Traffic Flow Models

CIVL 4162/6162
(Traffic Engineering)
Lesson Objective

• Demonstrate traffic flow characteristics using observed data

• Describe traffic flow models
  - Single regime
  - Multiple regime

• Develop and calibrate traffic flow models
Field Observations (1)

• The relationship between speed-flow-density is important to observe before proceeding to the theoretical traffic stream models.

• Four sets of data are selected for demonstration
  - High speed freeway
  - Freeway with 55 mph speed limit
  - A tunnel
  - An arterial street
High Speed Freeway

- Figure 10.3
High Speed Freeway (1)

- This data is obtained from Santa Monica Freeway (detector station 16) in LA
- This urban roadway incorporates
  - high design standards
  - Operates at nearly ideal conditions
- A high percentage of drivers are commuters who use this freeway on regular basis.
- The data was collected by Caltrans
High Speed Freeway (2)

- Measurements are averaged over 5 min period
- The speed-density plot shows
  - a very consistent data pattern
  - Displays a slight S-shaped relationship
High Speed Freeway: Speed-Density

- Uniform density from 0 to 130 veh/mi/lane
- Free flow speed little over 60 mph
- Jam density can not be estimated
- Free flow speed portion shows like a parabola
- Congested portion is relatively flat
High Speed Freeway: Flow-Density

- Maximum flow appears to be just under 2000 veh per hour per lane (vhl)
- Optimum density is approx. 40-45 veh/mile/lane (vml)
- Consistent data pattern for flows up to 1,800 vhl
High Speed Freeway: Flow-Speed

- Optimum speed is not well defined
  - But could range between 30-45 mph
- Relationship between speed and flow is not consistent beyond optimum flow
Break-Out Session (3 Groups)

- Find out important features from
  - Figure 10.4
  - Figure 10.5
  - Figure 10.6
Difficulty of Speed-Flow-Density Relationship (1)

- A difficult task
- Unique demand-capacity relationship vary
  - over time of day
  - over length of roadway
- Parameters of flow, speed, density are difficult to estimate
  - As they vary greatly between sites
Difficulty of Speed-Flow-Density Relationship (2)

• Other factors affect
  - Design speed
  - Access control
  - Presence of trucks
  - Speed limit
  - Number of lanes

• There is a need to learn theoretical traffic stream models
Individual Models

- Single Regime model
  - Only for free flow or congested flow

- Two Regime Model
  - Separate equations for
    - Free flow
    - Congested flow

- Three Regime Model
  - Separate equations for
    - Free flow
    - Congested flow
    - Transition flow

- Multi Regime Model
Single Regime Models

• Greenshield’s Model
  - Assumed linear speed-density relationships
  - All we covered in the first class
  - In order to solve numerically traffic flow fundamentals, it requires two basic parameters
    • Free flow speed
    • Jam Density

\[ u = u_f - \left( \frac{u_f}{k_j} \right) * k \]
Single Regime Models: Greenberg

- Second regime model was proposed after Greenshields
- Using hydrodynamic analogy he combined equations of motion and one-dimensional compressive flow and derived the following equation
  \[ u = u_0 \times \ln \left( \frac{k_j}{k} \right) \]
- Disadvantage: Free flow speed is infinite
Single Regime Models: Underwood

- Proposed models as a result of traffic studies on Merrit Parkway in Connecticut
- Interested in free flow regime as Greenberg model was using an infinite free flow speed
- Proposed a new model

\[ u = u_f \ast e^{-(\frac{k}{k_0})} \]
Single Regime Models: Underwood (2)

- Requires free flow speed (easy to compute)
- Optimum density (varies depending upon roadway type)
- Disadvantage
  - Speed never reaches zero
  - Jam density is infinite
Single Regime Models: Northwestern Univ.

\[ u = u_f \times e^{-\frac{1}{2\left(\frac{k}{k_0}\right)^2}} \]

- Formulation related to Underwood model
- Prior knowledge on free flow speed and optimum density
- Speed does not go to “zero” when density approaches jam density
Single Regime Model Comparisons (1)

• All models are compared using the data set of freeway with speed limit of 55mph (see fig. 10.4)

• Results are shown in fig. 10.7

• Density below 20vml
  - Greenberg and Underwood models underestimate speed

• Density between 20-60 vml
  - All models underestimate speed and capacity
Single Regime Model Comparisons (2)

- Density from 60-90 vml
  - all models match very well with field data

- Density over 90 vml
  - Greenshields model begins to deviate from field data

- At density of 125 vml
  - Speed and flow approaches to zero
### Single Regime Model Comparisons (3)

<table>
<thead>
<tr>
<th>Flow Parameter</th>
<th>Data Set</th>
<th>Greenshields</th>
<th>Greenberg</th>
<th>Underwood</th>
<th>Northwestern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Flow (qm)</td>
<td>1800-2000</td>
<td>1800</td>
<td>1565</td>
<td>1590</td>
<td>1810</td>
</tr>
<tr>
<td>Free-flow speed (uf)</td>
<td>50-55</td>
<td>57</td>
<td>--inf..</td>
<td>75</td>
<td>49</td>
</tr>
<tr>
<td>Optimum Speed (k0)</td>
<td>28-38</td>
<td>29</td>
<td>23</td>
<td>28</td>
<td>30</td>
</tr>
<tr>
<td>Jam Density (kj)</td>
<td>185-250</td>
<td>125</td>
<td>185</td>
<td>..inf..</td>
<td>..inf..</td>
</tr>
<tr>
<td>Optimum Density</td>
<td>48-65</td>
<td>62</td>
<td>68</td>
<td>57</td>
<td>61</td>
</tr>
<tr>
<td>Mean Deviation</td>
<td>-</td>
<td>4.7</td>
<td>5.4</td>
<td>5.0</td>
<td>4.6</td>
</tr>
</tbody>
</table>
Multiregime Models (1)

