Traffic Stream Model Relationship

Sabya Mishra

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Three basic flow characteristics are Flow (q), Speed (u), and Density (k). Let us first define them

Flow (q): defined as the number of vehicles passing a specific point or short section in a given period of time in a single lane (Unit: vehicles/hour/lane)

Speed (u): defined as the ratio of distance traversed per unit distance (Unit: miles/hour)

Density (k): defined as the number of vehicles occupying a section of roadway in a single lane (Unit: vehicles/mile/lane)

There are some unique parameters we need to define; essentially they are the boundary conditions.

q_m: Maximum flow or capacity

u_f : Free flow speed (speed which exists when flow approaches zero)

u_0: Optimum speed (speed which exists under maximum flow conditions)

k_j: Jam density (density when both flow and speed approaches zero)

k_0: Optimum density (density when flow is maximum)

Let us consider the speed-density relationship first in Figure 1.

Consider a linear speed-density relationship. The linear relationship can be interpreted as following

$$u = u_f - \left(\frac{u_f}{k_j}\right) k \tag{1}$$

Speed approaches free flow speed when $k \to 0$and $q \to 0$

Flow approaches maximum flow when $u \to u_0$and $k \to k_0$

Density approaches jam density when $u \to 0$and $q \to 0$

From the units flow-density-speed quation can be writen as

$$q = uk \tag{2}$$

Flow density equation can be written as

$$q = u_f k - \left(\frac{u_f}{k_j}\right) k^2 \tag{3}$$

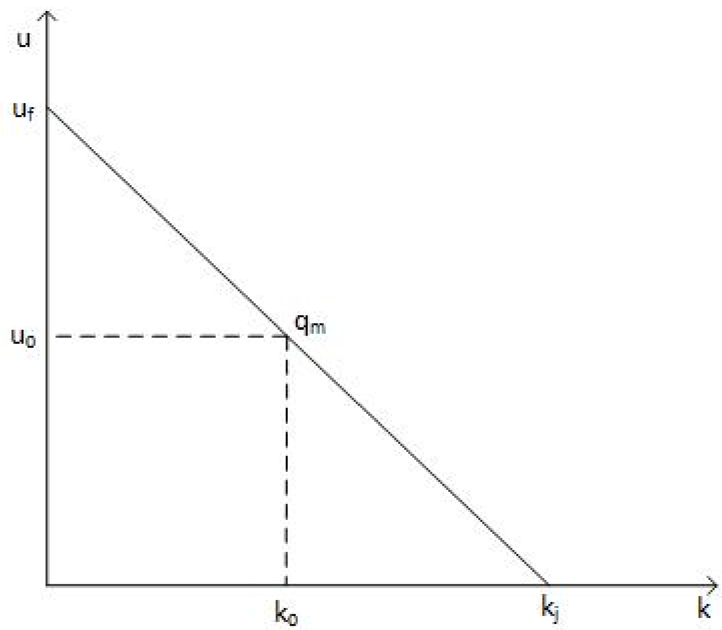

Figure 1: Speed Density Relationship

At $q = q_m$,$k = k_0, (\partial q/\partial k) = 0$,
taking derivative of equation (3), with respect to k
$0 = u_f - \left(\frac{u_f}{k_j}\right) 2k_0$
Solving the above produces

$$k_0 = \frac{k_j}{2} \tag{4}$$

Equation (4) suggestd that optimum speed is half of the jam density. Figure 2 shows the flow density relationship. The form of equation 3 provides the indication of parabolic shape. Similarly relationship between speed and flow can be determined and the relationship is demonstrated in Figure 3.

Now considering relationship between optimum and free flow speed, equation (3) can be rewritten as the following

