BIAXIAL BENDING

Consideration of Columns with Axial Load and Biaxial Bending

1. Occurrence. Will be presented in any situations where beams frame into column at right angles, or bridge pier, etc.

2. Criteria for determining nominal strength are the same as for columns in uniaxial bending (from material standpoint). Problem is complicated by the fact that the neutral axis of the bending is no longer parallel to a major axis. Optimum column dimensions are not likely to be equal if projected eccentricities vary.

3. Graphical presentation of the problem

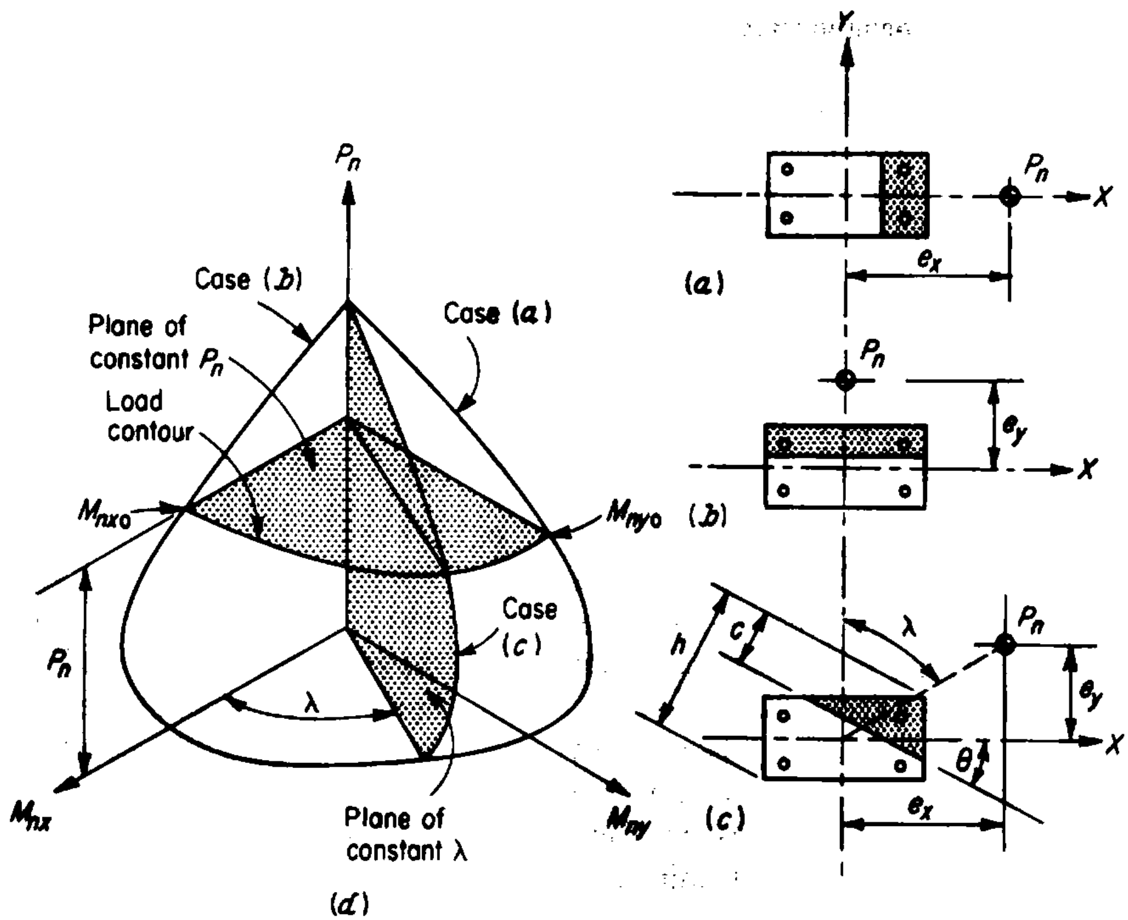
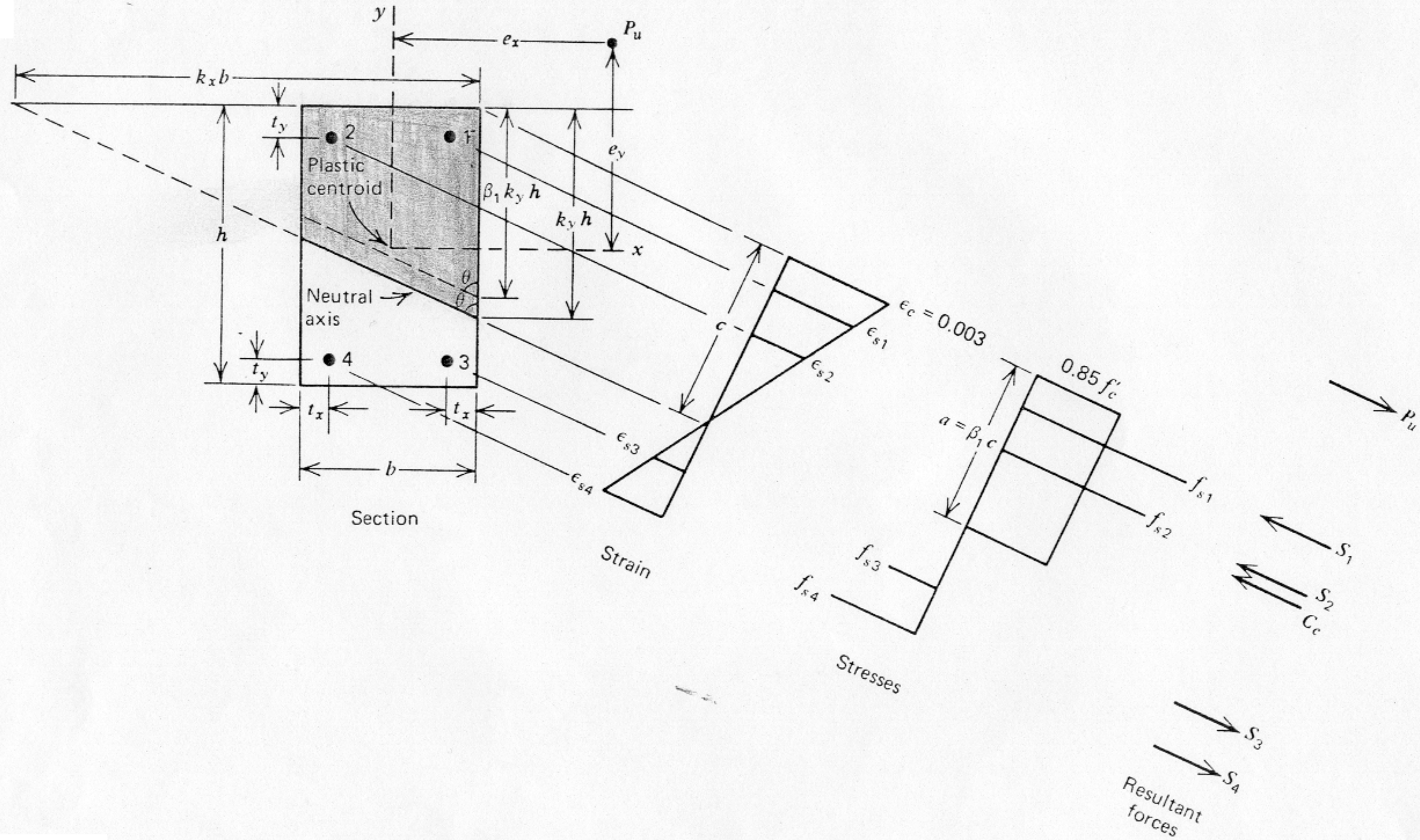


