CIVL 7111 - Special Modelling Project 2

Structural Mechanics - Torsion of Nonhomogeneous Prismatic Bars

Problem Statement - Consider the problem of torsion of a homogeneous isotropic prismatic bar. The general two-dimensional boundary-value problem is:

$$\nabla^2\phi(x,y) + 2G\theta = 0 \qquad \text{in } \Omega$$
$$\phi = 0 \qquad \text{on } \Gamma$$

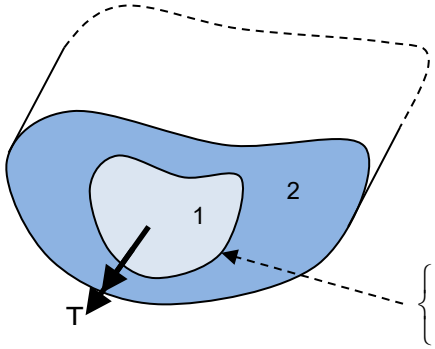
where the dependent variable ϕ is the Prandlt stress function, G is the shear modulus, and θ is the constant rate of twist along the axis of the bar. The stress components are given in terms of the derivatives of the Prandlt stress function.

$$\tau_{xz} = \frac{\partial\phi}{\partial y} \qquad \tau_{yz} = -\frac{\partial\phi}{\partial x}$$

The total torque transmitted along the bar is determined from:

$$T = 2\iint_{\Omega}\phi\, dA$$

Consider the a cross-section shown in the diagram below:

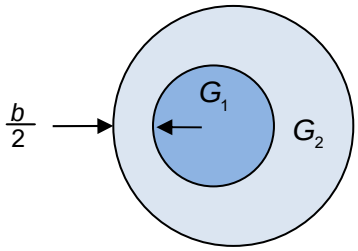


$$\nabla^2\phi_1(x,y) + 2G_1\theta = 0$$

$$\nabla^2\phi_2(x,y) + 2G_2\theta = 0$$

$$\left\{ \frac{1}{G_1}\frac{\partial\phi_1}{\partial n} = \frac{1}{G_2}\frac{\partial\phi_2}{\partial n} \qquad \phi_1 = \phi_2 \right.$$

Consider the a cross-section shown in the diagram below:



a = outer diameter = 1

b = inner diameter

$$\frac{a}{b} = 2 \text{ and } 4 \qquad \frac{G_1}{G_2} = 1 \text{ and } 5$$

Use **POIS36** to the model this problem for the various cases indicated by the ratios a/b and G_1/G_2. From these results plot the stress function and calculate the torque on the cross-section. *See* - J.F. Ely and O.C. Zienkiewicz, “Torsion of compound bars - a relaxation solution.” Int. J. Mech. Sci. Vol 1, pp. 356-365, 1960.