Rigid Pavement Mechanics

Sources of Stress

- Autogenous Shrinkage
- Thermal Contraction/Expansion
- Thermal Curling and Warping
- Moisture Curling and Warping
- Frost Heave
- Wheel Loads

Major Distress Conditions

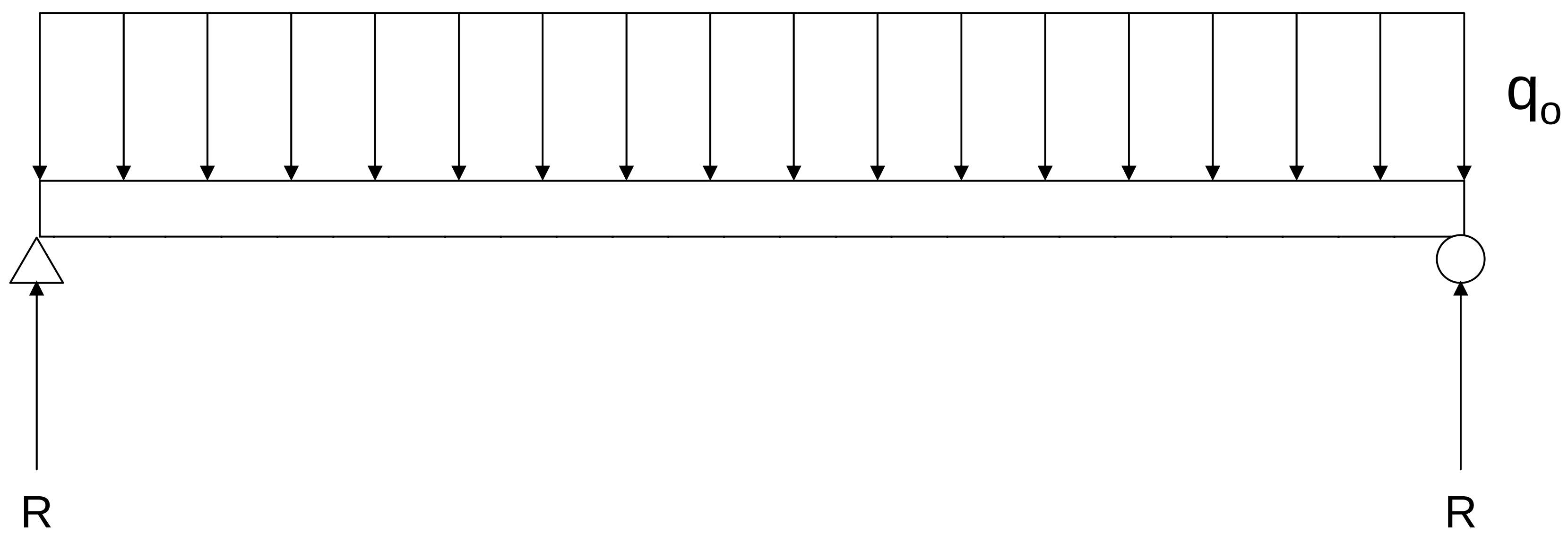
- Fatigue Cracking
  - Bottom-up transverse cracks
  - Top-down transverse cracks
  - Longitudinal cracks
  - Corner breaks
- Joint Faulting (JPCP and JRCP)
- Pumping (JPCP and JRCP)
- Punchouts (CRCP)

Design Considerations

- Slab Thickness
- Base Type and Thickness
- Joint Spacing
- Temperature Steel
- Dowel Bars
- Drainage

Beam Bending

$q_o = q \times b$ (applied load per unit length)



Beam Bending

$$\frac{dM}{dx} = V$$

$$\Rightarrow \quad \frac{d^2M}{dx^2} = -q_o$$

$$\frac{dV}{dx} = -q_o$$

Shear and Moment Relationships

Beam Bending

$$\kappa = \frac{1}{R} = \frac{d^2 w}{dx^2}$$

$$\Rightarrow \quad M = -EI\,\frac{d^2 w}{dx^2}$$

$$\kappa = -\frac{M}{EI}$$

Moment-Curvature Relationships

Beam Bending

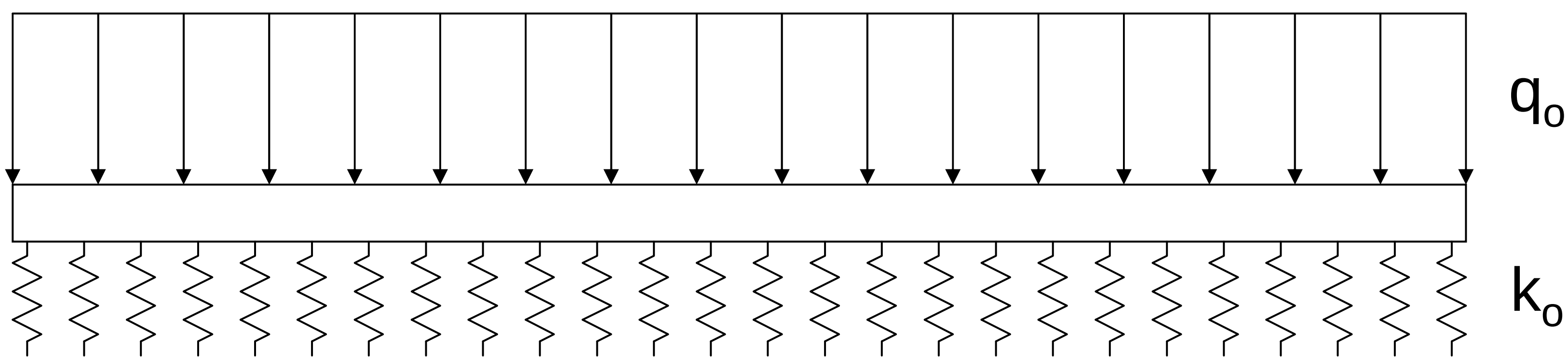
$$\frac{d^2}{dx^2}\left(-EI\frac{d^2w}{dx^2}\right)=-q_o$$

