4

Microscopic Speed Characteristics

Microscopic speed characteristics are those speed characteristics of *individual* vehicles passing a point or short segment during a specified period of time. Speeds and travel times over *longer* sections of roadways and statistical analysis between *groups* of vehicles will be considered as macroscopic speed characteristics and discussed in Chapter 5.

Speed is a fundamental measurement of the traffic performance on the highway system. Most analytical and simulation models of traffic predict speed as the measure of performance given the design, demand, and control on the highway system. More extensive models will then use speed as an input for the estimation of fuel consumption, vehicle emissions, and traffic noise. Speed is also used as an indication of level of service, in accident analysis, and in economic studies. Therefore, the traffic analyst must be familiar with speed characteristics and associated statistical analysis techniques.

This chapter contains five sections plus selected problems and references. The first section is devoted to speed trajectories of individual vehicles. The next section presents speed characteristics under uninterrupted flow conditions. The third and fourth sections are concerned with mathematical distributions and their evaluation. The final section describes procedures for estimating population means and sample size requirements.

4.1 VEHICULAR SPEED TRAJECTORIES

This section is about the trajectories of individual vehicles over space and time as influenced by interrupted flow and highway grade situations. Interrupted flow situations include sign- and signal-controlled intersections as well as railroad and pedestrian crossings. Highway grade situations will include the interactions between various types of vehicles and the length and steepness of grades.

First, equations are provided for determining vehicle speed trajectories over space and time based on specified acceleration and deceleration rates. Maximum and normal acceleration and deceleration rates are presented for various types of vehicles under various grade situations. Then the equations and rates of acceleration and deceleration are applied to several highway traffic situations.

4.1.1 Equations of Motion and Acceleration/Deceleration Rates

Two equations of motion that can be used to calculate distance traveled and elapsed time given the speed and acceleration (or deceleration) rates are*

$$t = \frac{\mu_e - \mu_b}{a} \tag{4.1}$$

$$d = 1.47\mu_b t + 0.733at^2 \tag{4.2}$$

where μ_b = speed (miles per hour) at the beginning of the acceleration (or deceleration) cycle

μ_e = speed (miles per hour) at the end of the acceleration (or deceleration) cycle

a = acceleration (or deceleration) rate (miles per hour/second)

t = time for vehicle to accelerate (or decelerate) at rate a from beginning speed (μ_b) to ending speed (μ_e)(seconds)

d = distance for vehicle to accelerate (or decelerate) at rate a from beginning speed μ_b to ending speed (μ_e) (feet)

As vehicles proceed along a highway, drivers may desire or be required to accelerate and/or decelerate their vehicles because of other vehicles in the traffic stream, interrupted flow situations, or highway design features. Acceleration and deceleration rates vary considerably between drivers, vehicles, traffic situations, roadway situations, and for different speed levels. One of the most comprehensive summaries of previous research and field studies of maximum and normal acceleration and deceleration rates is contained in the *Transportation and Traffic Engineering Handbook* [23] and is a starting point for materials covered in this chapter. The following two paragraphs are devoted to acceleration rates and deceleration rates.

Maximum and normal acceleration rates for various vehicle types, speed changes, and grade situations are summarized in Table 4.1. Maximum acceleration rates decrease with increased weight-to-horse-power ratios, with steeper grades, and with higher running speeds. Normal acceleration rates observed under typical driving conditions are considerably less than maximum acceleration rates. For example, normal acceleration rates for passenger vehicles in level terrain and in nonemergency situations are on the order of one-half to two-thirds of the maximum acceleration rates as shown in Table 4.1. Another important consideration is the acceleration capabilities (or the

*Coefficients are needed in equation (4.2) because speeds and acceleration rates are expressed in terms of miles per hour, while distances and time are expressed in feet and seconds.

TABLE 4.1 Maximum and Normal Acceleration Rates (Miles per Hour/Second)[a]

COMPOSITE PASSENGER VEHICLE

Grade	Speed Change 0–15	0–30	30–40	40–50	50–60
Level	8.0 (3.3)	5.0 (3.3)	4.7 (3.3)	3.8 (2.6)	2.8 (2.0)
+2%	7.8	4.6	4.2	3.4	2.4
+6%	6.7	3.7	3.4	2.5	1.5
+10%	5.8	2.8	2.5	1.6	0.6

PICKUP TRUCKS

Grade	0–15	0–30	30–40	40–50	50–60
Level	8.0	5.0	2.0	1.8	1.5
+2%	7.8	4.6	1.6	1.4	1.0
+6%	6.7	3.7	0.7	0.5	0.2
+10%	5.8	2.8	[30]	—	—

TWO-AXLE, SIX-TIRE TRUCK

Grade	0–15	0–30	30–40	40–50	50–60
Level	2.0	1.0	1.0	0.6	0.2
+2%	1.6	0.6	0.6	0.2	[50]
+6%	0.7	0.1	[30]	—	—
+10%	[14]	—	—	—	—

TRACTOR-SEMITRAILER TRUCK

Grade	0–15	0–30	30–40	40–50	50–60
Level	2.0	1.0	0.8	0.4	0.1
+2%	1.6	0.6	0.3	[45]	—
+6%	0.7	[23]	—	—	—
+10%	[4]	—	—	—	—

[a]Normal acceleration rates are shown in parentheses. Some vehicle types on steeper grades cannot exceed a performance-limiting speed. These speeds, shown in brackets, are called crawl speeds and cannot be exceeded.

Source: Reference 23.

lack thereof) of vehicles with higher weight-to-horsepower ratios on steeper grades. For example, a tractor-semitrailer truck on a sustained 6 percent upgrade will be unable to accelerate once a speed on the order of 23 miles per hour is reached. Thereafter, the speed will remain constant on the upgrade, and this speed is referred to as the crawl speed. Table 4.2, which summarizes acceleration rates, distances traveled, and elapsed times for passenger vehicles on level terrain and under normal operating conditions, has been prepared for later analysis. The distances traveled and the elapsed times were calculated using equations (4.1) and (4.2).

TABLE 4.2 Normal Acceleration Rates with Associated Distances Traveled and Elapsed Times for Passenger Vehicles in Level Terrain[a]

Initial Speed (miles/hr)	Final Speed (miles/hr)				
	15	30	40	50	60
0	3.3 4.5 49	3.3 9.1 200	3.3 12.1 354	3.1 15.9 574	2.9 20.9 929
30	—	—	3.3 3.0 154	2.9 6.8 374	2.5 11.8 729
40	—	—	—	2.6 3.8 220	2.3 8.8 575
50	—	—	—	—	2.0 5.0 355

[a]The first value given in each case is for acceleration rate in miles per hour/second, the second for elapsed time in seconds, and the third for distance traveled in feet.

Source: Reference 23.

Minimum safe stopping distances rather than maximum deceleration rates are considered because of the dependency on coefficient of friction rather than vehicle type and because most design issues are concerned with the distance requirements to stop a vehicle from some specified running speed. The equation for minimum stopping distance in terms of running speed, grade situation, and coefficient of friction between the tires and the pavement is

$$S = \frac{(\mu_b)^2}{30(f \pm g)} \tag{4.3}$$

where S = minimum stopping distance in (feet)
μ_b = running speed at beginning of the deceleration (miles per hour)
f = coefficient of friction between tires and pavement
g = grade situation expressed as a decimal (i.e., a 3 percent upgrade is expressed as +0.03)

A table of minimum stopping distances to bring a vehicle to a complete stop as a function of initial running speed, grade situation, and coefficient of friction is presented as Table 4.3. The three subtables are for various tire–pavement situations: dry pavement–good tires, dry pavement–poor tires, and wet pavement. In each table, minimum stopping distances are given for selected running speeds and for three grade situations: 6 percent downgrade, level, and 6 percent upgrade. Note that coefficient of

TABLE 4.3 Minimum Stopping Distances (feet)

DRY PAVEMENT, GOOD TIRES

Grade	Running Speed (miles/hr)						
	20	30	40	50	60	70	80
f Value →	0.75	0.75	0.75				
−6%	19	43	77	—	—	—	—
Level	18	40	71	—	—	—	—
+6%	16	37	66	—	—	—	—

DRY PAVEMENT, POOR TIRES

f Value →	0.58	0.53	0.43				
−6%	26	64	148	—	—	—	—
Level	23	57	127	—	—	—	—
+6%	21	51	111	—	—	—	—

WET PAVEMENT

f Value →	0.40	0.36	0.33	0.31	0.30	0.29	0.27
−6%	39	100	198	333	500	710	1016
Level	33	83	162	269	400	563	790
+6%	29	71	137	225	333	467	646

Source: Reference 29.

friction values are not only a function of the tire–pavement situation but also depend on running speeds (i.e., lower coefficient of friction values are encountered at higher speeds). Also note that the size and type of vehicle are not considered directly. Table 4.4 has been prepared for later analysis which summarizes deceleration rates, distances traveled, and elapsed times for passenger vehicles in level terrain and under normal operating conditions. The distances traveled and the elapsed times were calculated using equations (4.1) and (4.2).

4.1.2 Highway Traffic Applications

Three applications will be made to demonstrate how acceleration and deceleration performance of vehicles can be used to develop vehicle speed trajectories for travel through an intersection, during a passing maneuver, and for trucks on grades. Normal acceleration and deceleration performance will be used in these examples and are based on data provided in Tables 4.2 and 4.4. Special care should be exercised in selecting appropriate acceleration and deceleration performance in such types of analysis.

