10.2.1 Importance of Field Location

To obtain meaningful speed–flow–density measurements, the data collection must be taken at the right location during a selected time period. Putting it in another way, the location and time period of field measurements significantly affect the resulting speed–flow–density measurements. Consider the directional roadway depicted at the top of Figure 10.2 with traffic moving from left to right. There are no entrances or exits along the roadway, and in the middle portion of the study section the capacity has been reduced due to a lane drop. To describe the example clearly, assume three-lane sections at the upstream and downstream portions and a two-lane section in the middle portion.

Measurement stations are established at locations A, B, C, and D. Speeds, flows, and densities are measured independently at each station. Station A is at the upstream end of the study section, away from any influence due to the lane-drop location. Station
$B$ is in the three-lane section, but just a short distance upstream of the lane-drop location. Station $C$ is in the two-lane section and station $D$ is in the downstream three-lane section.

Theoretical speed–flow, flow–density, and speed–density curves are shown on Figure 10.2 directly below the four measurement stations. The curves are identical for stations $A$, $B$, and $D$; however, the curves for station $C$ are different. For station $C$, the roadway capacity is only two lanes (two-thirds of the capacity of stations $A$, $B$, and $D$) and the roadway jam density is two-thirds of stations $A$, $B$, and $D$.

Assume that this is an inbound section of roadway carrying traffic to the central business district during the morning peak period. From 6:00 to 7:30 A.M. the traffic demand increases from a very small flow up to a flow equivalent to two lanes of capacity. The hypothetical collected data measurements are plotted on all diagrams of Figure 10.2 as solid dots. While stations $A$, $B$, and $D$ are operating only at two-thirds of capacity and operating at relatively high speeds and low densities, station $C$ has reached its capacity with significantly lower speeds and higher densities.

Now assume that the demand from 7:30 to 8:00 A.M. increases to a flow equivalent of two and one-half lanes of capacity. What data measurements would be observed at the four stations? Station $A$ is fairly easy to predict since it is not influenced by the two-lane section and its capacity is greater than two and one-half lanes. The resulting data points are plotted as hollow circles. Now consider station $B$. Its data points would be identical to station $A$'s data points until it is affected by the two-lane section. When this occurs, the flow at station $B$ will be equal to the flow at station $C$ (two lanes of flow) and station $B$ would be congested with exhibited low speeds and high densities. The resulting data points are plotted as hollow circles.

Now consider station $C$. Although the traffic demand is equivalent to two and one-half lanes of traffic, its capacity is only two lanes, and thus the flow will remain fairly uniform right at capacity. Station $C$ would be operating at optimum speed and optimum density. The resulting data points are plotted as hollow circles. Finally, consider station $D$. Its capacity of three lanes exceeds its demand of two and one-half lanes of traffic. However, station $C$ can pass only two lanes of traffic, so that station $D$ will exhibit uniform data points of two lanes of traffic and relatively high speeds and low densities. The resulting data points are plotted as hollow circles.

Now assume that the demand after 8:00 A.M. begins to decrease. What data measurements would now be observed at the four stations? The previously plotted hollow circles would revert back to solid dots at all stations and further data collection would result in new data points being superimposed on previously collected data points.

Now review the original theoretical curves with the hypothetical measured data points. Stations $A$, $C$, and $D$ will only exhibit data points in the free-flow regime and only in the case of station $C$ are measurements available over the complete free-flow regime. The data point configuration for station $B$ does exhibit observations in the free-flow and congested-flow regimes. However, data points are not provided over the complete theoretical curves. In the free-flow regime, data points are obtained up to about two and one-half lanes of flow. In the congested-flow regime, data points are clustered at a congested flow rate approximately equivalent to two lanes of flow, which
is the downstream capacity. Note that some observations may be recorded in the region between the free-flow regime and the congested-flow regime.

None of the data sets from the four measurement stations covers the complete range of possible speed, flow, and density values. If the roadway at station B had a little lower capacity, say equivalent to two and one-half lanes of flow, the complete free-flow regime would be obtained. Even then, however, the measurements in the congested-flow regime would be limited to a relative small segment. The distribution of data points in the congested-flow regime could be improved if the capacity of the roadway at station C varied with time from a very small capacity to a capacity equivalent to two and one-half lanes of flow. Although in real life, capacity might change with time due to merging, weaving, and/or incidents, it is unlikely that a wide range in capacity changes would occur, particularly at the time of field measurements.

