CIVL 7012/8012

Practice Problems

Recap From Last Lecture

$$P(A \cup B) \equiv P(A \text{ or } B) \equiv P(A + B)$$

$$P(A \cap B) \equiv P(A \text{ and } B) \equiv P(AB)$$

Using this new shorthand, we can rewrite the basic axioms of probability as:

Negation (complement): $P(A)' \equiv P(A^c) \equiv P(\bar{A}) = 1 - P(A)$

Union (mutually exclusive events): $P(A + B) = P(A) + P(B)$

Union (general): $P(A + B) = P(A) + P(B) - P(AB)$

Conditional Probability: $P(A \mid B) = \dfrac{P(B \mid A) * P(A)}{P(B)}$ for $P(B) > 0$

Intersection (independent events): $P(AB) = P(A) \cdot P(B)$

Intersection (general): $P(A \cap B) = P(A \mid B) \cdot P(B)$

Examples

Aggregates for the construction of a reinforced concrete building are supplied by two companies, Company A and Company B. Orders are for Company A to deliver 600 truck loads a day and 400 truck loads a day from Company B. From prior experience, it is expected that 3% of the material from Company A will be substandard, whereas 1% of the material from Company B are substandard.

Let: A = Aggregates supplied by Company A

B = Aggregates supplied by Company B

E = Aggregates are substandard

A.) What is the probability of substandard aggregates?

B.) If a load of aggregates is found to be substandard, what is the probability that it is from company A?

Examples

A tower may be subjected to earthquake loads which could be of high intensity (event H) or of long duration (event L). It is estimated that if the load has long duration, the probability that its intensity is high is 0.7. Also, if the load has high intensity, there is 20% probability that it will be of short duration. Finally, the probability of having a long duration earthquake load is 0.3.

The designer estimated that the probability of failure when the tower is subjected to a short duration-high intensity earthquake is 0.05, whereas, this probability is doubled if the earthquake is of long duration but low intensity. Also, he is certain that the tower will fail if subjected to an earthquake with both high intensity and long duration, and that it will survive with certainty if subjected to an earthquake of low intensity and short duration. A) Are the events H and L mutually exclusive? B) Are they independent? C) collectively exhaustive? D) Calculate the probability of failure of this tower when subjected to an earthquake.

Examples

A particular landfill containment system is comprised of a thick layer of clay placed between the landfill and the surrounding soil stratum to prevent contaminants leaking from the landfill into the soil stratum due to rainfall infiltration. A layer of synthetic material (geomembrane) was also placed above the clay material to provide additional protection against leakage of contaminants. Nevertheless, the quality of workmanship during construction may not be completely satisfactory. First, the clay might have been compacted poorly; second, the geomembrane might have holes punctured by sharp stones that were not detected during inspection. Moreover, extremely heavy rainfall could happen during the operation of the landfill, which could induce excessive pore pressure on the geomembrane/clay layers.

The engineer in this case believes that leakage will happen "during extremely heavy rainfall, and either the clay was not well compacted or there were holes in the geomembrane" (event I). Leakage could also occur "under ordinary rainfalls, but only when the clay was not well compacted and the geomembrane contained holes" (event II). Let W= event of well-compacted clay; P(W)=0.9; H=event of geomembrane containing holes; P(H)=0.30; and E=event of extremely heavy rainfall, P(E)=0.20. If the geomembrane contained holes, the probability of a well-compacted clay is reduced to 60%.

a. Express event I in terms of the symbols defined above. Repeat for event II.
b. Determine the probability of event I. Repeat for event II.
c. Are W and H mutually exclusive? Are they statistically independent? Provide explanations to support your answers.
d. Are events I and II mutually exclusive? Are they collectively exhaustive? Explain.
e. Determine the probability of leakage for this landfill containment system.