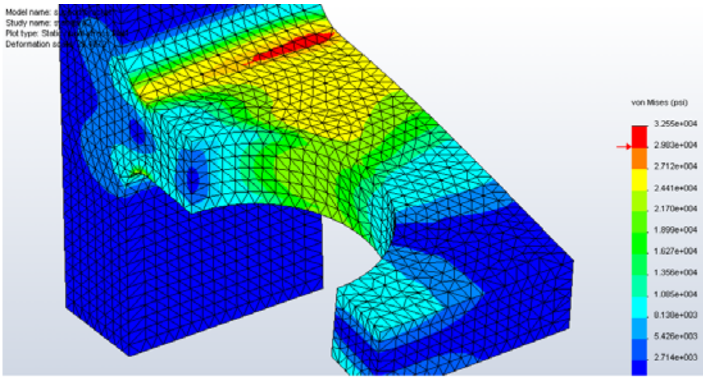


Practical Considerations in Modeling

Interpretation of Stresses

In the stiffness or displacement formulation of the finite element method, used in this course, the primary quantities determined are the interelement nodal displacements of the assemblage.

Secondary quantities, such as stress and strain, are computed based on these nodal displacements.

In the case of the bar and constant-strain triangles, stresses are constant over the element.

For these elements, it is common practice to assign the stress to the centroid of the element with acceptable results.

Practical Considerations in Modeling

Interpretation of Stresses

An alternative procedure sometimes is to use an average (possibly weighted) value of the stresses evaluated at each node of the element.

This averaging method is often based interpolating the element nodal values using the element shape functions.

The averaging method is called *smoothing*.

While the results from smoothing may be pleasing to the eye, they may not indicate potential problems with the model and the results.

Practical Considerations in Modeling

Interpretation of Stresses

You should always view the unsmoothed contour plots as well.

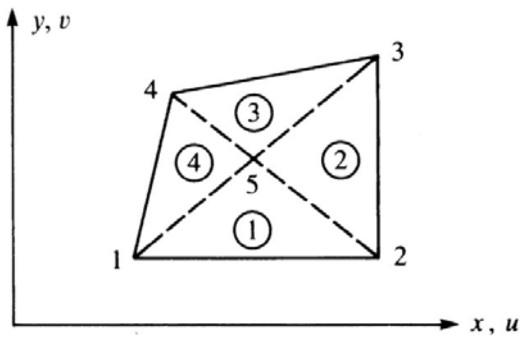
Highly discontinuous contours between elements in a region of an unsmoothed plot indicate modeling problems and typically require additional refinement of the element mesh in the suspect region.

Practical Considerations in Modeling

Static Condensation

Let's consider the concept of static condensation and used it to develop the stiffness matrix of a quadrilateral element.

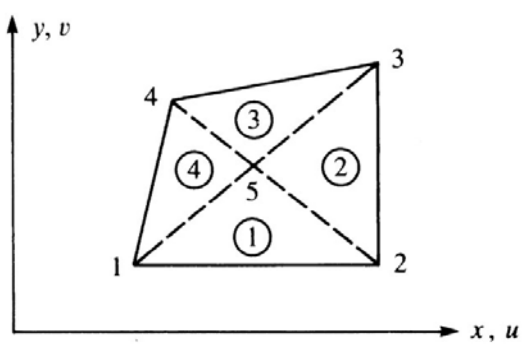
Consider a general quadrilateral element as shown below.



Practical Considerations in Modeling

Static Condensation

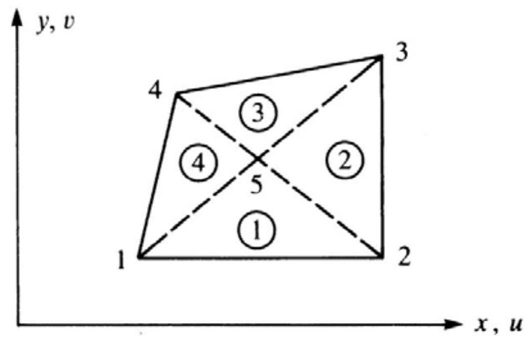
An imaginary node 5 is temporary introduced at the intersection of the diagonals of the quadrilateral to create four triangles.



