

CIVL 7/8012
In-class problem solutions
Discrete distributions

Binomial distribution

1. $p = \frac{3}{4}$

$$q = \frac{1}{4}$$

$$n = 4$$

$$x = 2$$

$$P(x) = \binom{n}{x} (p)^x (q)^{n-x}$$

$$P(2) = \binom{4}{2} \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2 = 0.21$$

2. $p = 1/50$

$$q = 49/50$$

$$n = 10$$

$$x = 2$$

$$P(x \leq 2) = P(1) + P(2)$$

$$= \binom{10}{2} \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^9 + \binom{10}{2} \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$$

$$= 0.166 + 0.015$$

$$= 0.181$$

Negative binomial distribution

3. $p = 0.1$

$$q = 0.9$$

$$n = 20$$

$$x = 3$$

$$P(x) = \binom{n-1}{x-1} (p)^x (q)^{n-x}$$

$$P(3) = \binom{10}{2} (0.1)^3 (0.9)^{17} \\ = 0.0285$$

Geometric distribution

$$4. \quad p = 0.01$$

$$q = 0.99$$

$$n = 5$$

$$P(n) = p q^{n-1}$$

$$P(5) = (0.01) (0.99)^4 = 0.0096$$

Hypergeometric distribution

$$5. \quad N = 1000$$

$$K = 100$$

$$n = 20$$

$$P(x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$$

$$P(x \leq 2) = P(0) + P(1) + P(2)$$

$$= \frac{\binom{100}{0} \binom{900}{20}}{\binom{1000}{20}} + \frac{\binom{100}{1} \binom{900}{19}}{\binom{1000}{20}} + \frac{\binom{100}{2} \binom{900}{18}}{\binom{1000}{20}}$$

$$= 0.119 + 0.27 + 0.288$$

$$= 0.677$$

Poisson distribution

$$6. \quad \lambda = 2 \text{ per year}$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(0) = \frac{e^{-2} 2^0}{0!} = 0.1353$$

$$P(2) = \frac{e^{-2}2^2}{2!} = 0.27$$

For 50 years, $\lambda = 100$ per 50 year

$$P(0) = \frac{e^{-100}100^0}{0!} = 3.72e^{-44}$$