In summary, there are two important points to keep in mind with traffic stream models based on field observations. First, the location limits the range of flow, speed, and density values. A corollary is that given a set of flow, speed, and density measurements, the type of location can be determined. (Review the diagrams shown in Figure 10.2 without reference to the roadway sketch. Could these locations be described?) The second important point is that in validating traffic stream models, the data sets used may influence the results and the comparison between models.

10.2.2 Field Observations

The results of field studies that show the relationships between flow, speed, and density are important to observe before proceeding to theoretical traffic stream models. Four sets have been selected to show flow—speed—density data point patterns for a high-speed freeway [2], a freeway with a 55-mile per hour speed limit [1], a tunnel [3], and an arterial street [4]. Particular attention will be given to consistency of data point patterns; ranges of flow, speed, and density observed values; and to flow, speed, and density parameters.

Relationships between flow, speed, and density observed values for a high-speed freeway are shown in Figure 10.3. These data were obtained from the Santa Monica Freeway (detector station 16) in Los Angeles. This urban roadway incorporates high design standards and operates under nearly ideal conditions. A high percentage of the drivers are commuters who use this freeway on a regular basis. The data were collected automatically by the California Department of Transportation as part of their freeway surveillance and control project. Measurements are averaged over 5-minute periods. The speed—density plot shows a very consistent data point pattern and displays a slight S-shaped relationship. The sample provides a rather uniform distribution of observed densities over the range from near zero to 130 vehicles per mile per lane, but no density values were observed over 130 vehicles per mile per lane. The free-flow speed is slightly over 60 miles per hour and the jam density cannot be estimated. The flow—density plot also shows a very consistent data point pattern, and while the free-flow portion appeared somewhat as a parabola, the congested-flow portion is relatively flat with a tail to the right. The maximum flow or capacity appears to be just under 2000 vehicles per hour per lane, and the optimum density is approximately 40 to 45
vehicles per mile per lane. The speed–flow plot shows a consistent data point pattern except when flows are over 1800 vehicles per hour per lane. It would appear that the optimum speed is not well defined and could lie anywhere between 30 and 45 miles per hour.
Relationships between flow, speed, and density observed values for a freeway having a speed limit of 55 miles per hour are shown in Figure 10.4. These data were obtained from the Eisenhower Expressway near Harlem Avenue in Chicago. This roadway incorporates high design standards, operates under nearly ideal conditions, and is

![Graphs showing relationships between flow, speed, and density.](image)

*Figure 10.4 Freeway Data (55-mile/hr Speed Limit) (From Reference 1)*
used by regular commuters. In comparison with the previous described data collection set, the Chicago data set was collected about 10 to 15 years earlier, and the speed limit of 55 miles per hour appeared to have a high degree of compliance. The data were automatically collected by the Illinois Department of Transportation as part of their freeway surveillance and control project. Measurements are averaged over 5-minute periods during the afternoon peak period. The speed–density plot shows a very consistent data point pattern and displays a S-shaped relationship with a discontinuity at a density of 50 to 55 vehicles per mile per lane. Although the sample provides a rather uniform distribution of observed densities over its range, observations were not recorded for density ranges below 15 or above 115 vehicles per mile per lane. The free-flow speed appears to be about 55 miles per hour, but the jam density cannot be estimated. The flow–density plot also shows a very consistent data point pattern. Like the first data set, while the free-flow portion is somewhat shaped like a parabola, the congested-flow portion is relatively flat with a tail to the right. The maximum flow or capacity appears to be about 2000 vehicles per hour per lane and occurs at an optimum density of about 50 vehicles per mile per lane. The speed–flow plot generally shows a consistent pattern except for two or three data points inside the enclosure curves and the absence of more than one data point at flows over 1850 vehicles per hour per lane. The optimum speed appears to be on the order of 40 miles per hour.