Figure 1. Interaction surface for compression plus biaxial bending: (a) uniaxial bending about Y- axis; (b) uniaxial bending about X-axis; (c) biaxial bending about diagonal axis; and (d) interaction surface.



Column section with biaxial bending at the ultimate load.

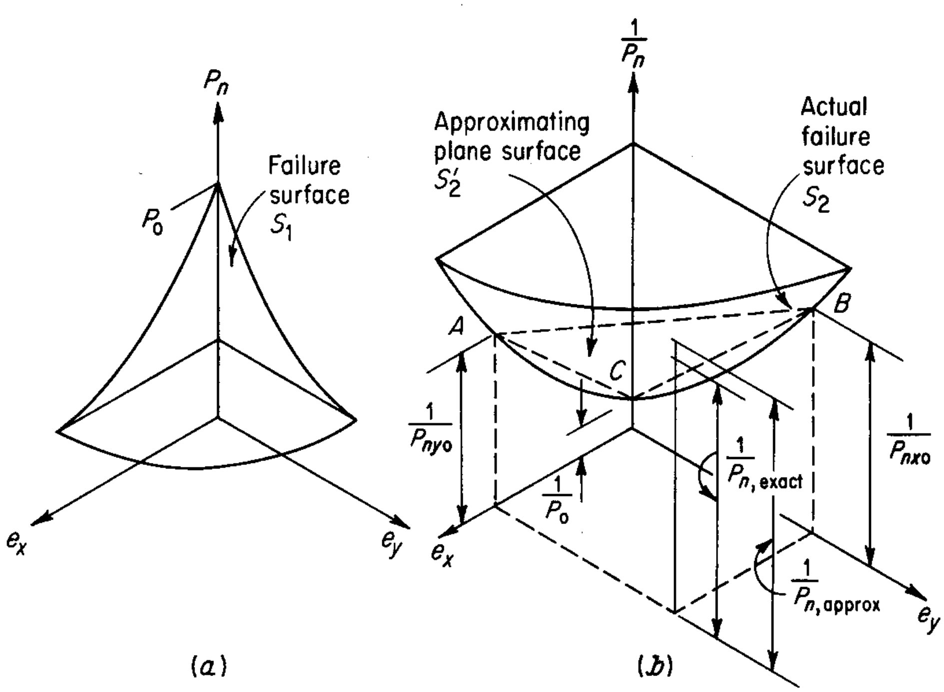
Current Methods of Analysis

Bresler, B. "Design Criteria for Reinforced Concrete Columns Under Axial Load and Biaxial Loading." *Journal of the American Concrete Institute,* Vol. 57, No. 5, November 1960, pp.481-490.

Bresler based his analysis on an assumption of a number of possible "Failure Surfcace" in three dimensions.

Failure Surface 1 - Failure point defined as a function of axial load and eccentricities.

Failure Surface 2 - Similar basis with 1 failure point defined as function of $1/p_n$, e_x, e_y



Interaction surfaces for the reciprocal load method.

Bresler reasoned:

1. The failure surface is too complicated to exactly define.
2. An acceptable approximation could be defined by a plane which passes through three points which could be found by conventional (uniaxial bending) analysis.

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Reciprocal Load Mehtod

Bresler's reciprocal load equation derived from the geometry of the approximate plane:

$$\frac{1}{p_n} = \frac{1}{p_{nx0}} + \frac{1}{p_{ny0}} - \frac{1}{p_0}$$

where

p_n = Approximate value of ultimate load in biaxial bending with eccentricity of e_x, e_y.
p_{nx0} = Ultimate load when only eccentricity e_y is present ($e_x = 0$)
p_{ny0} = Ultimate load when only eccentricity e_x is present ($e_y = 0$)
p_0 = Ultimate load for concentrically loaded column.

This procedure is acceptably accurate for design purposes provided $p_n \geq 0.1P_0$. If $P_n < 0.1P_0$, it would be more accurate to neglect the axial force entirely and to calculate the section for biaxial bending only.

ACI strength reduction factors do not change the development in any fundamental way as long as the ϕ factor is constant for all columns.

$$\frac{1}{\phi p_n} = \frac{1}{\phi p_{nx0}} + \frac{1}{\phi p_{ny0}} - \frac{1}{\phi p_0}$$

Note that:

- It is necessary to use the uniaxial curves without the horizontal cutoff in obtaining values for the above equation.
- $\phi p_n \leq \begin{cases} 0.8\phi p_0 & Tied \quad Columns \\ 0.85\phi p_0 & Spiral \quad Columns \end{cases}$

Load Contour Method

The load contour method is based on representing the failure surface of the Figure 1 give above by a family of curves corresponding to constant value of p_n. The general form of these curves can be approximated by a non-dimensional interaction equation:

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha_2} = 1.0$$

where:

$$M_{nx} = P_n e_y$$
$$M_{nx0} = M_{nx} \quad when \quad M_{ny} = 0$$

$$M_{ny} = P_n e_x$$
$$M_{ny0} = M_{ny} \quad when \quad M_{nx} = 0$$

This equation gives the surface of design strength. The parameter alpha is

$1.15 < \alpha < 1.55$ for square and rectangular columns where β is tabulated for specific