Practical Considerations in Modeling

Static Condensation

We can superimpose the stiffness matrices of the four triangles to create the stiffness matrix of the quadrilateral element, where the internal imaginary node 5 degrees of freedom are said to be ***condensed out*** so that they never enter into the final equations.



Practical Considerations in Modeling

Static Condensation

Let's start by partitioning the equilibrium equations:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} d_e \\ d_i \end{Bmatrix} = \begin{Bmatrix} F_e \\ F_i \end{Bmatrix}$$

where d_i is the vector of displacements corresponding to the imaginary internal node, F_i is the vector of loads at the internal node, and d_e and F_e are the actual displacements and loads, respectively.

Rewriting the above equations we gives:

$$K_{11}d_e + K_{12}d_i = F_e$$

$$K_{21}d_e + K_{22}d_i = F_i$$

Practical Considerations in Modeling

Static Condensation

Solving the second equations for d_i gives:

$$d_i = -K_{22}^{-1}K_{21}d_e + K_{22}^{-1}F_i$$

Substituting the above equation, we obtain the condensed equilibrium equation: $k_c d_c = F_c$

where
$$k_c = K_{11} - K_{12}K_{22}^{-1}K_{21}$$
$$F_c = F_e - K_{12}K_{22}^{-1}F_i$$

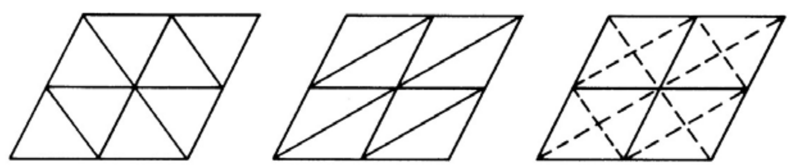
where k_c and F_c are called the ***condensed stiffness matrix*** and the ***condensed load vector***, respectively.

Practical Considerations in Modeling

Static Condensation

An advantage of the four-CST quadrilaterals is that the solution becomes less dependent on the ***skew*** of the subdivision mesh.

The ***skew*** means a *directional stiffness bias* that is built into a model through certain discretization patterns.



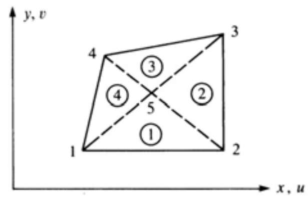
(a) (b) (c)

Practical Considerations in Modeling

Static Condensation

The stiffness matrix of a typical triangular element, call it element 1, labeled with nodes 1, 2, and 5 is given as:

$$\left[k^{(1)}\right] = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} & k_{15}^{(1)} \\ k_{21}^{(1)} & k_{22}^{(1)} & k_{25}^{(1)} \\ k_{51}^{(1)} & k_{52}^{(1)} & k_{55}^{(1)} \end{bmatrix}$$

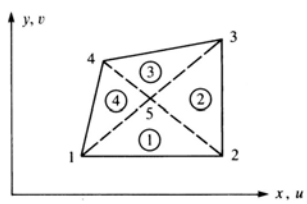


where $k_{ij}^{(1)}$ is a 2 x 2 matrix.