- Eddie first proposed two-regime models because
  - Used Underwood model for Free flow conditions
  - Used Greenberg model for congested conditions

- Similar models are also developed in the era

- Three regime model
  - Free flow regime
  - Transitional regime
  - Congested flow regime
## Multiregime Models (2)

<table>
<thead>
<tr>
<th>Multiregime Model</th>
<th>Free Flow Regime</th>
<th>Transitional Flow Regime</th>
<th>Congested Flow Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddie Model</td>
<td>$u = 54.9e^{-k/163.9}$ \quad (k \leq 50)</td>
<td>NA</td>
<td>$u = 26.8\ln\left(\frac{162.5}{k}\right)$ \quad (k \geq 50)</td>
</tr>
<tr>
<td>Two-regime Model</td>
<td>$u = 60.9 - 0.515k$ \quad (k \leq 65)</td>
<td>NA</td>
<td>$u = 40 - 0.265k$ \quad (k \geq 65)</td>
</tr>
<tr>
<td>Modified Greenberg Model</td>
<td>$u = 48$ \quad (k \leq 35)</td>
<td>NA</td>
<td>$u = 32\ln\left(\frac{145.5}{k}\right)$ \quad (k \geq 35)</td>
</tr>
<tr>
<td>Three-regime Model</td>
<td>$u = 50 - 0.098k$ \quad (k \leq 40)</td>
<td>$u = 81.4 - 0.91k$ \quad (40 \leq k \leq 65)</td>
<td>$u = 40 - 0.265k$ \quad (k \geq 65)</td>
</tr>
</tbody>
</table>
Multiregime Models (3)

• Challenge
  - Determining breakeven points

• Advantage
  - Provide opportunity to compare models
  - Their characteristics
  - Breakeven points
Summary

• Multiregime models provide considerable improvements over single-regime models
• But both models have their respective
  - Strengths
  - weaknesses
• Each model is different with continuous spectrum of observations
Model Calibration (1)

• In order calibrate any traffic stream model, one should get the boundary values,
  - free flow speed () and jam density ()

• Although it is difficult to determine exact free flow speed and jam density directly from the field, approximate values can be obtained

• Let the linear equation be \( y = ax + b \); such that is
  - \( Y \) denotes density (speed) and \( x \) denotes the speed (density)
Model Calibration (2)

• Using linear regression method, coefficient $a$ and $b$ can be solved as

$$
b = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}
$$

$$
a = \bar{y} - b\bar{x}
$$
Example

• For the following data on speed and density, determine the parameters of the Greenshields' model.

• Also find the maximum flow and density corresponding to a speed of 30 km/hr.

<table>
<thead>
<tr>
<th>$k$ (veh/km)</th>
<th>$u$ (km/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>5</td>
</tr>
<tr>
<td>129</td>
<td>15</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>70</td>
<td>25</td>
</tr>
</tbody>
</table>
Model Calibration (1)

<table>
<thead>
<tr>
<th>x(k)</th>
<th>y(u)</th>
<th>(xᵢ - x̄)</th>
<th>(yᵢ - ȳ)</th>
<th>(xᵢ - x̄) * (yᵢ - ȳ)</th>
<th>(xᵢ - x̄)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>5</td>
<td>73.5</td>
<td>-16</td>
<td>-1198</td>
<td>5402.3</td>
</tr>
<tr>
<td>129</td>
<td>15</td>
<td>31.5</td>
<td>-6.3</td>
<td>-198.5</td>
<td>992.3</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>-78</td>
<td>18.7</td>
<td>-1449</td>
<td>6006.3</td>
</tr>
<tr>
<td>70</td>
<td>25</td>
<td>-28</td>
<td>3.7</td>
<td>-101.8</td>
<td>756.3</td>
</tr>
<tr>
<td>390</td>
<td>85</td>
<td></td>
<td></td>
<td>-2948.7</td>
<td>13157.2</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum x}{n} = \frac{390}{4} = 97.5
\]

\[
\bar{y} = \frac{\sum y}{n} = \frac{85}{4} = 21.3
\]

\[
b = \frac{\sum_{i=1}^{n} (xᵢ - \bar{x})(yᵢ - \bar{y})}{\sum_{i=1}^{n} (xᵢ - \bar{x})^3} = -0.2
\]

\[
a = \bar{y} - b\bar{x} = 21.3 + 0.2 * 97.5 = 40.8
\]

\[
u = 40.8 - 0.2k
\]
Model Calibration (2)

\[ u = 40.8 - 0.2k \Rightarrow u_f = 40 \text{ and } \frac{u_f}{k_j} = 0.2 \]

\[ k_j = \frac{40.8}{0.2} = 204 \text{ veh/mi} \]

\[ q_m = \frac{u_f k_j}{4} = \frac{40.8 \times 204}{4} = 2080.8 \text{ veh/hr} \]

Density corresponding to speed of 30 km/hr is given by

\[ 30 = 40.8 - 0.2k \Rightarrow k = \frac{40.8 - 30}{0.2} = 54 \text{ veh/km} \]