Relationships between observed values of flow, speed, and density in a tunnel are shown in Figure 10.5. These data were obtained in the Holland Tunnel in New York. This two-lane directional roadway is a tunnel underneath the Hudson River that connects New Jersey and New York. The design features are somewhat restrictive, with 11-foot lanes, no shoulders, and a typical tunnel alignment consisting of a downgrade followed by an upgrade. The data were automatically collected by the Port of New York Authority as part of their tunnel surveillance and control project. Measurements have been averaged over 5-minute periods. Drivers in the tunnel are regular commuters. The speed–density plot shows a very consistent data point pattern, but unlike the two earlier data sets, displays almost a linear configuration rather than a S-shaped relationship. Like earlier data sets, densities over 110 vehicles per mile per lane were not obtained. The free-flow speed appeared to be about 45 miles per hour, and the jam density could not be estimated. The shape of the flow–density plot is similar to the earlier flow–density plots with a parabola-type shape in the free-flow regime and a relative flat slope with a tail to the right in the congested-flow regime. The maximum flow or capacity was about 1350 vehicles per hour per lane and occurred at optimum densities of 50 to 60 vehicles per mile per lane. The speed–flow plot generally showed a consistent pattern in the free-flow regime, but the data points were clustered in the congested regime. There appears to be a bottleneck downstream which has a capacity on the order of 1100 to 1200 vehicles per hour per lane. Optimum speed appears to be about 25 miles per hour.

Relationships between observed values of flow, speed, and density for an arterial street are shown in Figure 10.6. This data set was obtained from a Newcastle University research team in the United Kingdom who are studying incident detection techniques on arterial streets. Data were collected from a detector located at the upstream
end of a link that had a signal at the downstream end. This detector was part of a traffic-responsive signal control system called "SCOOT." The data were collected for short periods, and unlike the earlier described controlled access freeway and tunnel sites, the environment around the detector site could be classified as uncontrolled. The
speed–density plot shows a fairly consistent data point pattern except for the infrequent observations in the density range 40 to 90 vehicles per mile per lane. There appeared to be two distinctly different operational modes. One operational mode was when the site was unaffected by the downstream signal and free-flow conditions existed, and the other
occurred when the queue from the signal backed into the detector site. Free-flow speeds were about 30 miles per hour and jam densities were not observed. The data point pattern was not S-shaped but exhibited a continuous decreasing slope with increased densities. The flow-density plot in the free-flow regime is similar to earlier flow-density plots with a parabola-type shape and the congested flow regime is relatively flat with a tail to the right. The maximum flow is between 600 and 700 vehicles per hour per lane. The scarcity of observations with densities between 40 and 90 vehicles per mile per lane provides suspicion that the site can never reach its capacity because of queue backups from the downstream signal. The speed-flow plot provides a less consistent pattern than that of the three earlier data sets. The highest flows can be identified, but the optimum speed is difficult to estimate.

The analysis of flow-speed-density relationships based on field measurements can be a very difficult task. Unique demand-capacity relationships over time of day and over length of roadway must be present. Even then the complete range of flow, speed, and density values will probably not be recorded. Parameter values of flow, speed, and density are often difficult to estimate and can greatly vary between sites. Many other factors affect flow-speed-density relationships, such as design speed, access control, presence of trucks, speed limits, number of lanes, and so on. Bridging the gap between theory and practice is a challenge to the theoretician and the professional. Having observed real-world flow-speed-density relationships, it is now appropriate to learn of proposed theoretical traffic stream models.

10.3 PROPOSED INDIVIDUAL MODELS

Over the years a number of traffic stream models have been proposed. The earlier models assumed a single regime phenomenon over the complete range of flow conditions including free-flow and congested flow situations. Later models attempted to improve on the earlier models by considering two separate regimes (free-flow regime and congested-flow regime) and attempted to generalize by introducing additional parameters that could be used to distinguish between roadway environments.

10.3.1 Single-Regime Models

The first single-regime model was developed by Greenshields in 1934, based on observing speed-density measurements obtained from an aerial photographic study [5]. These measurements were presented in Figure 7.1 and Greenshields concluded that speed was a linear function of density. Section 10.1 was based on the assumption of a linear speed-density relationship, so all equations, illustrations, and derivations are for the Greenshields model. The model requires knowledge of the free-flow speed and jam density parameters in order to solve numerically for the speed-density relationship. The free-flow speed is relatively easy to estimate in the field and generally lies between the speed limit and the design speed of the roadway. On the other hand, the estimation and use of jam density present problems. Jam density values are difficult to obtain in
the field, but a general value of 185 to 250 vehicles per mile per lane can be calculated assuming that a stopped vehicle occupies 21 to 28 feet of roadway space. The use of this jam density value also presents a problem because according to this model, optimum density is equal to one-half of the jam density value. This is not compatible with observed optimum density values on the order of 40 to 70 vehicles per mile per lane.