TABLE 4.4 Normal Deceleration Rates with Associated Distances Traveled and Elapsed Times for Passenger Vehicles in Level Terrain[a]

Initial Speed (miles/hr)	Final Speed (miles/hr)				
	0	30	40	50	60
15	5.3 2.8 30	—	—	—	—
30	4.6 6.5 143	—	—	—	—
40	4.2 9.5 297	3.3 3.0 154	—	—	—
50	4.0 12.5 495	3.3 6.0 352	3.3 3.0 198	—	—
60	3.9 15.5 737	3.3 9.0 594	3.3 6.0 440	3.3 3.0 242	—
70	3.8 18.5 1023	3.3 12.0 880	3.3 9.0 726	3.3 6.0 528	3.3 3.0 286

[a]The first value given in each case is for the deceleration rate in miles per hour per second, the second for elapsed time in seconds, and the third for distance traveled in feet.

Source: Reference 23.

A graphical illustration of vehicle trajectories in the vicinity of a signalized intersection in a rural area is shown in Figure 4.1. In each case shown, the running speed some distance upstream and downstream of the signal is assumed to be 50 miles per hour. The speeds at the stop line at the intersection are assumed to be 50, 30, and 0 miles per hour. The vehicle trajectories during constant running speed are shown as heavy lines while the vehicle trajectories during deceleration and acceleration are shown as lighter lines. Note that the lost time due to slowing to 30 miles per hour is only a few seconds, whereas a required stop (even without stopped time) is considerably more.

A graph of vehicle trajectories during a passing maneuver is shown in Figure 4.2. In this example a vehicle traveling 60 miles per hour approaches a vehicle ahead in the same lane which is traveling at a constant 40 miles per hour. The higher-speed vehicle decelerates, maintains a distance headway of 100 feet for 4 seconds, and then passes while accelerating back to 60 miles per hour. Note that the passing vehicle encroaches

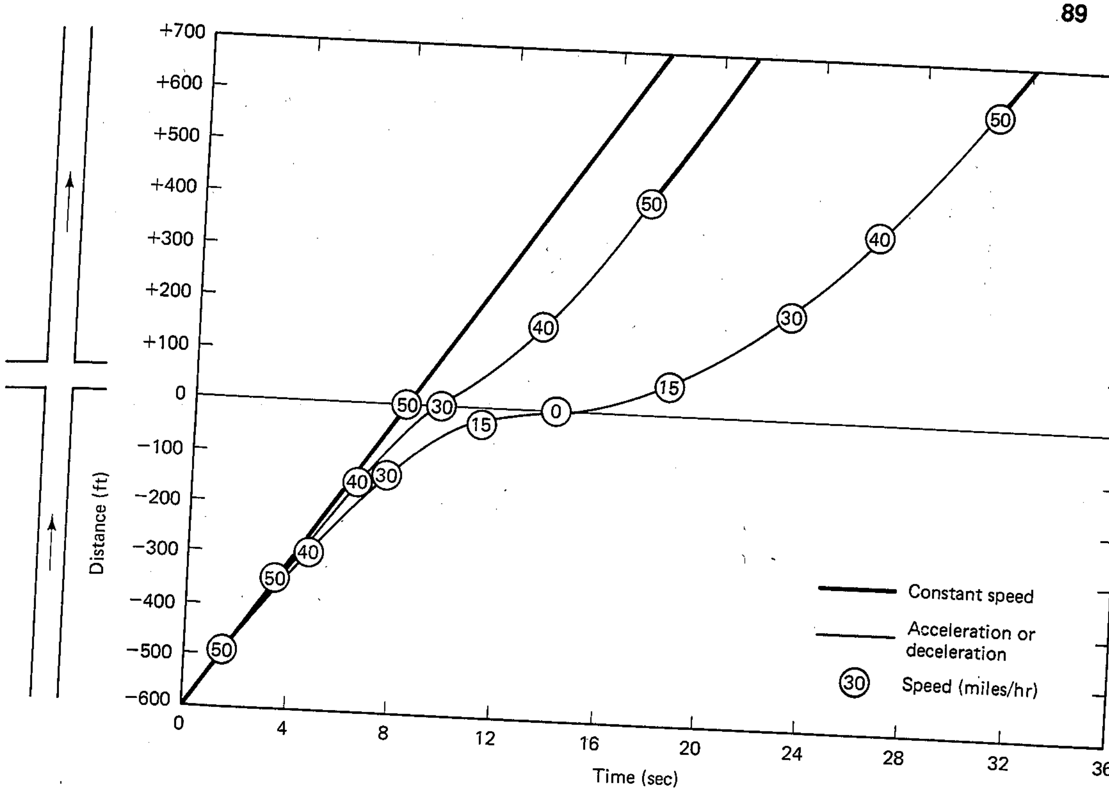


Figure 4.1 Typical Vehicle Trajectories in the Vicinity of a Signalized Intersection

in the adjacent lane for about 900 feet and 11 seconds. The vehicle trajectory of the second vehicle during encroachment is shown as a heavy line in the figure. The vehicle trajectory of the following vehicle is shown as a dashed line if passing was not required and a constant 60-mile per hour running speed was maintained.

The final example is concerned with the trajectory of trucks on grades. In this example maximum rather than normal acceleration and deceleration (due to limited vehicle performance not to braking) are considered. The performance of trucks are grouped on the basis of weight-to-horsepower ratios, and 100, 200, and 300 pounds per horsepower are used to represent light, typical, and heavy trucks, respectively. Special nomographs are prepared to aid in the analysis of truck projectories on grades. An example nomograph contained in the 1985 *Highway Capacity Manual* [20] for typical trucks [200 pounds per horsepower (lb/hp)] is reproduced here as Figure 4.3. Two situations are presented in this figure: acceleration on upgrades and downgrades, and deceleration on upgrades. In the acceleration situation, the initial speed is assumed to be zero and the vehicle accelerates to various speeds as a function of the length and steepness of grade. The dashed lines represent the acceleration performance of trucks on grades varying from +8 percent upgrade to -5 percent downgrade. Note the leveling off of speed with length of grade. For example, on a 6 percent upgrade after about

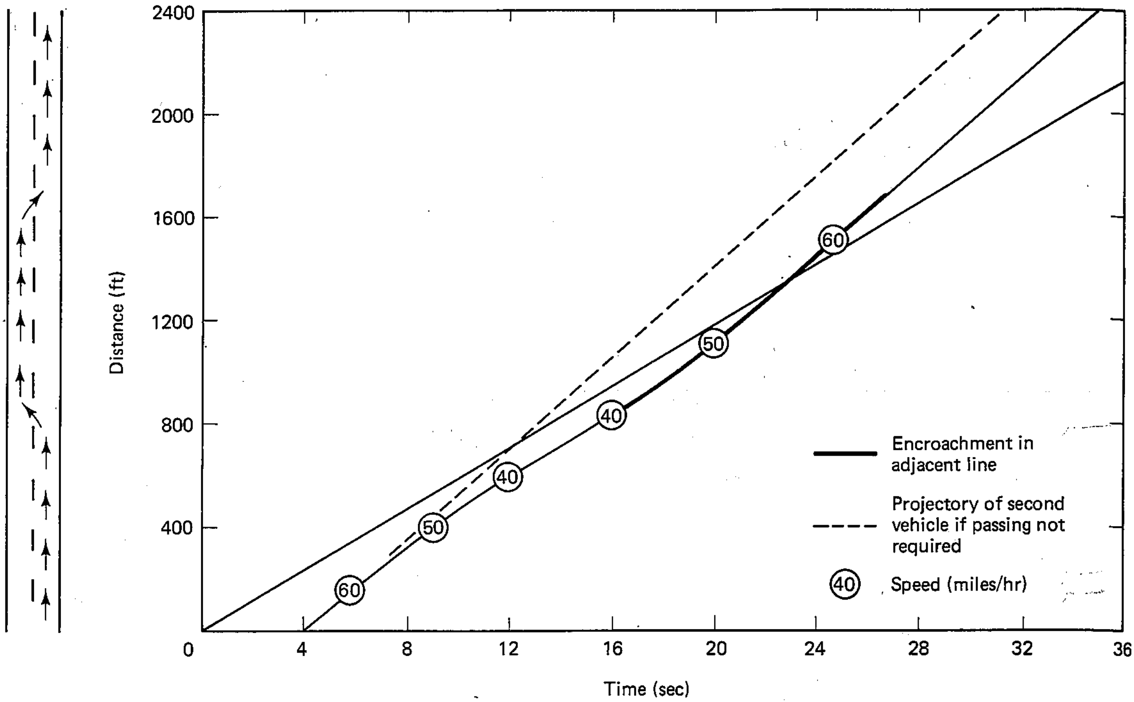


Figure 4.2 Vehicle Trajectories during a Passing Maneuver

3000 feet, the truck speed remains constant at about 23 miles per hour. This is because this particular type of truck cannot accelerate further once this speed is reached, and this speed is referred to as the crawl speed. In fact, in all cases there will be a leveling off of speed with length of grade because of performance limitations (upgrades) or because of drivers not using the full performance of the vehicle as the desired running speeds are approached (downgrades). In the deceleration situation, the initial speed is assumed to be 55 miles per hour, and the vehicle decelerates to various speeds as a function of length of grade and steepness of upgrade. The solid lines represent the deceleration performance for grades varying from +1 percent upgrade to +8 percent upgrade. Again, note the leveling off of speed with length of grade.