- Strength
- Geometry
- Material strength

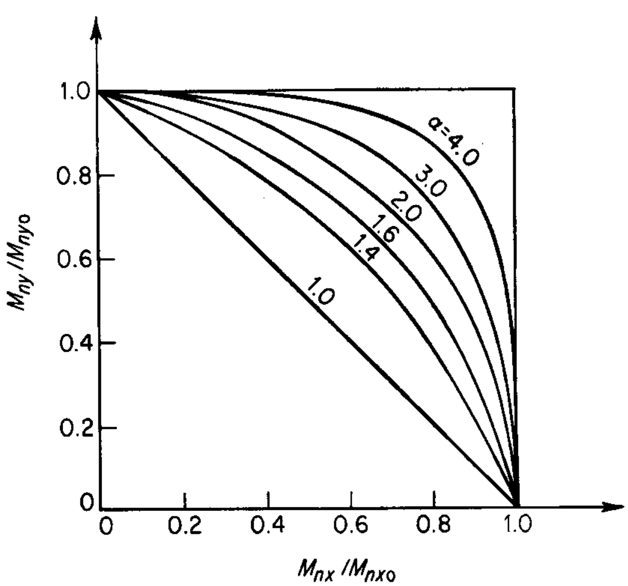


FIGURE 8.16
Interaction
varying α. (*Adapted from Ref. 8.10.*)

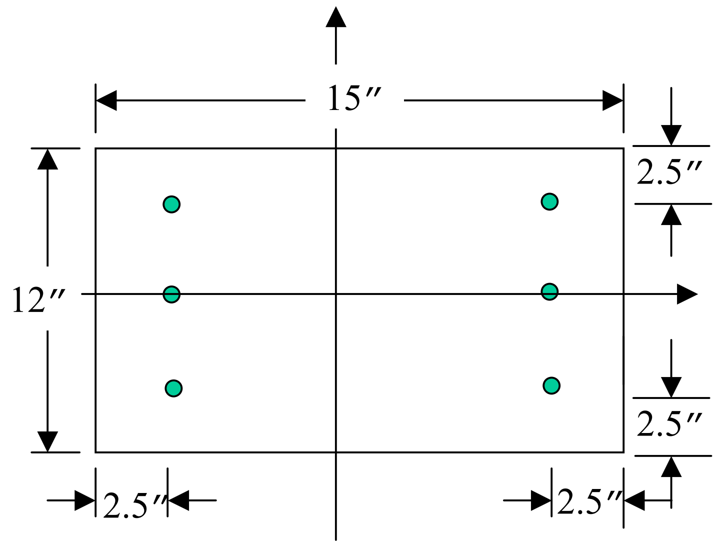
Design Example for Biaxial Bending

$e_x = 6"$

$e_y = 3"$ $\qquad f'_c = 4\ ksi$

$A_{st} = 8\ in^2$ $\qquad f_y = 60\ ksi$

$P_u = 275 \quad kips$



Check the adequacy of the trial design

(a) using the reciprocal load method

(b) using load contour method

(a) Using the reciprocal load method - Solution:

About Y-axis

$$\left.\begin{aligned} &\gamma = \frac{15}{16} = 0.75 \\ &\rho_t = \frac{A_s}{bh} = \frac{8}{240} = 0.033 \\ &\frac{e}{h} = \frac{6}{20} = 0.30 \end{aligned}\right\} \text{ Use Graph A.7}$$

Read from graph and simplify

$$\begin{cases} \dfrac{\phi P_n}{A_g} = 1.75 \rightarrow \phi P_n = 1.75 \times 240 = 420\ kips \\ \dfrac{\phi P_n}{A_g} = 3.65 \rightarrow \phi P_n = 3.65 \times 240 = 876\ kips \end{cases}$$

About X-axis

$$\gamma = \frac{7}{12} = 0.58 \text{ say } 0.6$$

$$\left.\begin{array}{l}\gamma = \dfrac{7}{12} = 0.58 \\ \rho_t = \dfrac{A_s}{bh} = \dfrac{8}{240} = 0.033 \\ \dfrac{e}{h} = \dfrac{3}{12} = 0.25\end{array}\right\} \text{ Use Graph A.6}$$