Practical Considerations in Modeling

Static Condensation

The assembled stiffness matrix for the quadrilateral is:

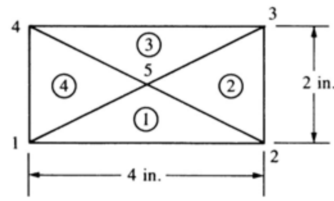


$$\begin{array}{c} \begin{array}{ccccc} (u_1,v_1) & (u_2,v_2) & (u_3,v_3) & (u_4,v_4) & (u_5,v_5) \end{array} \\ [k] = \left[\begin{array}{cccc|c} k_{11}^{(1)}+k_{11}^{(4)} & k_{12}^{(1)} & 0 & k_{14}^{(4)} & k_{15}^{(1)}+k_{15}^{(4)} \\ k_{21}^{(1)} & k_{22}^{(1)}+k_{22}^{(2)} & k_{23}^{(2)} & 0 & k_{25}^{(1)}+k_{25}^{(2)} \\ 0 & k_{32}^{(2)} & k_{33}^{(2)}+k_{33}^{(3)} & k_{34}^{(3)} & k_{35}^{(2)}+k_{35}^{(3)} \\ k_{41}^{(4)} & 0 & k_{43}^{(3)} & k_{44}^{(3)}+k_{44}^{(4)} & k_{45}^{(3)}+k_{45}^{(4)} \\ \hline k_{51}^{(1)}+k_{51}^{(4)} & k_{52}^{(1)}+k_{52}^{(2)} & k_{53}^{(2)}+k_{53}^{(3)} & k_{54}^{(3)}+k_{54}^{(4)} & \begin{array}{c} k_{55}^{(1)}+k_{55}^{(2)} \\ +k_{55}^{(3)}+k_{55}^{(4)} \end{array} \end{array}\right] \end{array}$$

Practical Considerations in Modeling

Example Problem

Consider the quadrilateral with internal node 5 and dimensions as shown below. Apply the static condensation technique.



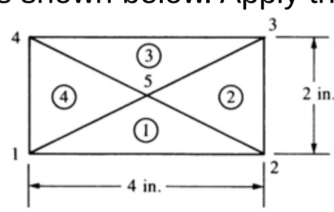
Using the CST stiffness matrix for plain strain, we get:

$$[k^{(1)}] = [k^{(3)}] = \frac{E}{4.16}\begin{array}{c} \begin{array}{cccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{5} \\ \multicolumn{2}{c}{3} & \multicolumn{2}{c}{4} & \multicolumn{2}{c}{5} \end{array} \\ \begin{bmatrix} 1.5 & 1.0 & 0.1 & 0.2 & -1.6 & -1.2 \\ 1.0 & 3.0 & -0.2 & 2.6 & -0.8 & -5.6 \\ 0.1 & -0.2 & 1.5 & -1.0 & -1.6 & 1.2 \\ 0.2 & 2.6 & -1.0 & 3.0 & 0.8 & -5.6 \\ -1.6 & -0.8 & -1.6 & 0.8 & 3.2 & 0.0 \\ -1.2 & -5.6 & 1.2 & -5.6 & 0.0 & 11.2 \end{bmatrix} \end{array}$$

Practical Considerations in Modeling

Example Problem

Consider the quadrilateral with internal node 5 and dimensions as shown below. Apply the static condensation technique.



Using the CST stiffness matrix for plain strain, we get:

$$[k^{(2)}] = [k^{(4)}] = \frac{E}{4.16}\begin{array}{c} \begin{array}{cccccc} \multicolumn{2}{c}{4} & \multicolumn{2}{c}{1} & \multicolumn{2}{c}{5} \\ \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} & \multicolumn{2}{c}{5} \end{array} \\ \begin{bmatrix} 1.5 & 1.0 & 0.1 & 0.2 & -1.6 & -1.2 \\ 1.0 & 3.0 & -0.2 & 2.6 & -0.8 & -5.6 \\ 0.1 & -0.2 & 1.5 & -1.0 & -1.6 & 1.2 \\ 0.2 & 2.6 & -1.0 & 3.0 & 0.8 & -5.6 \\ -1.6 & -0.8 & -1.6 & 0.8 & 3.2 & 0.0 \\ -1.2 & -5.6 & 1.2 & -5.6 & 0.0 & 11.2 \end{bmatrix} \end{array}$$