The nomograph is specially constructed so that initial speeds other than zero and 55 miles per hour can be considered. For example, if a 200-lb/hp truck approaches a +4 percent upgrade that is 2000 feet long and is followed by a +6 percent upgrade that is 1000 feet long, the anticipated speeds at the end of each individual grade can be predicted. Using the nomograph shown in Figure 4.3, the speed at the end of the 2000-foot-long +4 percent upgrade is found to be 39 miles per hour. Projecting this point horizontally to the left from the +4 percent to the +6 percent upgrade curve, and then moving along the +6 percent upgrade curve down and to the right for an additional distance of 1000 feet, the speed at the end of the second grade is found to be 26 miles per hour. Note that speeds above 55 miles per hour are not considered, and vehicle

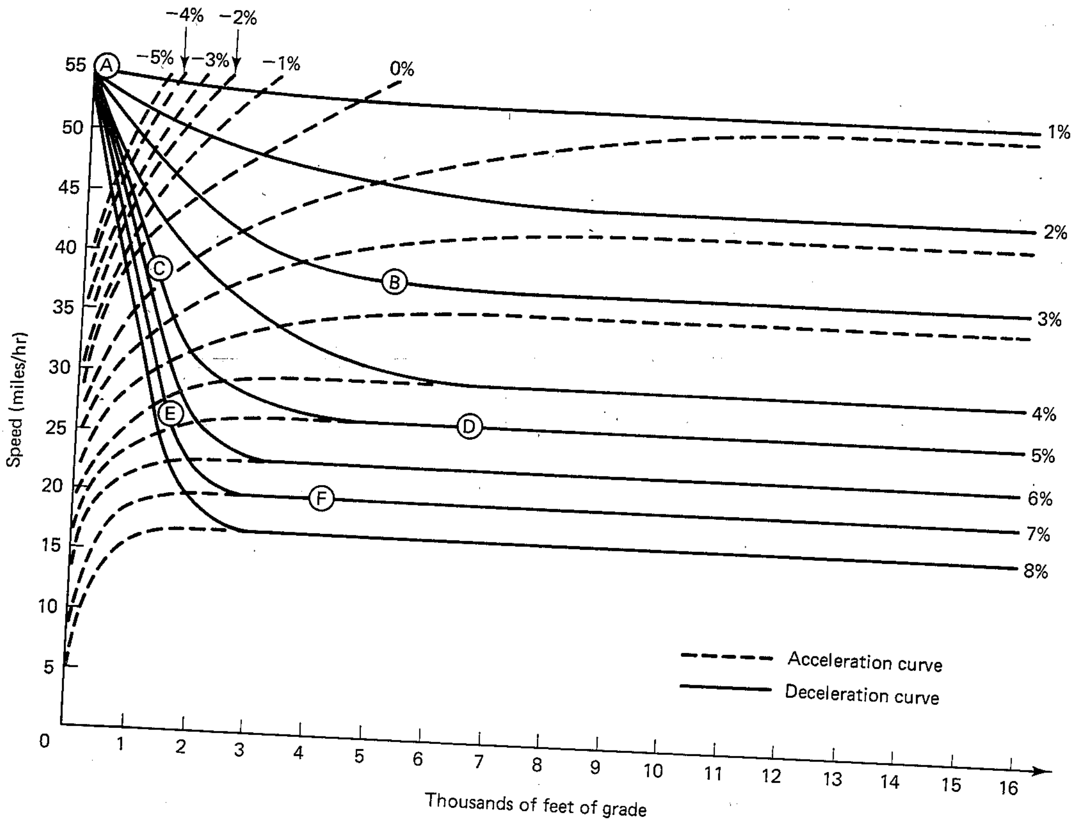


Figure 4.3 Performance Curves for a Typical Truck (200 lb/hp) (From Reference 20)

trajectories when speeds approach 55 miles per hour require some adjustments if the driver does not use the maximum acceleration performance of the truck.

For illustrative purposes, consider the speed trajectory of a 200-lb/hp truck along a highway that has a profile as shown in the top portion of Figure 4.4. The truck speeds at various points along the upgrades and downgrades are shown in the lower portion of Figure 4.4, and the results for the 200-lb/hp truck were obtained using the performance curves shown in Figure 4.3. The speed of the truck at the beginning of subsection 1 is assumed to be 55 miles per hour, and since the grade in subsection 1 is 0 percent, the truck speed of 55 miles per hour is maintained for the first 3 miles. The truck speed at the end of subsection 2, which is 1 mile long and has a grade of +3 percent, is reduced to 38 miles per hour (point B on Figure 4.3). The 1 mile, +5 percent grade reduced the truck speed further, to 26 miles per hour (point D on Figure 4.3), and the 1/2 mile, +7 percent grade reduced the truck speed even further, to 20 miles per hour (point F on Figure 4.3). At this point the summit is reached and a 1 mile, −3 percent downgrade is encountered. The truck is able to accelerate and has the performance capability to return to 55 miles per hour within approximately 2000 feet. However, in practice as the

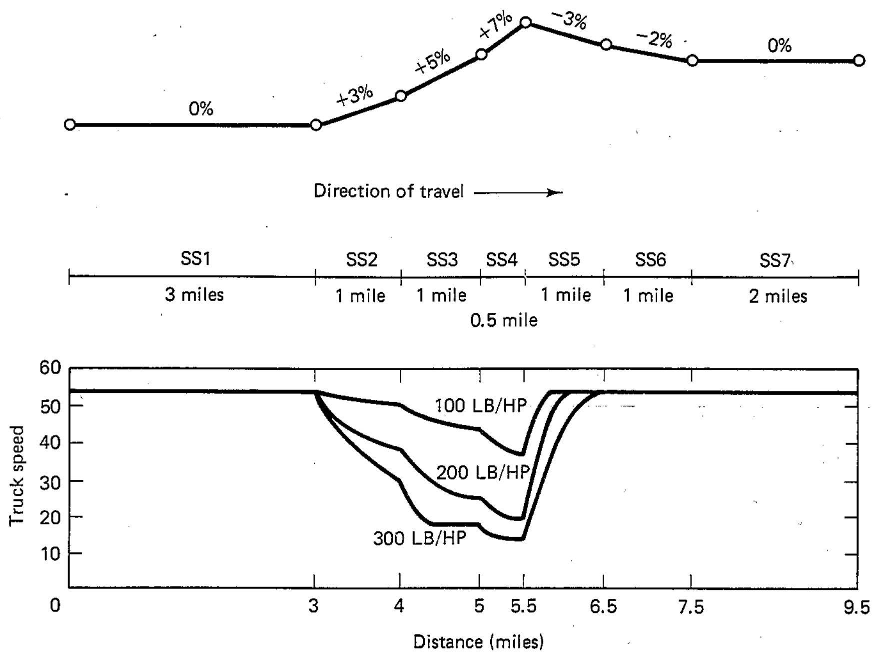


Figure 4.4 Truck Speeds as Affected by Grades

driver approaches his desired speed, the maximum acceleration is not maintained and so the truck will more likely reach a 55 mile-per hour running speed farther downstream as shown in Figure 4.4. For comparison purposes, truck speed profiles are also shown for light trucks (100 lb/hp) and heavy trucks (300 lb/hp) in the same example.

4.2 SPEED CHARACTERISTICS UNDER UNINTERRUPTED FLOW CONDITIONS

Consider standing at a point along a highway facility during a relatively short period of time under uninterrupted flow conditions: that is, a location away from intersections which has little or no roadside development. The speeds of individual vehicles are measured and recorded. This would result in a series of individual vehicular speeds such as 50, 46, 48, 55, 48, and so on, miles per hour. The sample mean and sample variance of these "ungrouped" speed observations would be

$$\bar{\mu} = \frac{\sum_{i=1}^{N} \mu_i}{N} \tag{4.4}$$

$$s^2 = \frac{\sum_{i=1}^{N} (\mu_i - \bar{\mu})^2}{N - 1} \tag{4.5}$$

$$s = \sqrt{s^2} \tag{4.6}$$

where $\bar{\mu}$ = sample mean speed (miles per hour)
μ_i = speed of vehicle i
N = total number of speed observations
s^2 = sample variance
s = sample standard deviation

In most cases the speed observations are "grouped." The frequencies of each speed level or speed interval are determined from the series of individual vehicular speeds observed. An example of grouped speed data is shown in the first two columns of Table 4.5 based on 200 individual speed observations. The cumulative frequency and percentile are shown in the next two columns and will be utilized a little later. The last two columns are calculations required in determining the sample mean and sample variance using the following equations:

$$\bar{\mu} = \frac{\sum_{i=1}^{g}\left(f_i\mu_i\right)}{N} \tag{4.7}$$

$$s^2 = \frac{\sum_{i=1}^{g} f_i\left(\mu_i\right)^2 - \frac{1}{N}\left(\sum_{i=1}^{g} f_i\mu_i\right)^2}{N-1} \tag{4.8}$$

$$s = \sqrt{s^2} \tag{4.9}$$

where $\bar{\mu}$ = mean speed of sample
g = number of speed groups
i = speed group i
f_i = number of observations in speed group i
μ_i = midpoint speed of group i
N = total number of speed observations
s^2 = sample variance
s = sample standard deviation

