CIVL 7111 - Special Modelling Project 2

Structural Mechanics - Torsion of Nonhomogeneous Prismatic Bars

**Problem Statement** - Consider the problem of torsion of a homogeneous isotropic prismatic bar. The general two-dimensional boundary-value problem is:

\[
\nabla^2 \phi(x,y) + 2G \theta = 0 \quad \text{in } \Omega
\]

\[
\phi = 0 \quad \text{on } \Gamma
\]

where the dependent variable \( \phi \) is the Prandtl stress function, \( G \) is the shear modulus, and \( \theta \) is the constant rate of twist along the axis of the bar. The stress components are given in terms of the derivatives of the Prandtl stress function.

\[
\tau_{xz} = \frac{\partial \phi}{\partial y} \quad \tau_{yz} = \frac{\partial \phi}{\partial x}
\]

The total torque transmitted along the bar is determined from:

\[
T = 2 \oint_{\Gamma} \phi dA
\]

Consider the a cross-section shown in the diagram below:

\[
\nabla^2 \phi_1(x,y) + 2G_1 \theta = 0
\]

\[
\nabla^2 \phi_2(x,y) + 2G_2 \theta = 0
\]

\[
\frac{1}{G_1} \frac{\partial \phi_1}{\partial n} = \frac{1}{G_2} \frac{\partial \phi_2}{\partial n} \quad \phi_1 = \phi_2
\]

Consider the a cross-section shown in the diagram below:

\[
a = \text{outer diameter} \quad b = \text{inner diameter}
\]

\[
\frac{a}{b} = 2 \text{ and } 4 \quad \frac{G_1}{G_2} = 1 \text{ and } 3
\]

Use **POIS36** to model this problem for the various cases indicated by the ratios \( a/b \) and \( G_1/G_2 \). From these results plot the stress function and calculate the torque on the cross-section. See - J.F. Ely and O.C. Zienkiewicz, “Torsion of compound bars - a relaxation solution.” Int. J. Mech. Sci. Vol 1, pp. 356-365, 1960.