Practical Considerations in Modeling

Example Problem

The resulting assembled matrix before static condensation is:

$$[k] = \frac{E}{4.16}\left[\begin{array}{cccccccc|cc}
3.0 & 2.0 & 0.1 & 0.2 & 0.0 & 0.0 & -0.1 & -0.2 & -3.0 & -2.0 \\
 & 6.0 & -0.2 & 2.6 & 0.0 & 0.0 & 0.2 & -2.6 & -2.0 & -6.0 \\
 & & 3.0 & -2.0 & -0.1 & 0.2 & 0.0 & 0.0 & -3.0 & 2.0 \\
 & & & 6.0 & -0.2 & -2.6 & 0.0 & 0.0 & 2.0 & -6.0 \\
 & & & & 3.0 & 2.0 & 0.1 & 0.2 & -3.0 & -2.0 \\
 & & & & & 6.0 & -0.2 & 2.6 & -2.0 & -6.0 \\
 & & & & & & 3.0 & -2.0 & -3.0 & 2.0 \\
 & & & & & & & 6.0 & 2.0 & -6.0 \\
\hline
 & & & & & & & & 12.0 & 0.0 \\
\text{Symmetry} & & & & & & & & & 24.0
\end{array}\right]$$

Practical Considerations in Modeling

Example Problem

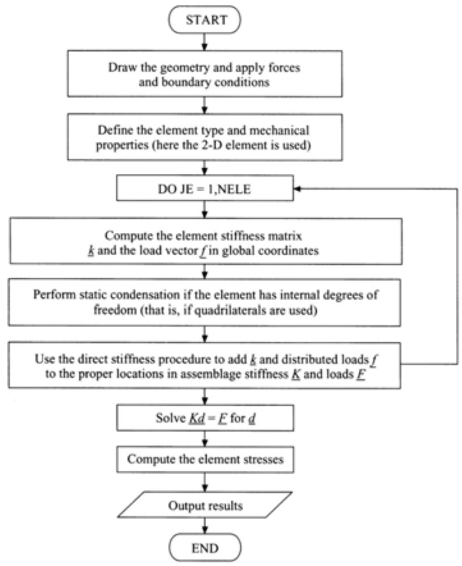
The resulting assembled matrix before static condensation is:

$$[k_c] = \frac{E}{4.16}\begin{array}{c}
\begin{array}{cccccccc} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 & u_4 & v_4 \end{array} \\
\begin{bmatrix}
2.08 & 1.00 & -0.48 & 0.20 & -0.92 & -1.00 & -0.68 & -0.20 \\
 & 4.17 & -0.20 & 1.43 & -1.00 & -1.83 & 0.20 & -3.77 \\
 & & 2.08 & -1.00 & -0.68 & 0.20 & -0.92 & 1.00 \\
 & & & 4.17 & -0.20 & -3.77 & 1.00 & -1.83 \\
 & & & & 2.08 & 1.00 & -0.48 & 0.20 \\
 & & & & & 4.17 & -0.20 & 1.43 \\
 & & & & & & 2.08 & -1.00 \\
\text{Symmetry} & & & & & & & 4.17
\end{bmatrix}
\end{array}$$

Practical Considerations in Modeling

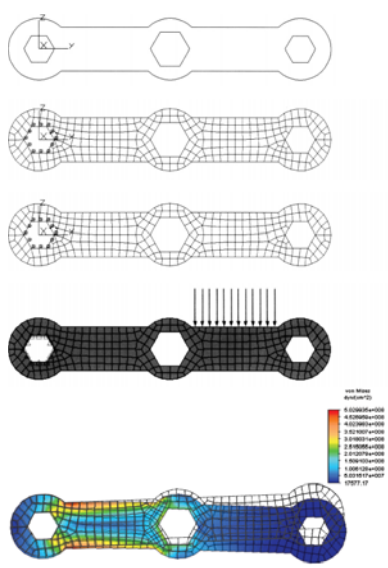
Flowchart for the Solution of Place Stress/Strain Problems