$$q_m = u_f k_0 - \left(\frac{u_f}{k_j}\right) k_0^2 \tag{5}$$

In equation (5) replace, $q_m = u_0 k_0$

$$u_0 k_0 = u_f k_0 - \left(\frac{u_f}{k_j}\right) k_0^2 \tag{6}$$

Solving equation (6) we get

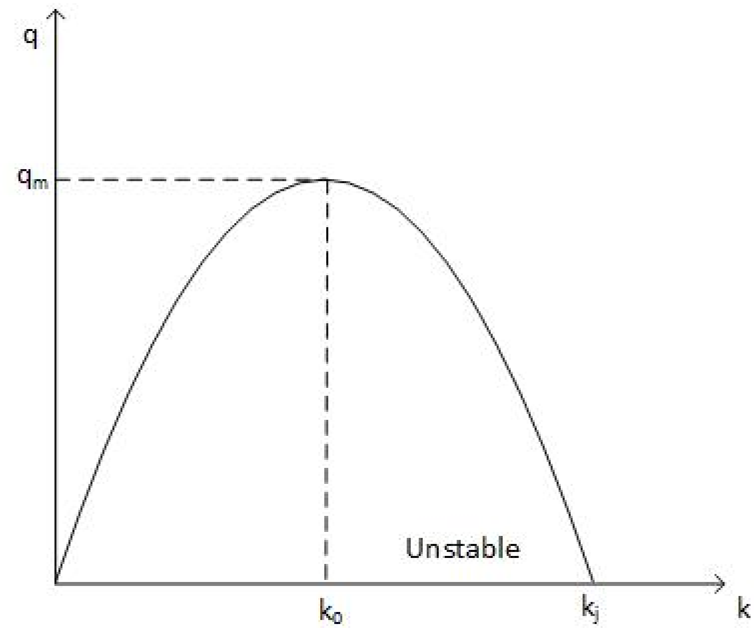

Figure 2: Flow Density Relationship

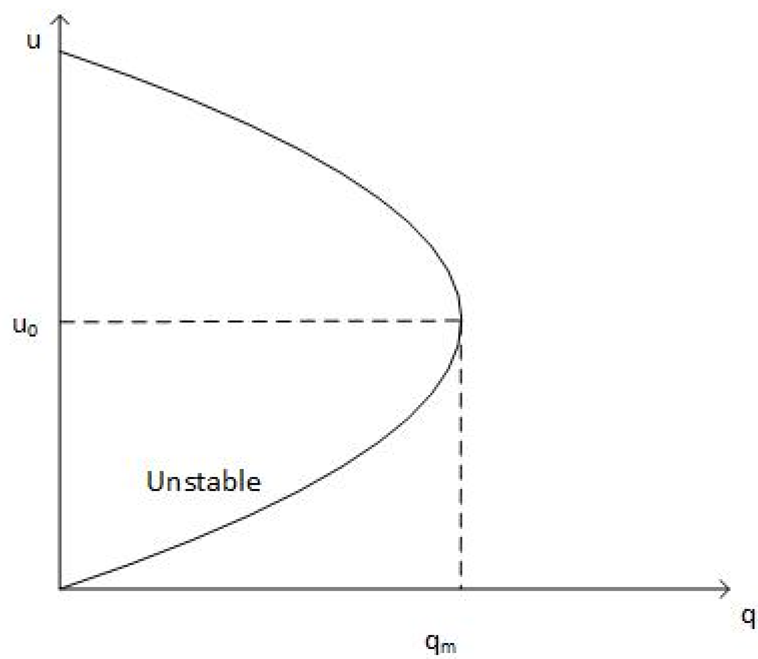

Figure 3: Speed Flow Relationship

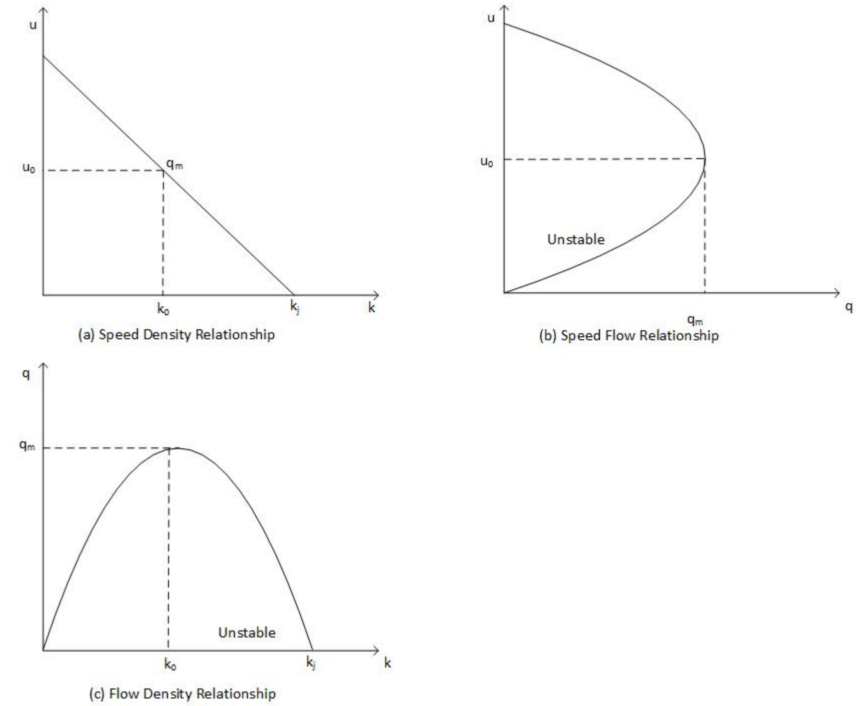

Figure 4: Speed Flow and Density Relationship

$$u_0 = u_f - \left(\frac{u_f}{k_j}\right) k_0 \tag{7}$$

We know from equationIn equation replace $k_0 = \frac{k_j}{2}$ in equation (7), and solving we will get

$$u_0 = \frac{u_f}{2} \tag{8}$$

By combining equation (7), and (8),

$$q_0 = k_0 u_0 = \frac{u_f k_j}{4} \tag{9}$$

All the above forumation of traffic stream models are derived using the assumption that the relationship between speed and density is linear. These equations may not hold true if this assumption is violated. All three plots and their respective relationship is shown in Figure 4