CIVL 7/8012
In-class problem solutions
Continuous distributions

Normal distribution

1. $P(z<2.55)$
   $= 0.9946$

2. $P(z>1.26)$
   $= 1- P(z<1.26)$
   $= 1-0.8962$
   $= 0.1038$

3. $P(z>-1.32)$
   $= 1- P(z<-1.32)$
   $= 0.9066$

4. $P(-1.25<z<0.37)$
   $= 1- P(z<-1.25) – P(z>0.37)$
   $= 1 - 0.1056 – 0.3557$
   $= 0.5987$

5. $\mu = 2.03$
   $\sigma = 0.44$
   a. $x = 2.5$
      $z = \frac{z-\mu}{\sigma} = 1.06$
      $P(z>2.5) = 0.0062$
   b. $x = 1.59 \Rightarrow z = -1$
      $x = 2.47 \Rightarrow z = +1$
      $P(-1<z<1) = 0.6826$
6. $\mu = 3$
   $\sigma = 0.01$
   $z = \pm \frac{0.01}{0.005} = \pm 2$
   Proportion of balls within the specification
   $P(-2<z<2) = 0.9544$
   Therefore $1-0.9544 = 0.0456$ or 4.56% of ball bearings will be scrapped.

7. $P(-1.96<z<1.96) = 0.95$
   So, $1.96 = \frac{(1.5+d)-1.5}{0.2} \Rightarrow \mathrm{d} = 0.392$

Exponential distribution

8. $\lambda = 1/\mu$
   $= 1/6$

   $P(X>x) = e^{-\lambda x} = e^{-\frac{1}{6}*6} = 0.367$

Gamma distribution

9. $r = \alpha = 8;\ 1/\lambda = \beta$
   a. $E(X) = 8*15 = 120$
   b. $V(X) = 8*15^2 = 1800 \Rightarrow \sigma = 42.43$
   c. $P(X \le x) = F\left(\frac{x}{\beta}; \alpha\right)$

      $P(60 \le X \le 120) = P(X \le 120) - P(X \le 60)$
      $= F(8:8) - F(4:8)$
      $= 0.547 - 0.051$
   d. $P(X \ge 30) = 1 - P(X<30)$
      $= 1 - F(2;8)$
      $= 0.999$

10. COV = 0.4
    E(X) = 80
    a. COV = $\frac{\sqrt{Var(X)}}{E(X)}$

       $\Rightarrow \sqrt{Var(X)} = 32$

       Variance, $\sigma_x^2 = \ln\left(1 + \frac{Var(X)}{\{E(X)\}^2}\right) = \ln\left(1 + \frac{32^2}{80^2}\right) = 0.148$

       Mean, $\mu_x = \ln\left(E(X) - \frac{1}{2}\sigma_x^2\right) = \ln\left(80 - \frac{0.148}{2}\right) = 4.3$

    b. $\Phi\left(\frac{\ln(X)-\mu_x}{\sigma_x}\right) = \Phi\left(\frac{\ln(20)-4.3}{\sqrt{0.148}}\right) = \Phi(-3.39) = 0.0003$

    c. $P(X < 101 | X > 100) = \frac{P(100<X<101)}{P(X>100)} = \frac{\Phi\left(\frac{\ln(101)-4.3}{\sqrt{0.148}}\right) - \Phi\left(\frac{\ln(100)-4.3}{\sqrt{0.148}}\right)}{1-\Phi\left(\frac{\ln(100)-4.3}{\sqrt{0.148}}\right)}$

$$= \frac{\Phi(0.8) - \Phi(0.77)}{1 - \Phi(0.77)}$$
$$= \frac{0.788 - 0.779}{1 - 0.779}$$
$$= 0.04$$