CIVL 7/8012
In-class problem solutions
Confidence intervals

1.
   a. $56.8 \pm \frac{1.96*3}{\sqrt{25}} = 56.8 \pm 1.176 = (55.624, 57.976)$
   b. $56.8 \pm \frac{2.58*3}{\sqrt{75}} = 56.8 \pm 0.893 = (55.907, 57.693)$
   c. $n = \left[\frac{2*2.58*3}{1}\right]^2 = 239.62 \Rightarrow n = 240$

2. $s = 1.578, n = 15,\ t_{0.025,14} = 2.145$

   $$25.313 \pm 2.145 * \frac{1.578}{\sqrt{15}} = 25.313 \pm 0.874 = (24.439, 26.187)$$

3. $\bar{x}_1 = 92.5, \sigma_1 = 1, n_1 = 30$
   $\bar{x}_2 = 78.5, \sigma_2 = 1.5,\ n_2 = 40$
   $z_{0.05} = 1.645$

   $$CI = (92.5 - 78.5) \pm 1.645\sqrt{\frac{1}{30} + \frac{2.25}{40}} = 14 \pm 0.492$$

   $13.508 \le \mu_1 - \mu_2 \le 14.492$

4. $\bar{x}_1 = 2.007, n_1 = 18, s_1 = 0.01$
   $\bar{x}_2 = 2.001, n_2 = 10, s_2 = 0.012$

   $$S_p = \sqrt{\frac{17 * 0.01^2 + 9 * 0.012^2}{18 + 10 - 2}} = 0.0107$$

   99% CI :

   $$(\bar{x}_1 - \bar{x}_2) \pm t_{0.005,26} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

   $$= (2.007 - 2.001) \pm 2.779 * 0.0107 * \sqrt{\frac{1}{18} + \frac{1}{10}}$$

   $$= 0.006 \pm 0.0117$$

   Therefore, $-0.0057 \le \mu_1 - \mu_2 \le 0.0177$

5. $Mean\ of\ difference, \bar{d} = 1.21, n = 14,\ s_D = 12.68$
   90% CI:

   $$\bar{d} - t_{0.05,13}\frac{s_D}{\sqrt{n}} \le \mu_1 - \mu_2 \le \bar{d} + t_{0.05,13}\frac{s_D}{\sqrt{n}}$$

   $$= 1.21 - 1.771\frac{12.68}{\sqrt{14}} \le \mu_1 - \mu_2 \le 1.21 + 1.771\frac{12.68}{\sqrt{14}}$$

   $$= -4.79 \le \mu_1 - \mu_2 \le 7.21$$

6. $n = 20, s^2 = 0.0225$

$$90\%\ CI: \frac{(n-1)s^2}{\chi^2_{0.05,19}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{0.95,19}}$$

$$= \frac{19 * 0.0225}{30.14} \leq \sigma^2 \leq \frac{19 * 0.0225}{10.12}$$

$$= 0.0141 \leq \sigma^2 \leq 0.0422$$
$$\Rightarrow 0.1187 \leq \sigma \leq 0.205$$

7. $\bar{x}_1 = 24.6, n_1 = 12, s_1 = 0.85, v_1 = 11$
$\bar{x}_2 = 22.1, n_2 = 15, s_2 = 0.98, v_2 = 14$
90% CI:

$$\frac{{s_1}^2}{{s_2}^2}\frac{1}{\mathrm{F}_{0.05,11,14}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{{s_1}^2}{{s_2}^2}\mathrm{F}_{0.05,14,11}$$

$$= \frac{0.85^2}{0.98^2}\frac{1}{2.565} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{0.85^2}{0.98^2}2.755$$

$$= 0.293 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq 2.072$$

8. $\bar{x} = 2259.92, n = 12, s = 35.57$
90% PI:

$$\bar{x} - t_{0.05,11}s\sqrt{1+\tfrac{1}{n}} \leq X_{n+1} \leq \bar{x} + t_{0.05,11}s\sqrt{1+\tfrac{1}{n}}$$
$$= 2259.92 - 1.796 * 35.57 * \sqrt{1+\frac{1}{12}} \leq X_{n+1} \leq 2259.92 + 1.796 * 35.57 * \sqrt{1+\frac{1}{12}}$$
$$= 2193.42 \leq X_{n+1} \leq 2326.41$$