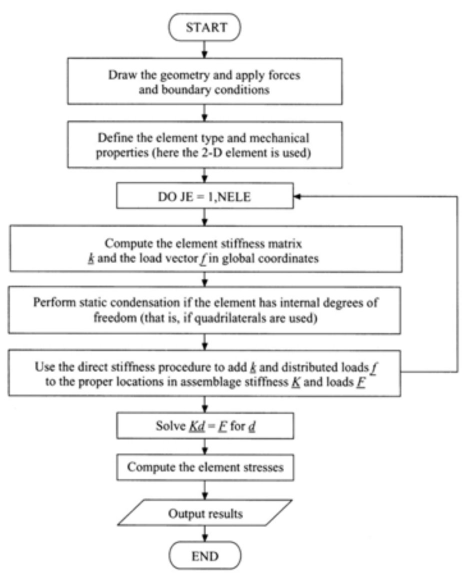
The following flowchart is typical for a finite element process used for the analysis of plane stress and plane strain problems.



Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems



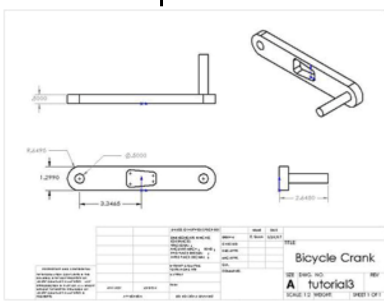


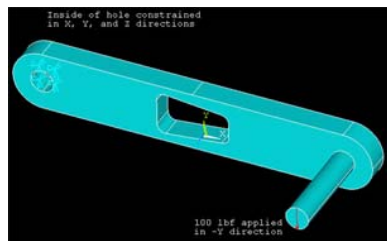
Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems

A bicycle company is disappointed with the negative feedback they have received on their latest model, and they have pinpointed the problem to an outdated bicycle crank design.

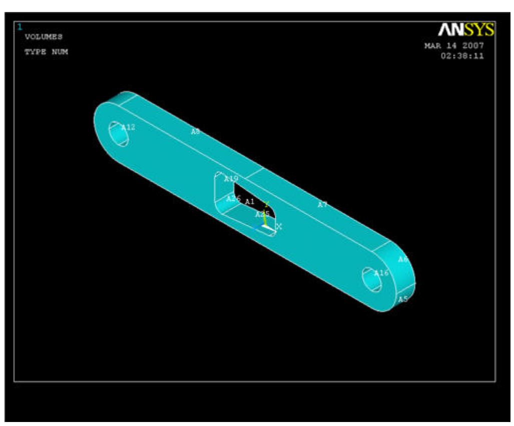
They have outsourced the task of analyzing the crank to you, providing you with the geometry of the bicycle crank and attached pedal shaft shown below.

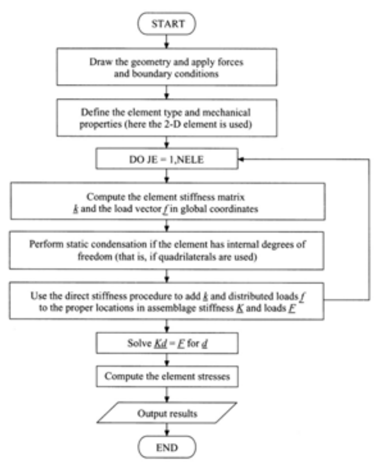




Practical Considerations in Modeling

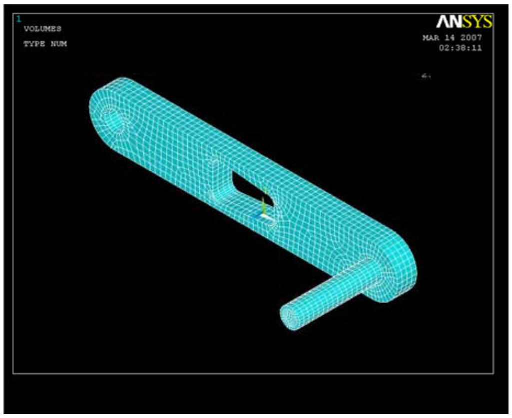
Flowchart for the Solution of Place Stress/Strain Problems