Note that in calculating the variance the term $N-1$ rather than N is used in the demonstration because of dependency between the last $f_i\mu_i$ value and $\bar{\mu}$. However, for moderate sample sizes, using N or $N-1$ will have little effect on the numerical results. For the grouped speed data shown in Table 4.5, the sample mean and the sample variance are calculated as follows:

$$\bar{\mu} = \frac{\sum_{i=1}^{g}(f_i\mu_i)}{N} = \frac{10{,}460}{200} = 52.3 \text{ miles/hour} \tag{4.10}$$

$$s^2 = \frac{\sum_{i=1}^{g} f_i\left(\mu_i\right)^2 - 1/N\left(\sum_{i=1}^{g} f_i\mu_i\right)^2}{N-1} \tag{4.11}$$

$$= \frac{554{,}882 - 1/200(10{,}460)^2}{199}$$

$$= 39.3 \text{ (miles/hour)}^2$$

$$s = \sqrt{s^2} = 6.3 \text{ miles/hour} \quad (4.12)$$

TABLE 4.5 Grouped Speed Data

u_i	f_i	Cumulative Frequency	Cumulative %	$f_i\mu_i$	$f_i(\mu_i)^2$
30	—	0	0	—	—
31	—	0	0	—	—
32	0	0	0	—	—
33	1	1	1	33	1,089
34	2	3	2	68	2,312
35	1	4	2	35	1,225
36	1	5	2	36	1,296
37	—	5	2	—	—
38	1	6	3	38	1,444
39	1	7	4	39	1,521
40	2	9	4	80	3,200
41	1	10	5	41	1,681
42	5	15	8	210	8,820
43	4	19	10	172	7,396
44	1	20	10	44	1,936
45	7	27	14	315	14,175
46	4	31	16	184	8,464
47	8	39	20	376	17,672
48	8	47	24	384	18,432
49	15	62	31	735	36,015
50	8	70	35	400	20,000
51	8	78	39	408	20,808
52	10	88	44	520	27,040
53	23	111	56	1,219	64,607
54	15	126	63	810	43,740
55	16	142	71	880	48,400
56	9	151	76	504	28,224
57	14	165	82	798	45,486
58	6	171	86	348	20,184
59	3	174	87	177	10,443
60	9	183	92	540	32,400
61	3	186	93	183	11,163
62	6	192	96	372	23,064
63	3	195	98	189	11,907
64	3	198	99	192	12,288
65	2	200	100	130	8,450
66	—	200	100	—	—
67	—	200	100	—	—
68	—	200	100	—	—
69	—	200	100	—	—
70	—	200	100	—	—
	200			10,460	554,882

Two types of distributions are often plotted in regard to measured speed distributions as shown in Figure 4.5. Figure 4.5a is a straight frequency distribution in which the vertical scale represents the number of observations in each speed group and the horizontal scale is speed in miles per hour. Figure 4.5b is a cumulative percentile distribution in which the vertical scale represents the percent of vehicles traveling at or less than the indicated speed group and the horizontal scale is speed in miles per hour. These distributions are helpful in showing important measures of central tendencies and dispersion as well as qualitatively observing the shape of the speed distribution.

The sample mean was calculated in equation (4.10) to be 52.3 miles per hour. The sample mode can be observed in Figure 4.5a to be 53 miles per hour. The sample median can be observed in Figure 4.5b to be 52.5 miles per hour. Hence the mean, mode, and median of the sample are the three measures of central tendency of the sample and are 52.3, 53.0, and 52.5 miles per hour, respectively.

The standard deviation of the sample was calculated in equation (4.12) to be 6.3 miles per hour. Figure 4.5 can be used to determine the total range (0 to 100 percent) and is found to be 65 miles per hour minus 33 miles per hour or 32 miles per hour. Figure 4.5b can be used to determine the 15 to 85 percentile range and is found to be 57.8 miles per hour minus 45.5 miles per hour, or 12.3 miles per hour. Another measure of dispersion used uniquely in speed distributions is the 10-mile per hour pace. This measure is defined as the highest percentile of vehicles in a 10-mile per hour range. Inspection of Table 4.5 and Figure 4.5 indicates that the 10-mile per hour pace is 47 to 57 or 48 to 58 miles per hour, each of which includes 62 percent of all vehicles in the sample. In summary, the four measures of dispersion: standard deviation, total range, 15 to 85 percentile range, and 10-mile per hour pace were found to be 6.3 miles per hour, 32 miles per hour, 12.3 miles per hour, and 62 percent (47 to 57 or 48 to 58 miles per hour), respectively.

The distributions graphically shown in Figure 4.5, and the calculated measures of central tendency and dispersion provide insights into the type of mathematical distribution that may represent the speed distribution. Inspection of the top portion of Figure 4.5a indicates a fairly symmetrical bell-shaped distribution while the corresponding cumulative distribution in the bottom portion indicates an S-shaped distribution. The three calculated measures of central tendency are all almost numerically equal. The calculated ranges and the pace which are measures of dispersion are nearly symmetrical about the mean. These observations would lead the analyst to consider strongly the possibility of a normal distribution. As measured distributions became nonsymmetrical or bimodal, other distributions, such as the log-normal and composite distributions, might be considered. These three distributions are discussed in the following section.

4.3 MATHEMATICAL DISTRIBUTIONS

Mathematical distributions most commonly employed to represent measured speed distributions are normal distributions, log-normal distributions, and composite distributions. Initial and primary attention is given to normal distributions and near the end of this section log-normal and composite distributions are described briefly.

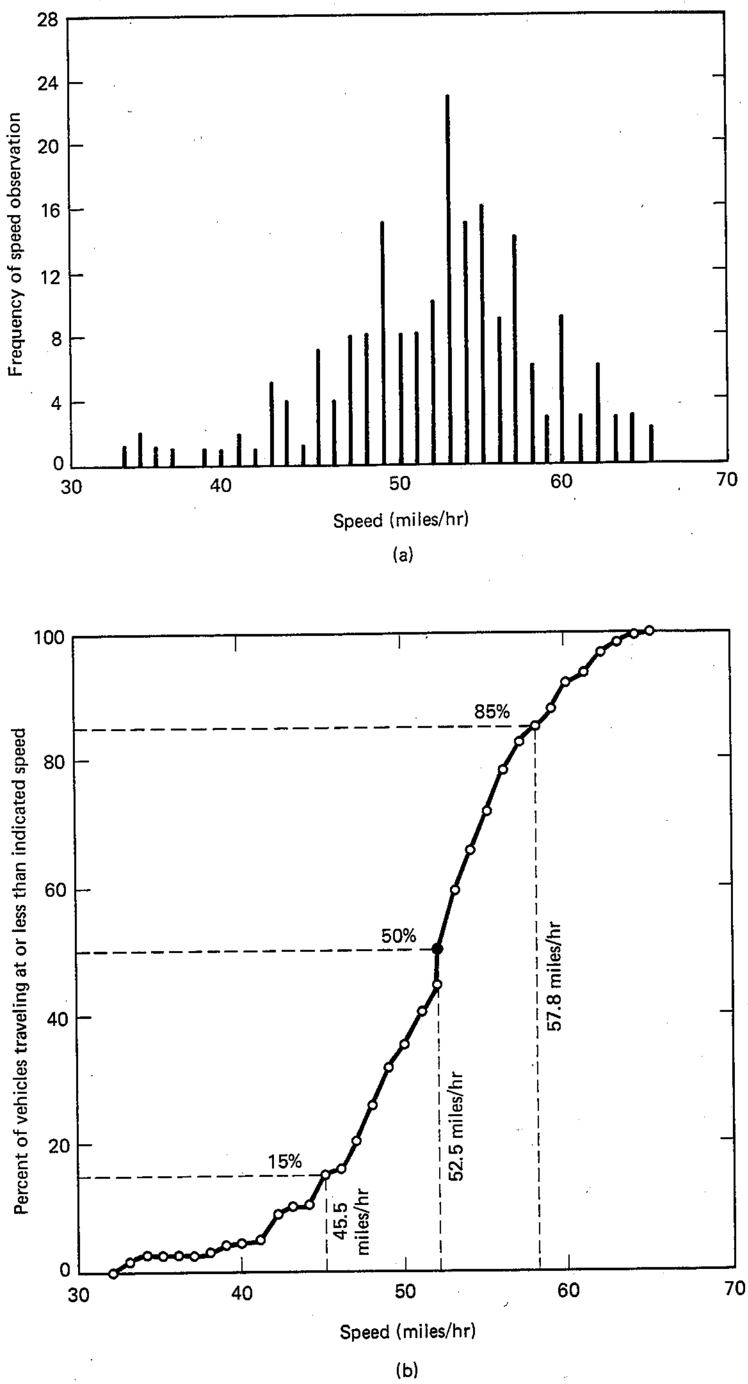


Figure 4.5 Measured Speed Distribution Example

It is desirable to find appropriate mathematical distributions to represent speed distributions for two major reasons. First, each mathematical distribution has unique attributes, and if a measured distribution can be represented by a mathematical distribution, the measured distributions can be said to have similar attributes and hence greater knowledge of the measured distribution can be inferred. It is also desirable to find appropriate distributions for purposes of computer simulation for which individual vehicle speeds are needed as input. Although measured speed distributions can be used within such models, it is easier and more flexible to use mathematical distributions.