Read from graph and simplify

$$\begin{cases}\dfrac{\phi P_n}{A_g} = 1.8 \rightarrow \phi P_n = 1.8 \times 240 = 432 \ kips \\ \dfrac{\phi P_n}{A_g} = 3.65 \rightarrow \phi P_n = 3.65 \times 240 = 876 \ kips\end{cases}$$

The reciprocal method gives:

$$\frac{1}{\phi p_n} = \frac{1}{\phi p_{nx0}} + \frac{1}{\phi p_{ny0}} - \frac{1}{\phi p_0}$$

$$\frac{1}{\phi p_n} = \frac{1}{432} + \frac{1}{420} - \frac{1}{876} = 0.00356$$

$$\phi p_n = 281 \quad kips > \quad P_u = 275 \quad kips$$

Therefore the design is adequate.

Using load contour method Solution

About Y-axis

$$\left.\begin{aligned} P_u = \phi P_n = 275 \quad kips \\ \frac{\phi P_n}{A_g} = \frac{275}{240} = 1.15 \end{aligned}\right\} \text{Use Graph A.7}$$

$$\frac{\phi M_{nx0}}{A_g h} = 0.62$$

$$\phi M_{nx0} = 0.62 \times 240 \times 20 = 2980 \quad in - kips$$

About X-axis

$$\left.\begin{aligned} P_u = \phi P_n = 275 \quad kips \\ \frac{\phi P_n}{A_g} = \frac{275}{240} = 1.15 \end{aligned}\right\} \text{Use Graph A.6}$$

$$\frac{\phi M_{nx0}}{A_g h} = 0.53$$

$$\phi M_{nx0} = 0.53 \times 240 \times 12 = 1530 \quad in - kips$$

$$M_{uy} = P_u e_x = 275 \times 6 = 1650 \quad in - kips$$

$$M_{ux} = P_u e_y = 275 \times 3 = 825 \quad in - kips$$

The reciprocal method:

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha_2} = 1.0$$

$$\left(\frac{825}{1530}\right)^{1.15} + \left(\frac{1650}{2980}\right)^{1.15} = 0.491 + 0.507 = 0.998 < 1.000 \quad \text{This column is adequate.}$$

From Bresler $\alpha = \frac{\log 0.5}{\log \beta} \quad \beta = 0.56 \rightarrow \alpha = \frac{\log 0.5}{\log 0.56} = 1.19$

Note: Consider consider biaxial bending when estimated eccentricity ratio approaches or exceed 0.2.

Design Example

Problem: Select a tied column cross-section to resist factored loads and moments of P_u = 420 kips, M_{ux} = 70 ft-kips, and M_{uy} = 80 ft-kips. Use 8#8 bars in each face and No. 3 ties.

$$f'_c = 4\ ksi$$
$$f_y = 60\ ksi$$

Solution:

8 No. 8 bars $\rightarrow A_s = 8 \times 0.79 = 6.32 \quad in^2$

$$P_u = 0.8\phi\left[0.85 f'_c A_c + A_s f_y\right]$$
$$P_u = 0.8 \times 0.65\left[0.85(4)(A_g - 6.32) + (6.32)(60)\right] = 420$$

Solve for gross cross sectional area

$$A_g = 132 \quad in^2$$

Select:

$$b = h = 11.45 \rightarrow use \quad b = h = 12 \quad in$$

Try a 12 inch by 12 inch column, use 1.5 inch cover, No. 3 ties, and assume No. 8 bars:

$$\left.\begin{aligned} \gamma &= \frac{12 - 2(1.5 + 0.375 + 0.5)}{12} = 0.60 \\ \rho_t &= \frac{A_s}{bh} = \frac{6.32}{12 \times 12} = 0.044 \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{M_{ux}}{A_g h} &= \frac{70 \times 12}{144 \times 12} = 0.486 \\ \rho_t &= 0.044 \\ \gamma &= 0.60 \end{aligned}\right\} \text{Graph A.6} \rightarrow \begin{cases} \dfrac{P_{ux}}{A_g} = 2.4 \Rightarrow P_{ux} = 2.4 \times 144 = 345.6 \quad kips \\ \dfrac{\phi P_0}{A_g} = 4.15 \Rightarrow P_0 = 4.15 \times 144 = 517.6 \quad kips \end{cases}$$

$$\left.\begin{array}{l}\frac{M_{uy}}{A_g h}=\frac{80\times 12}{144\times 12}=0.555\\ \rho_t=0.044\\ \gamma=0.60\end{array}\right\}\text{Graph A.5}\rightarrow\left\{\begin{array}{l}\frac{P_{uy}}{A_g}=1.75\Rightarrow P_{ux}=1.75\times 144=252\quad kips\\ \frac{\phi P_0}{A_g}=4.15\Rightarrow P_0=4.15\times 144=517.6\quad kips\end{array}\right.$$