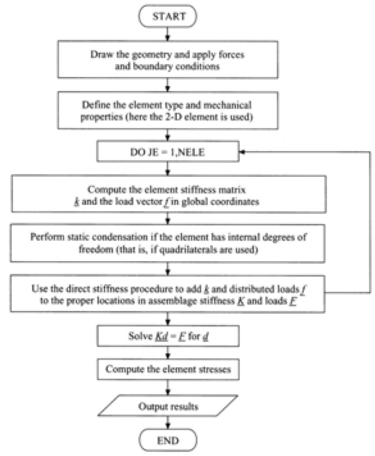




Practical Considerations in Modeling

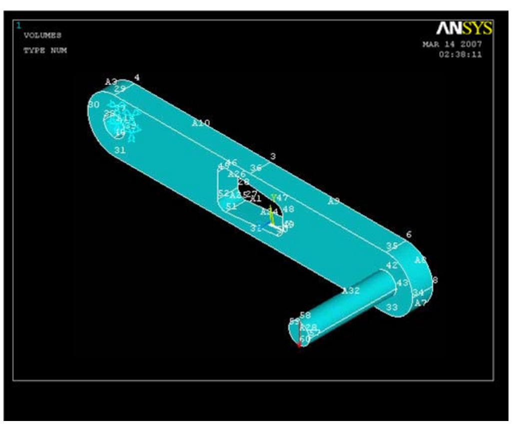
Flowchart for the Solution of Place Stress/Strain Problems

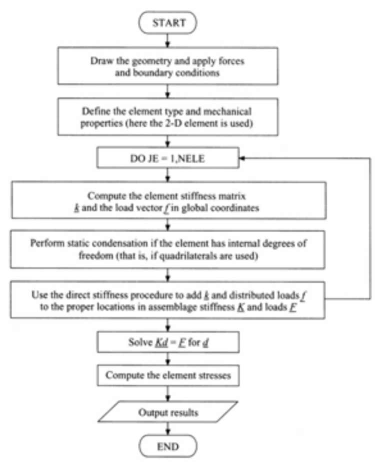




Practical Considerations in Modeling

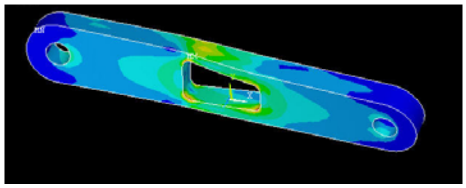
Flowchart for the Solution of Place Stress/Strain Problems

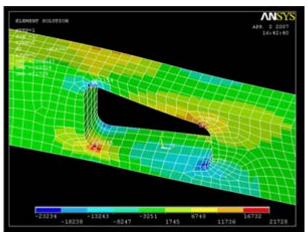


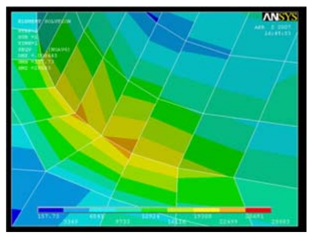


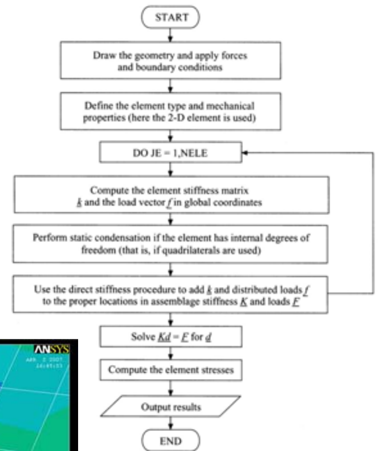
Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems



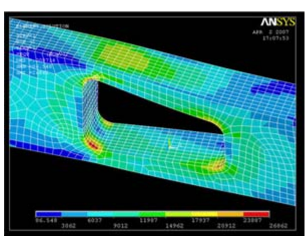




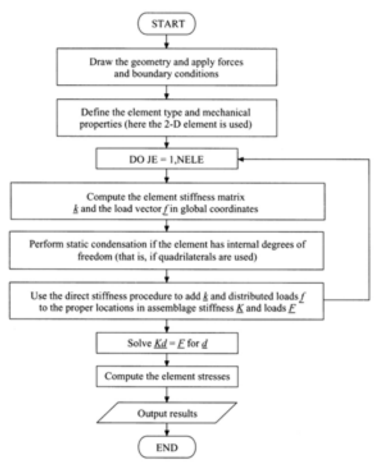


Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems



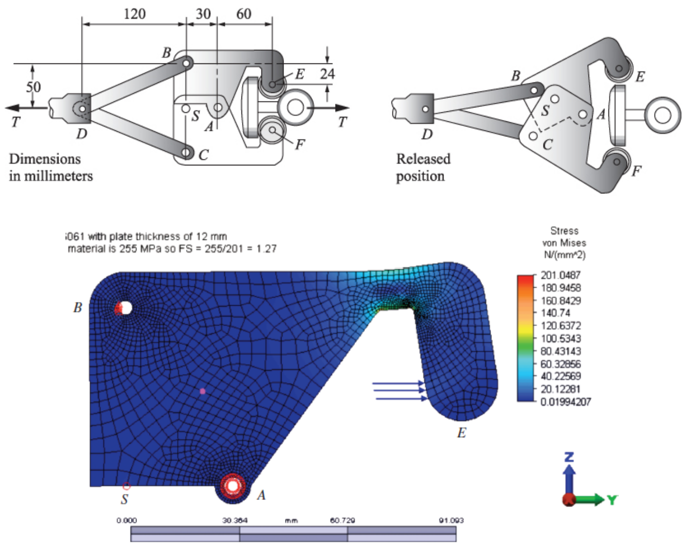
	Coarser Mesh	Finer Mesh
DMX	0.026148 *in*	0.026651 *in*
SMX	25,308 *psi*	27,942 *psi*



The maximum displacement at the tip of shaft is 1.9% greater and the maximum stress is 10% greater

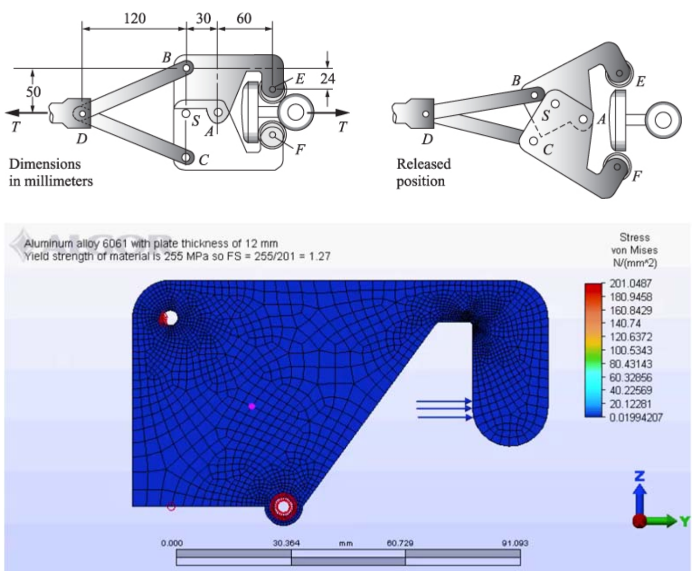
Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems



Practical Considerations in Modeling

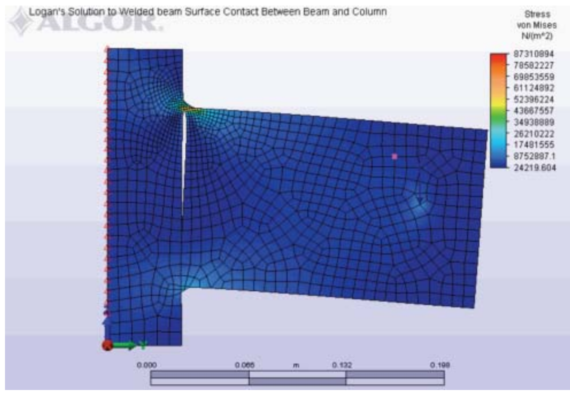
Flowchart for the Solution of Place Stress/Strain Problems



Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems

Below is a finite element model and the von Mises stress plot for a beam welded to a column by top and bottom fillet welds. The material is steel with E = 205 *GPa* and $\nu = 0.25$



Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems

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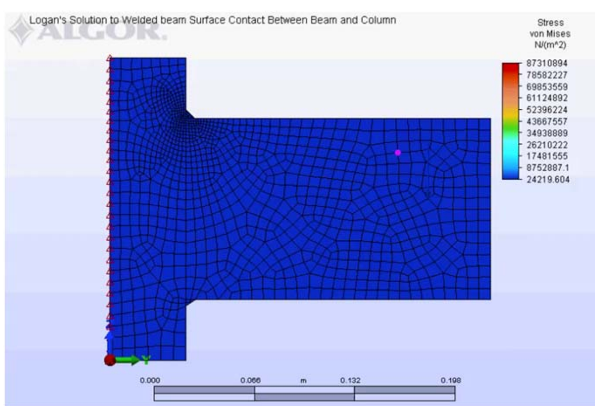
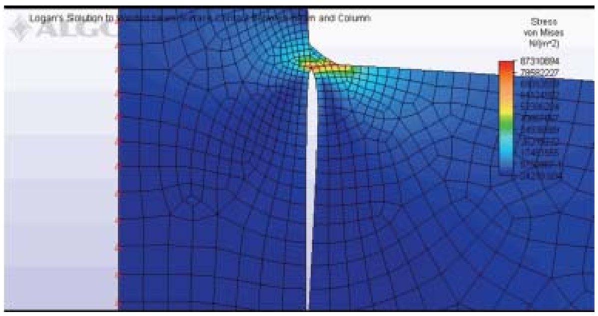


Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems

After mesh refinement around the top weld to double the number of elements in the weld, the maximum Von Mises stress was determined to be 87.3 Mpa compares reasonably well with that obtained by the classical method where a value of 94 MPa was obtained.



Practical Considerations in Modeling

Flowchart for the Solution of Place Stress/Strain Problems

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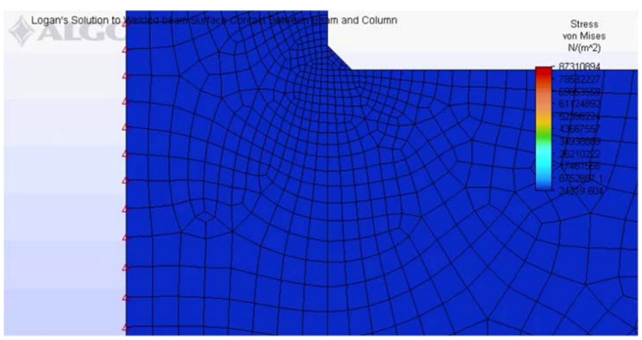

Practical Considerations in Modeling

Problems

14. Do problems ***7.2, 7.3,*** and ***7.4*** on pages 421 - 436 in your textbook “A First Course in the Finite Element Method” by D. Logan.

15. Work problems ***7.13, 7.16,*** and ***7.22*** on pages 421 - 436 in your textbook “A First Course in the Finite Element Method” by D. Logan.

End of Chapter 7