A unique normal distribution is defined when the mean and standard deviation are specified. Thé normal distribution is symmetrical about the mean and the dispersion or spread is a function of the standard deviation. For example, three unique normal distributions are shown in Figure 4.6. In comparing normal distribution 1 with normal distribution 2, $s_1 = s_2$ but $\mu_1 < \mu_2$. In comparing normal distributions 2 and 3, $\mu_2 = \mu_3$ but $s_2 < s_3$.

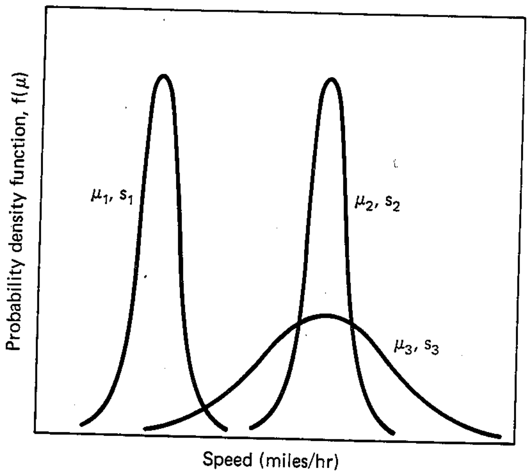


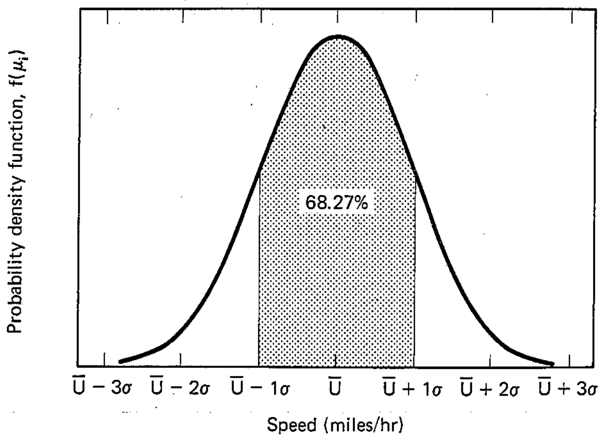
Figure 4.6 Examples of Normal Distributions

An additional attribute of the normal distribution is that the mean, mode, and median (the three measures of central tendency) are numerically equal. The dispersion is such that when the standard deviation is specified, 68.27 percent of the observations will be within 1 standard deviation of the mean, 95.45 percent within 2 standard deviations of the mean, and 99.73 percent within 3 standard deviations of the mean. These attributes are shown graphically in Figure 4.7.

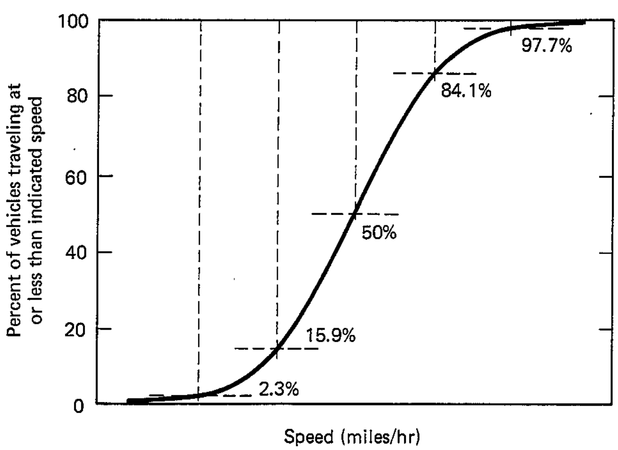
The probability density function of the normal distribution is shown in Figure 4.7a and the general equation is

$$f(\mu_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\mu_i-\bar{U})^2/2\sigma^2} \tag{4.13}$$

where $f(\mu_i)$ = probability density function of individual speeds
π = a constant, 3.1416
e = a constant Napierian base of logarithms ($e = 2.71828$)



(a)



(b)

Figure 4.7 Attributes of Normal Distributions

μ_i = speed value being investigated

$\overline{U}$ = population mean speed (miles per hour)

σ = population standard deviation

σ^2 = population variance

The total area under the probability density curve includes all possible outcomes and therefore is equal to unity (or 100 percent). The area under the curve between a speed value of, say, $\overline{U} - 1\sigma$ and $\overline{U} + 1\sigma$ (the shaded area in of Figure 4.7a) represents the probability of a speed between these two speed values, and in this case has a probability of 0.6827 (68.27 percent). In a similar way the area between $\overline{U} - 2\sigma$ and $\overline{U} + 2\sigma$ represents a probability of 0.9545 (95.45 percent) and the area between $\overline{U} - 3\sigma$ and

$\overline{U} + 3\sigma$ represents a probability of 0.9973 (99.73 percent). Note that one could ask What is the probability of an individual speed over $\overline{U} + 1\sigma$? The total unshaded area represents a probability of 0.3173 (1.0000-0.6827), the normal distribution is symmetrical and therefore the unshaded areas representing the probability of $\mu_i > \overline{U} + 1\sigma$ is 0.3173/2, or 0.1586. That is, if individual speeds are normally distributed, 15 to 16 vehicles of every 100 vehicles would be expected to travel at speeds greater than $\overline{U} + 1\sigma$. The cumulative form of the normal distribution is shown in Figure 4.7b. Note the S-shaped distribution and the indicated percentages of vehicles traveling at or less than indicated speeds.

Two issues must now be addressed in order to apply the normal distribution to a measured speed distribution. Equation (4.13) specifies that population mean ($\overline{U}$) and standard deviation of the population (σ) are required for the normal distribution. On the other hand, in most speed studies only the sample mean $\bar{\mu}$ and the standard deviation of the sample (s) are known. The sample mean is the best single estimate of the population mean and is used in the normal distribution. Estimating the standard deviation of the population from the standard deviation of the sample is more complex. The relationship between the two is a function of the sample size, N, and as N increases the difference between the two standard deviations becomes smaller and smaller. As N goes to infinity, the two standard deviations are identical. As a practical matter, if $N > 30$, the standard deviation of the sample is numerically substituted for the standard deviation of the population. For sample sizes less than 30, the t-distribution rather than the normal distribution is used.

The other issue involves the calculation procedures for the normal distribution. Inspection of equation (4.13) indicates that calculating the probability density function is rather tedious, and observing Figure 4.7a indicates that converting the probability density function calculations into probabilities for various speed ranges is very cumbersome. One solution is to "normalize" the normal distribution and integrate the probability density function between the mean speed value and other speed values. The resulting probabilities can be placed in a tabular form such as shown in Table 4.6. Then the analyst can calculate the X/σ values and determine the probabilities from the table. The probabilities can then be determined as a function of X/σ.

$$P = f\left(\frac{X}{\sigma}\right) \tag{4.14}$$

where P = probability of an observation between some speed, μ_X, and the mean speed, $\overline{U}$

X = speed range or deviation between μ_x and $\overline{U}$ ($X = \mu_x - \overline{U}$) (absolute value)

σ = standard deviation of the population (s is best estimate for large samples)

As an illustration, consider the example shown in Table 4.5. The sample mean is 52.3 miles per hour and the standard deviation of the speed sample is 6.3 miles per hour. The best estimate of the population mean is 52.3 miles per hour and since $N > 30$ (actually, $N = 200$), the standard deviation of the population is estimated to be 6.3 miles per hour and the normal distribution is used. Pose the question: What is the

TABLE 4.6 Calculating Probabilities from a Normal Distribution

	Second Decimal Place in x/s									
x/s	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2468	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3413	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4031	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998									
4.0	0.49997									
4.5	0.499997									
5.0	0.4999997									

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probability of individual speeds between 35 and 40 miles per hour? Since the mean lies above both 35 and 40 miles per hour, two calculations are required: X between 35.0 and 52.3, and X between 40.0 and 52.3.

$$\left(\frac{X}{\sigma}\right)_{35\to 52.3} = \frac{52.3 - 35.0}{6.3} = 2.75 \quad (4.15)$$

$$\left(\frac{X}{\sigma}\right)_{40\to 52.3} = \frac{52.3 - 40.0}{6.3} = 1.95 \quad (4.16)$$

Entering Table 4.6 with X/σ values of 2.75 and 1.95, the corresponding probability values are 0.4970 and 0.4744. That is, 49.70 percent of the vehicles are traveling at speeds between 35 and 52.3 miles per hour and 47.44 percent are traveling at speeds between 40 and 52.3 miles per hour. Therefore, the probability of a speed between 35 and 40 miles per hour is the difference between these two probabilities and numerically equal to 0.0226. With a sample size of 200, the expected frequency would be 4 or 5. Note in Table 4.5 that the measured frequency was 4.

Two other distributions have been suggested to represent measured speed distributions: log-normal and composite distributions. The log-normal distribution is presented by Gerlough and Huber [1] and the earlier work of Haight and Mosher [2] is identified. It is particularly appropriate for unimodal-shaped distributions like the normal distribution, except the distribution is skewed with a larger tail of the distribution extending to the right. In the next section, graphical techniques are discussed that provide insights as to when the log-normal distribution might be considered. Several statistical references are available for detailed analysis of log-normal distributions [3, 4, 5].