The reciprocal method gives:

$$\frac{1}{\phi p_n}=\frac{1}{\phi p_{nx0}}+\frac{1}{\phi p_{ny0}}-\frac{1}{\phi p_0}$$

$$\frac{1}{\phi p_n}=\frac{1}{345.6}+\frac{1}{252}-\frac{1}{597.6}$$

$\phi p_n = 195 \quad kips < \quad P_u = 420 \quad kips$ ---- This column is not adequate.

Try a 14 inch by 14 inch column, use 1.5 inch cover, No. 3 ties, and assume No. 8 bars:

$$\left.\begin{array}{l}\gamma=\frac{14-2(1.5+0.375+0.5)}{14}=0.66\\ \rho_t=\frac{A_s}{bh}=\frac{6.32}{14\times 14}=0.032\end{array}\right\}$$

$$\left.\begin{array}{l}\left.\begin{array}{l}\frac{M_{ux}}{A_g h}=\frac{70\times 12}{14\times 14\times 14}=0.3\\ \rho_t=0.032\\ \gamma=0.75\end{array}\right\}\rightarrow Fig\ A.7\rightarrow\left\{\begin{array}{l}\frac{P_{ux}}{A_g}=2.9\\ \frac{\phi P_0}{A_g}=3.6\end{array}\right.\\ \left.\begin{array}{l}\frac{M_{ux}}{A_g h}=\frac{70\times 12}{14\times 14\times 14}=0.3\\ \rho_t=0.032\\ \gamma=0.60\end{array}\right\}\rightarrow Fig\ A.6\rightarrow\left\{\begin{array}{l}\frac{P_{ux}}{A_g}=2.8\\ \frac{\phi P_0}{A_g}=3.6\end{array}\right.\end{array}\right\}\left\{\begin{array}{l}\gamma=0.66\\ \frac{P_{ux}}{A_g}=2.84\rightarrow P_{ux}=2.84\times 14\times 14=557\ kips\\ \frac{\phi P_0}{A_g}=3.6\rightarrow P_{ux}=3.6\times 14\times 14=713.4\ kips\end{array}\right.$$

$$\left.\begin{array}{l}\frac{M_{uy}}{A_g h}=\frac{80\times12}{14\times14\times14}=0.35\\ \rho_t=0.032\\ \gamma=0.75\end{array}\right\}\rightarrow Fig\ A.7\rightarrow\left\{\begin{array}{l}\frac{P_{ux}}{A_g}=2.75\\ \frac{\phi P_0}{A_g}=3.6\end{array}\right.$$

$$\left.\begin{array}{l}\frac{M_{uy}}{A_g h}=\frac{80\times12}{14\times14\times14}=0.35\\ \rho_t=0.032\\ \gamma=0.60\end{array}\right\}\rightarrow Fig\ A.6\rightarrow\left\{\begin{array}{l}\frac{P_{ux}}{A_g}=2.65\\ \frac{\phi P_0}{A_g}=3.6\end{array}\right.$$

$$\left\{\begin{array}{l}\gamma=0.66\\ \frac{P_{ux}}{A_g}=2.69\rightarrow P_{ux}=2.69\times14\times14=528\ kips\\ \frac{\phi P_0}{A_g}=3.6\rightarrow P_{ux}=3.6\times14\times14=713.4\ kips\end{array}\right.$$

The reciprocal method gives:

$$\frac{1}{\phi p_n}=\frac{1}{\phi p_{nx0}}+\frac{1}{\phi p_{ny0}}-\frac{1}{\phi p_0}$$

$$\frac{1}{\phi p_n}=\frac{1}{557}+\frac{1}{528}-\frac{1}{717.4}$$

$\phi p_n = 435 \quad kips > \quad P_u = 420 \quad kips$ ---- This column is adequate.

Need to finish detailing