$$EI\frac{d^4w}{dx^4}=q_o$$

Governing Differential Equation for Beam Bending

Beams on Elastic Foundations

$q_o = q\,b$ (applied load per unit length)



$p = k_o\,w = k\,b\,w$ (resistance per unit length)

Modulus of Subgrade Reaction

Beams on Elastic Foundations

$$\frac{dM}{dx} = V$$

$$\Rightarrow \quad \frac{d^2M}{dx^2} = k_o w - q_o$$

$$\frac{dV}{dx} = k_o w - q_o$$

Shear and Moment Relationships

Beam Bending

$$\kappa = \frac{1}{R} = \frac{d^2 w}{dx^2}$$

$$\Rightarrow \quad M = -EI \frac{d^2 w}{dx^2}$$

$$\kappa = -\frac{M}{EI}$$

Moment-Curvature Relationships

Beams on Elastic Foundations

$$\frac{d^2}{dx^2}\left(-EI\frac{d^2w}{dx^2}\right) = k_o w - q_o$$

$$EI\frac{d^4w}{dx^4} + k_o w = q_o$$

Governing Differential Equation for Beam Bending

Beams on Elastic Foundations

$$EI \frac{d^4 w}{dx^4} + k_o w = 0$$

$$w = e^{\beta x}\left(C_1 \sin \beta x + C_2 \cos \beta x\right) + e^{-\beta x}\left(C_3 \sin \beta x + C_4 \cos \beta x\right)$$

$$\beta = \sqrt[4]{\frac{k_o}{4EI}}$$

Solution of the Homogeneous Differential Equation

Slabs on Elastic Foundations

$$M = -EI \frac{d^2 w}{dx^2}$$ Beams

$$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right)$$ Slabs

Moment-Curvature Relationships

Slabs on Elastic Foundations

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)}$$

Slab Stiffness

Slabs on Elastic Foundations

$$EI\frac{d^4w}{dx^4} + k_o w = q_o$$ Beams

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + kw = q$$ Slabs

Modulus of Subgrade Reaction

Governing Differential Equation for Slab Bending

Slabs on Elastic Foundations

$$\beta = \sqrt[4]{\frac{k_o}{4EI}}$$

Beams

Radius of Relative Stiffness

$$\ell = \sqrt[4]{\frac{D}{k}} = \sqrt[4]{\frac{Eh^3}{12\left(1-\nu^2\right)k}}$$

Slabs

Solution of the Homogeneous Differential Equation

Stresses Due to Curling

Slab Curling

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$$

Hooke's Law for an Infinite Elastic Plate

Slab Curling

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} = 0$$

$$\sigma_y - \nu\sigma_x = 0$$

$$\sigma_y = \nu\sigma_x$$

If slab is bent about the y axis, $\varepsilon_y = 0$

Slab Curling

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\nu \sigma_x}{E} = \frac{\sigma_x}{E}\left(1 - \nu^2\right)$$

$$\sigma_x = \frac{E\varepsilon_x}{1 - \nu^2}$$

Slab bent about y axis

Slab Curling

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} = 0$$

$$\sigma_x - \nu\sigma_y = 0$$

$$\sigma_x = \nu\sigma_y$$

If slab is bent about the x axis, $\varepsilon_x = 0$

Slab Curling

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\nu \sigma_y}{E} = \frac{\sigma_y}{E}\left(1 - \nu^2\right)$$

$$\sigma_y = \frac{E\varepsilon_y}{1-\nu^2}$$

Slab bent about x axis

Slab Curling

Coefficient of
Thermal Expansion

$$\varepsilon_x = \varepsilon_y = \frac{\alpha \Delta t}{2}$$

Strains due to temperature difference Δt through slab depth

Slab Curling

$$\sigma_x = \frac{E\varepsilon_x}{1-\nu^2} = \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

$$\sigma_y = \nu\sigma_x = \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Stresses due to curling about y axis due to temperature difference

Slab Curling

$$\sigma_y = \frac{E\varepsilon_y}{1-\nu^2} = \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

$$\sigma_x = \nu\sigma_y = \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Stresses due to curling about x axis due to temperature difference

Slab Warping

$$\sigma = \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)} + \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Superposition of Stresses

Slab Warping

$$\sigma = C_1 \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)} + C_2 \frac{\nu E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Correction for Finite Slab

Slab Warping

$$\sigma_y = \nu\sigma_x = 0 \quad \Rightarrow \quad \nu = 0$$

$$\sigma = C_1 \frac{E\alpha\Delta t}{2\left(1-\nu^2\right)}$$

Stresses at Edge of Finite Slab