The composite distribution has been proposed when the traffic stream consists of two classes of vehicles (or drivers), and there is little interference between these two classes. An example would be a relatively lightly traveled rural freeway with passenger cars and trucks each having different speed limits. There are several possible combinations of distributions that could be used in a composite distribution, that is, normal or log-normal for one or the other subpopulations. Subpopulations on a per-lane basis might be considered. The traffic analyst may consider some form of a composite distribution when the measured speed distribution is bimodal; that is, two modes some distance apart are clearly identified.

4.4 EVALUATION AND SELECTION OF MATHEMATICAL DISTRIBUTIONS

The suggested procedure is to assume initially that the measured speed distribution can be represented by a normal distribution. Numerical checks are made of the measures of central tendency and dispersion from the measured speed distribution. If the numerical checks appear to support the assumption of a normal distribution, it is suggested that a graphical plot be made of the measured speed distribution on normal probability paper. If the distribution appears on the normal probability paper as a straight line, then the measured speed distribution is evaluated using the chi-square test. If the hypothesis is accepted, then the search for a mathematical distribution is completed and the normal distribution is selected. On the other hand, if there is significant evidence to eliminate the normal distribution, the process is repeated assuming a log-normal and/or a composite distribution.

The procedure described above will be applied to the measured speed distribution shown in Table 4.5 and Figure 4.5. Numerical checks are made of the measures of central tendency. The mean, median, and mode are 52.3, 52.5, and 53.0 miles per hour, respectively, which are in very close agreement. The frequency distribution shown in Figure 4.5a displays a fairly strong single mode. Numerical checks are made of the measures of dispersion: standard deviation, total range, 15 to 85 percentile range, and 10 miles per hour pace are 6.3 miles per hour, 32 miles per hour, 12.3 miles per hour, and 62 percent (47 to 57 or 48 to 58 miles per hour), respectively. Some of the more important checks are shown below.

- The variance of a measured speed distribution normally should be less than the variance of a random distribution ($s_R^2 = m$):

$$s^2 = (6.3)^2 = 39.3 \text{ miles/hour}$$

$$s_R^2 = m = 52.3 \text{ miles/hour}$$

$$s^2 < s_R^2$$

- The standard deviation should be approximately one-sixth of the total range since the mean plus and minus three standard deviations encompasses 99.73 percent of the observations of a normal distribution:

$$s_{\text{EST}} = \frac{\text{total range}}{6} \tag{4.17}$$

$$= \frac{32}{6} = 5.3 \text{ miles/hour}$$

$$s \approx s_{\text{EST}}$$

- The standard deviation should be approximately one-half of the 15 to 85 percent range since the mean plus and minus 1 standard deviation encompasses 68.27 percent of the observation of a normal distribution

$$s_{\text{EST}} = \frac{\text{15–85 percentile range}}{2} \tag{4.18}$$

$$= \frac{12.3}{2} = 6.15$$

$$s \approx s_{\text{EST}}$$

- The 10-mile per hour pace should be approximately straddling the sample mean

$$\bar{\mu} = 52.3 \text{ miles/hour}$$

$$\text{10-mile per hour pace} = 47 \text{ to } 57 \text{ or } 48 \text{ to } 58$$

$$\text{midpoint of pace} = 52 \text{ or } 53$$

$$\text{midpoint of pace} \sim \bar{\mu}$$

- The normal distribution has little skewness and the coefficient of skewness calculated below should be close to zero.

$$\text{coefficient of skewness} = \frac{\text{mean - mode}}{s} \tag{4.19}$$

$$= \frac{52.3 - 53.0}{6.3} = 0.1$$

or

$$\text{coefficient of skewness} = 3\left(\frac{\text{mean - median}}{s}\right) \tag{4.20}$$

$$= 3\left(\frac{52.3 - 52.5}{6.3}\right) = 0.1$$

The numerical checks appear to support the assumption of a normal distribution, so a graphical plot is made of the measured speed distribution on normal probability paper. The graphical plot is shown in Figure 4.8. The solid straight line represents a normal distribution in which the population mean ($\overline{U}$) is 52.3 miles per hour, and the population standard deviation of speed (σ) is 6.3 miles per hour. The plotted data points are taken from the measured speed distribution. There appears to be a very good fit in the 10 to 90 percentile range and as might be expected, a poor fit at the extreme tails of the distribution (less than 2 percent and more than 98 percentile ranges). Note that the slope of the line on the graph represents the standard deviation, with flatter slopes representing larger standard deviation values and steeper slopes representing smaller standard deviation values. The data points reveal that below the mean speed, the distribution is a little more spread out (a larger indicated standard deviation), while above the mean speed, the distribution is a little more dense (a smaller indicated standard deviation). Overall the graphical fit appears reasonable, so the next and last step is to perform a chi-square test.

The chi-square test was described in Section 2.6, so only the calculations and the conclusions will be presented here. Inspection of the observed frequencies in Table 4.5 reveals that the 1-mile per hour class interval is too small considering the number of observations (N = 200). Neiswanger [21] has proposed that there normally should be between 10 and 25 class intervals, depending on the range of the observation values and the number of observations. Sturges [22] proposed that the following equation be used to estimate the size of the class interval:

$$I = \frac{\text{Range}}{1 + (3.322)\log N} \tag{4.21}$$

where I = size of the class interval
Range = total range (largest observed value minus smallest observed value)
N = number of observations

Substituting 32 miles per hour for Range and 200 for number of observations, the size of the class can be determined.

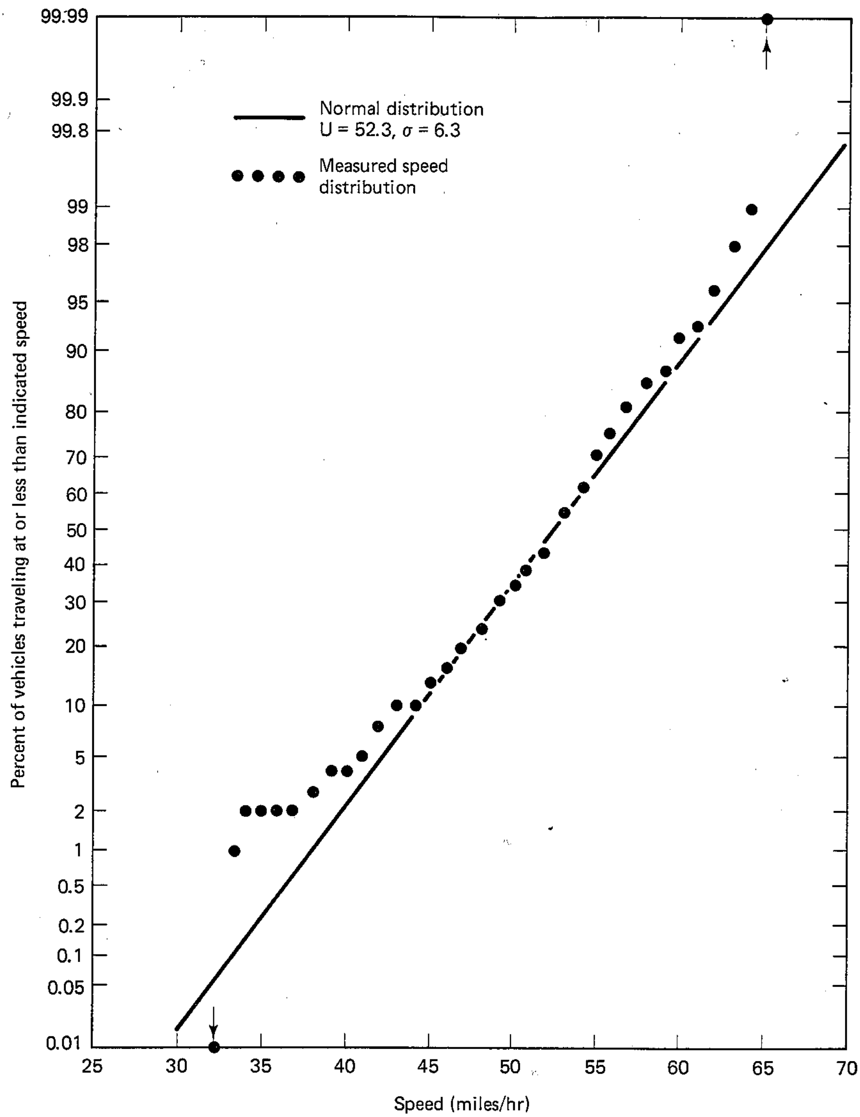

Figure 4.8 Measure Speed Distribution on Normal Probability Paper

$$I = \frac{32}{1 + (3.322)\log 200}$$

$$= \frac{32}{1 + 7.64} = 3.7$$

Considering this estimate of size of the class interval and the desire to have at least 10 class intervals, a class interval of 3 miles per hour is selected. Before performing the chi-square test, it is necessary to calculate the theoretical frequencies for the normal distribution having a mean of 52.3 miles per hour and a standard deviation of 6.3 miles per hour. This assumes that the population mean and standard deviation are numerically equal to the sample mean and standard deviation. The normal distribution calculations are shown in Table 4.7. The class interval limits are selected based on a class interval size of 3 miles per hour. The deviation (X) from each class interval limit to the mean is calculated and shown in the second column. The deviation is "normalized" by dividing it by the standard deviation. The probabilities of occurrence (P) between each class interval limit speed and the mean speed is obtained from Table 4.6 and entered in the fourth column. The theoretical probabilities for each class interval is then calculated. Finally, the theoretical frequency is determined by multiplying the theoretical probabilities by the total frequency ($N = 200$).

