

# CIVL 7111 - Special Modelling Project 4

## Structural Mechanics - Torsion of Nonhomogeneous Prismatic Bars

**Problem Statement** - Consider the problem of torsion of a homogeneous isotropic prismatic bar. The general two-dimensional boundary-value problem is:

$$\begin{aligned}\nabla^2 \phi(x, y) + 2G\theta &= 0 & \text{in } \Omega \\ \phi &= 0 & \text{on } \Gamma\end{aligned}$$

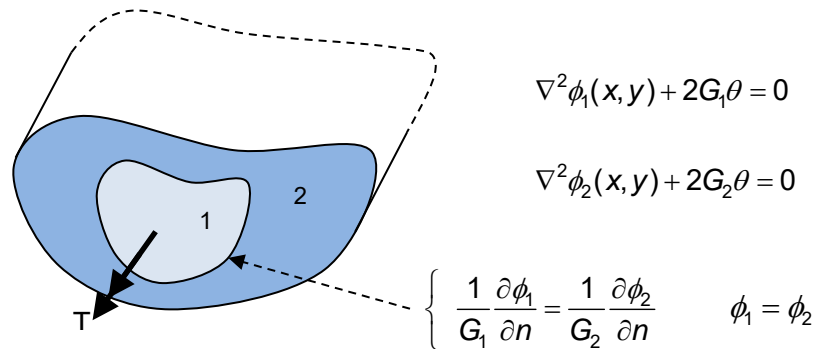
where the dependent variable  $\phi$  is the Prandtl stress function,  $G$  is the shear modulus, and  $\theta$  is the constant rate of twist along the axis of the bar. The stress components are given in terms of the derivatives of the Prandtl stress function.

$$\tau_{xz} = \frac{\partial \phi}{\partial y} \quad \tau_{yz} = -\frac{\partial \phi}{\partial x}$$

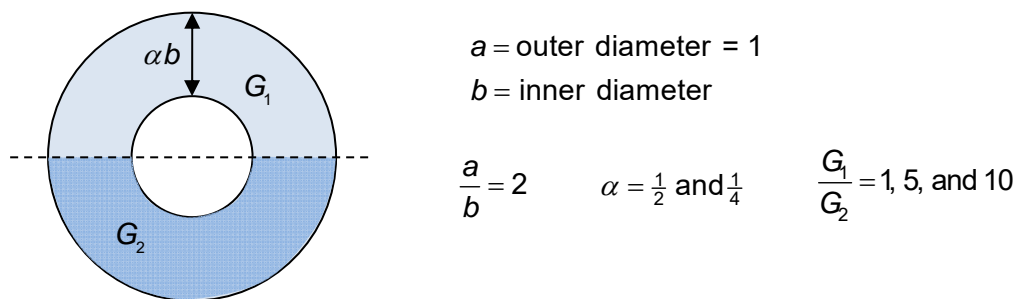
The total torque transmitted along the bar is determined from:

$$T = 2 \iint_{\Omega} \phi \, dA$$

Consider the a cross-section shown in the diagram below:



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Use **POIS36** to the model this problem for the various cases indicated by the ratios  $a/b$  and  $G_1/G_2$ . From these results plot the stress function and calculate the torque on the cross-section. See - J.F. Ely and O.C. Zienkiewicz, "Torsion of compound bars - a relaxation solution." Int. J. Mech. Sci. Vol 1, pp. 356-365, 1960.