TABLE 4.7 Normal Distribution Calculations

Class Interval Limit	X	$\frac{X}{\sigma}$	P	P_t	f_t
				0.0038	0.76
< 35.5	16.8	2.67	0.4962		
				0.0105	2.10
38.5	13.8	2.19	0.4857		
				0.0293	5.86
41.5	10.8	1.71	0.4564		
				0.0639	12.78
44.5	7.8	1.24	0.3925		
				0.1161	23.22
47.5	4.8	0.76	0.2764		
				0.1623	32.46
50.5	1.8	0.29	0.1141		
				0.1894	37.88
53.5	1.2	0.19	0.0753		
				0.1733	34.66
56.5	4.2	0.67	0.2486		
				0.1243	24.86
59.5	7.2	1.14	0.3729		
				0.0745	14.90
> 62.5	10.2	1.62	0.4474		
				0.0526	10.52

Table 4.8 summarizes the chi-square test calculations. The observed frequencies are taken from Table 4.5 and the theoretical frequencies are taken from Table 4.7. The resulting calculated value of chi-square is found to be 4.09. The degrees of freedom is equal to the number of class intervals minus three degrees of freedom. The three degrees of freedom lost are due to the normal distribution assuming the same mean,

TABLE 4.8 Chi-Square Test Calculations

Class Interval	f_o	f_t	$f_o - f_t$	$(f_o - f_t)^2$	$\frac{(f_o - f_t)^2}{f_t}$
< 35.5	4	0.76			
35.5–38.5	2	2.10	+1.28	1.64	0.19
38.5–41.5	4	5.86			
41.5–44.5	10	12.78	−2.78	7.73	0.60
44.5–47.5	19	23.22	−4.22	17.81	0.77
47.5–50.5	31	32.46	−1.46	2.13	0.07
50.5–53.5	41	37.88	+3.12	9.73	0.26
53.5–56.5	40	34.66	+5.34	28.52	0.82
56.5–59.5	23	24.86	−1.86	3.46	0.14
59.5–62.5	18	14.90	+3.10	9.61	0.64
> 62.5	8	10.52	−2.52	6.35	0.60
	200	200			$\chi^2_{CALC} = 4.09$

standard deviation, and frequency of the measured speed distribution. Note that it was necessary to combine the first three class intervals to obtain a minimum theoretical frequency of 5 or more. The resulting degrees of freedom is 6, and selecting an α value of 0.05, the table value of chi-square was found to be 12.59 from Appendix G. Since the calculated value is less than the table value of chi-square, the hypothesis is accepted and the concluding statement would be "There is no evidence of a statistical difference between the two distributions and the measured speed distribution could be identical to the normal distribution."

The measured speed distribution provided numerical checks, graphical plot, and chi-square test results which support the use of the normal distribution. If on the other hand, contrary results were found, attention would be directed to the log-normal and/or composite distributions. If there was no strong evidence of a bimodal distribution by inspection of a frequency distribution, as shown in the top portion of Figure 4.5, the log-normal distribution would be considered. The first step would be to plot the cumulative distribution of the speed measurements on logarithm-probability paper, as shown in Figure 4.9. The previously analyzed normal distribution is shown as a solid line, and the measured speed distribution is shown as a series of data points. A log-normal distribution would appear as a straight line in the figure. Therefore by inspection, the measured speed distribution is closer to a normal distribution than any log-normal distribution.

If, on the other hand, a measured speed distribution appears as a sloping straight line on Figure 4.9, the log-normal distribution would appear promising. For more quantitative evaluation, the theoretical frequency of the log-normal distribution could be determined and a chi-square test performed.

The composite distribution could be considered but usually after both the normal and log-normal distributions are rejected. Again the key characteristic for the composite

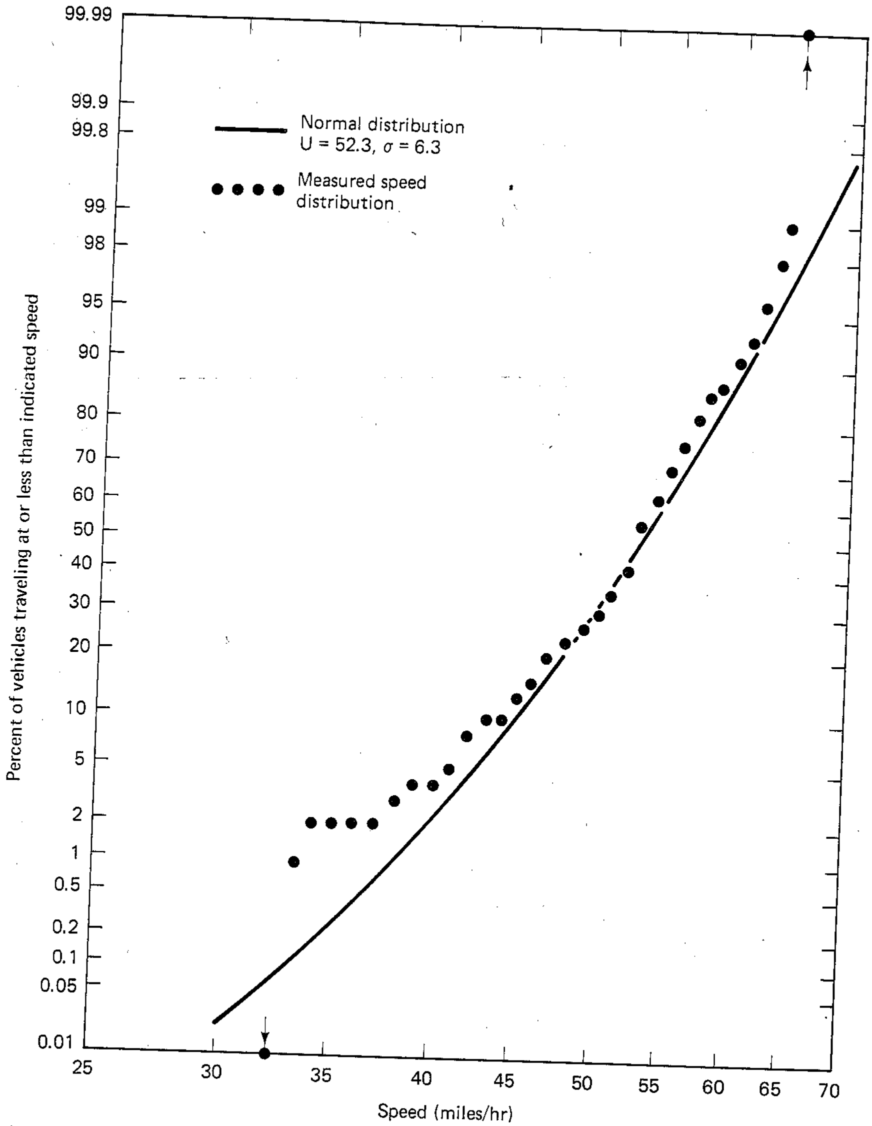

Figure 4.9 Measured Speed Distribution on Logarithm Probability Paper

distribution is a bimodal-appearing distribution. Two approaches may be considered. The speed measurements and field site could be restudied to attempt to identify the two subpopulations: that is, driver types, vehicle types, lane usage, and so on. If subpopulations can be identified, a second speed study should be undertaken and individual speeds of vehicles in the two subpopulations measured separately. Then the normal or log-normal distributions could be considered for each subpopulation. In simple, clear cases it might be possible to develop a composite distribution based on the original set of speed measurements if the mean, standard deviation, and frequency of each subpopulation can be determined.

4.5 ESTIMATION OF POPULATION MEANS AND SAMPLE SIZES

The best single estimate of the population mean is the sample mean. However, if several samples were taken under similar conditions and the sample means computed, there would be some numerical differences between sample mean values and hence differences in estimating the population mean. As more and more samples were taken and sample means computed, a distribution of sample means about the population mean would emerge in a fashion similar to a distribution of individual speeds about its sample mean. Of course, the distribution of sample means would be much more compact than the distribution of individual speeds, and standard statistics references [4, 5, 21] would show that the dispersion measure called the "standard error of the mean" would be

$$s_{\bar{x}} = \frac{s}{\sqrt{N}} \tag{4.22}$$

where $s_{\bar{x}}$ = standard error of the mean (miles per hour)
s = standard deviation of the sample of individual speeds
N = number of individual speeds observed

Further, the distribution of sample means would be normally distributed about the population mean even if the distribution of individual speed measurements were not normally distributed. It is significant to observe from equation (4.22) that only one sample of observations is required in order to calculate the standard error of the mean ($s_{\bar{x}}$). Using equation (4.22) and the speed observations presented in Table 4.5 and Figure 4.5, the standard error of the mean is

$$s_{\bar{x}} = \frac{s}{\sqrt{N}} = \frac{6.3}{\sqrt{200}} = 0.45 \text{ miles per hour}$$

Assuming the population mean equal to the sample mean (52.3 miles per hour) and using the calculated value of the standard error of the mean, a distribution of sample means can be calculated and the results are shown in Figure 4.10. Figure 4.10 is plotted in a similar fashion as Figure 4.5a except that the vertical scale is the probability density function of sample mean (μ's) rather than individual speeds (μ_i's). As mentioned before, the best single estimate of the population mean is the sample mean (52.3 miles per hour). However now with Figure 4.10, probability statements can be made about the population mean such as shown at the bottom of Figure 4.10. For example, with a probability of 0.9973, the population mean is expected to lie between 50.95 and 53.65 miles per hour. Other probability statements can be made as indicated on the figure.

This concept can be carried further in determining sample size requirements for speed studies. In speed studies a set of observations are made, the sample mean is calculated, and the population mean which is the real objective is inferred. Although larger samples may result in better estimates of the population mean, larger samples

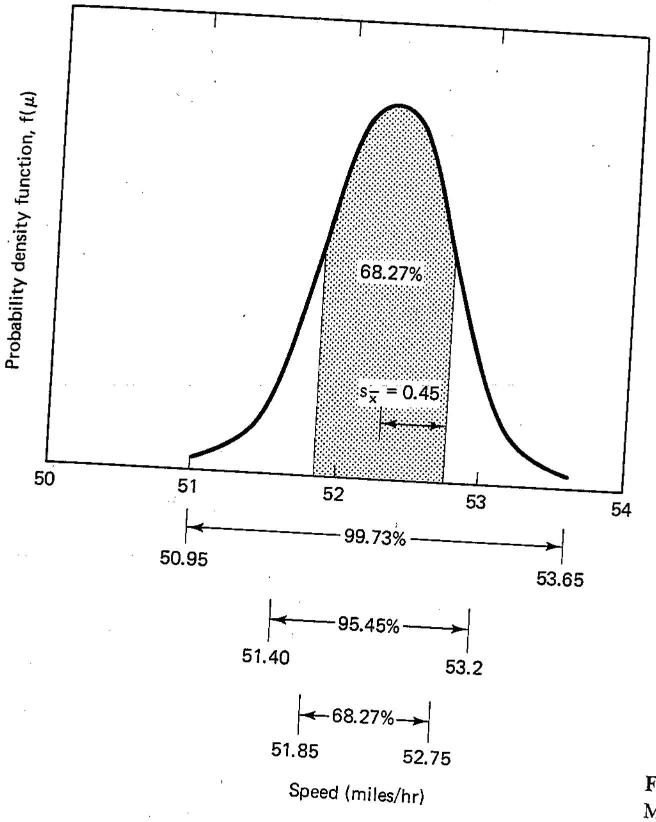

Figure 4.10 Distribution of Sample Means

require more time and effort in collection and analysis. Consequently, it is desirable to develop a technique that will provide the analyst with a means of selecting the smallest sample size possible while providing a limit on a prespecified probability that the population mean will be within a specified allowable error.

Rearranging equation (4.22) in order to solve for the required sample size (n), the equation becomes

$$n = \left(\frac{s}{s_{\bar{x}}}\right)^2 \tag{4.23}$$

Equation (4.23) has four variables, three that are clearly specified and one that is implied. The required sample size that is to be determined is represented by n. The standard deviation is a measure of dispersion of individually observed speeds that is obtained from a speed study or is known historically. The standard error of the mean, is a measure of the dispersion of sample means about the population mean, but in addition, probabilities are connected with different levels of the standard error of the mean. For example, in Figure 4.10, the population mean is expected to lie between 51.85 and 52.75 ($\pm 1s_{\bar{x}}$) with a probability of 0.6827. Or putting it another way, the error in the population mean is expected to be within ±0.45 miles per hour ($\pm 1s_{\bar{x}}$) with a probability of 0.6827. In most speed studies, probabilities or confidence levels closer to 0.95 or

0.99 are selected. In these cases instead of using $\pm 1 s_{\bar{x}}$, a coefficient on the order of 2 or 3 is used for the standard error of the mean. More specifically, if a probability level of 0.95 is selected, the coefficient can be obtained from Table 4.6. Since the distribution is symmetrical, a value of 0.4750 (one-half of 0.95) is entered and the coefficient is found to be 1.96. Equation (4.20) is now modified in two ways: the coefficient t is entered on the right side of the equation, and ε, the allowable error, is substituted for $ts_{\bar{x}}$. Equation (4.23) becomes

$$n = \left(\frac{ts}{ts_{\bar{x}}}\right)^2 = \left(\frac{ts}{\varepsilon}\right)^2 \tag{4.24}$$

where n = required sample size
s = standard deviation
ε = user-specified allowable error
t = coefficient of the standard error of the mean that represents user specified probability level

Figure 4.11 contains nomographs that can be used for the determination of sample size requirements based on equation (4.24). Figure 4.11a is for a 0.95 probability level, Figure 4.11b is for a 0.99 probability level. The analyst can enter the desired nomograph with the expected standard deviation value and the user-selected allowable error, and the minimum sample size can be determined. For example, the analyst may want to estimate the population mean within ±1 mile per hour with 99 percent confidence, and based on previous speed studies the standard deviation is expected to be 4 miles per hour. Figure 4.11b would indicate a minimum sample size of 100 observations. Another example would be the situation where the standard deviation is unknown and a pilot speed study is undertaken to estimate the standard deviation. The analyst then wishes to check to see if the pilot speed study sample is adequate or if a further speed study is required. Consider the example given in Table 4.5, in which 200 speeds are observed and the standard deviation is found to be 6.3 miles per hour. Assuming that the population mean is to be estimated within ±1 mile per hour with 95 percent confidence, Figure 4.11a would indicate a minimum sample size of approximately 152 observations. Since the number of observations in the original sample (n = 200) was larger than the determined minimum sample size (N = 152), no further observations are required.

4.6 SELECTED PROBLEMS

1. Undertake a library study of microscopic simulation models that require individual vehicular speeds to be generated within the model. What type(s) of mathematical distributions are employed?
2. Estimate the lost time due to a vehicle stopping (but with stopped delay being equal to zero) assuming cruise or running speeds upstream and downstream of the stop location of 30, 40, 50, and 60 miles per hour. Plot on graph similar to Figure 4.1.
3. There is considerable concern about traffic safety on high-speed approaches to rural signalized intersections. That is, when the signal changes to amber, the approaching vehicle may

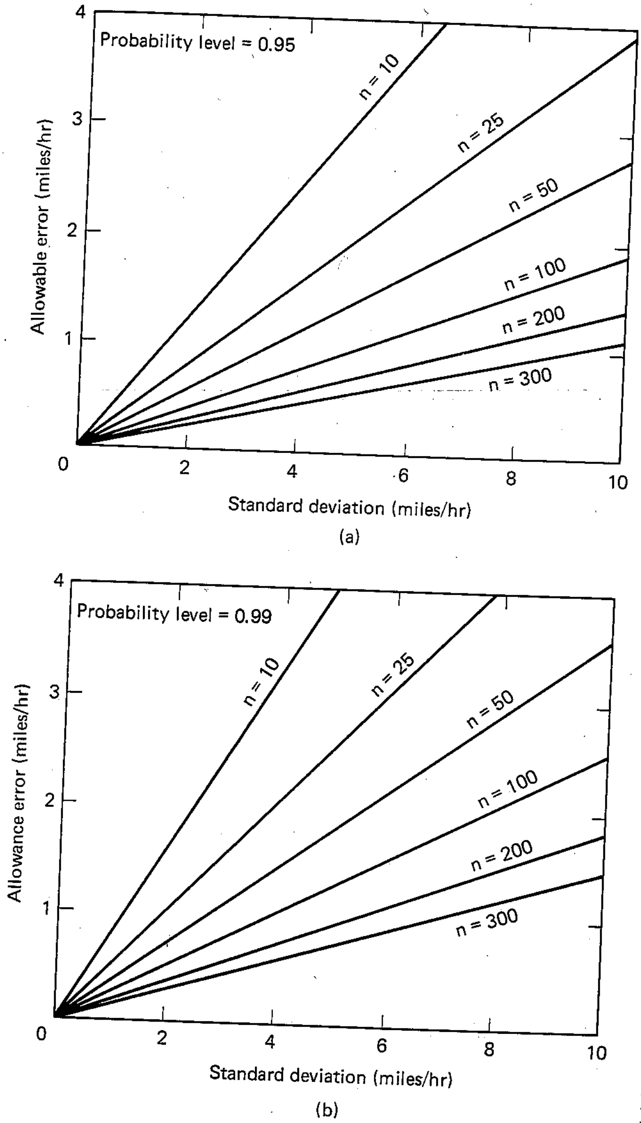

Figure 4.11 Sample Size Requirement Nomographs

be too far away from the stop line to accelerate and enter the intersection before the ending of the amber phase but too close to the intersection to decelerate and stop at the stop line. Consider the case of passenger vehicles in level terrain under wet pavement conditions. Assume approach speeds of 30, 40, 50, and 60 miles per hour and use maximum acceleration and deceleration rates. Determine the minimum amber phase to eliminate the so-called "dilemma" zone. (*Hint:* Plot a diagram in which the vertical scale is speed and the horizontal scale is distance to the stop line. Plot one curve for the minimum stopping distance and another for the maximum distance that can be traversed to the stop line as a function of approach speed and the length of the amber phase.) (See Reference 30 for more details.)