The Greenberg model was the second single-regime model that was proposed [6]. Observing speed–density data sets for tunnels, with particular attention to the congested portion such as shown in Figure 10.5, he concluded that a nonlinear model might be more appropriate. Using a hydrodynamic analogy he combined the equations of motion and continuity for one-dimensional compressible flow and derived the following equation:

\[ u = u_o \ln \left( \frac{k_j}{k} \right) \]  

(10.9)

One of the important results of Greenberg's work was the bridge that was discovered between his proposed macroscopic model and the third General Motors car-following model. This bridge was described earlier in Chapter 6 and was the foundation for later discovery that almost all developed car-following theories could be related mathematically to most macroscopic traffic stream models [7, 8]. The Greenberg model requires knowledge of the optimum speed and jam density parameters. Like the Greenshields model, jam density is difficult to observe in the field, and estimating optimum speed is even more difficult than estimating free-flow speed. A crude estimate is that the optimum speed is approximately one-half of the design speed. Another disadvantage of this model is that free-flow speed is infinity. Later, Edie, recognizing this disadvantage, proposed a two-regime modeling approach with the Greenberg model being used for the congested regime [9].

The third single-regime model was proposed by Underwood as a result of traffic studies on the Merritt Parkway in Connecticut [10]. Underwood was particularly interested in the free-flow regime and was disturbed by free-flow speed going to infinity in the Greenberg model. Hence a new model was proposed as shown in the following equation:

\[ u = u_f e^{-k/k_o} \]  

(10.10)

This formulation requires knowledge of the free-flow speed, which is fairly easy to observe, and the optimum density, which is difficult to observe and varies depending on the roadway environment. Another disadvantage of this model is that speed never reaches zero and jam density is infinity. Again Edie, recognizing this disadvantage, proposed a two-regime modeling approach with the Underwood model being used for the free-flow regime [9].

A fourth model was proposed by a group of researchers at Northwestern University when they observed that most speed–density curves appear as S-shaped curves [1]. The Northwestern group proposed the following equation:

\[ u = u_f e^{-1/2(k/k_o)^2} \]  

(10.11)
This formulation appears related to the Underwood model in that knowledge of the free-flow speed and optimum density are required and also, speed does not go to zero when density goes to jam density.

Further development of single-regime models was directed toward the introduction of a parameter in the formulation which would provide for a more generalized modeling approach. For example, Drew proposed a formulation based on Greenshields' model, but with the introduction of an additional parameter $n$ as shown in the following equation \([11]\).

$$u = u_f \left[ 1 - \left( \frac{k}{k_f} \right)^{n+1/2} \right]$$  \hspace{1cm} (10.12)

When the parameter $n$ is set equal to 1, the formulation converts to the Greenshields model. However, varying the parameter $n$, a family of models can be developed. Drew suggested varying $n$ from $-1$ to $+1$ and called these models a linear model ($n = +1$), a parabolic model ($n = 0$), and an exponential model ($n = -1$). Pipes-Munjal proposed a somewhat similar formulation that would provide a more generalized approach to single-regime models [12]. Their proposed formulation is shown by the following equation.

$$u = u_f \left[ 1 - \left( \frac{k}{k_f} \right)^n \right]$$  \hspace{1cm} (10.13)

Again in this formulation when $n = 1$, the Greenshields model can readily be identified.

To study further the attributes of these single-regime models and to provide an opportunity to compare model characteristics, the initial four models were applied to the freeway data set (55-mile per hour speed limit) that was shown in Figure 10.4. The models could be applied in two different ways. The way the models were applied in this example was to determine the best regression fit for each model separately and then to observe the resulting flow parameter values. Another way would have been to select parameter values based on the inspection of the freeway data set, use these parameter values in applying each model, and observe how well each model fit the freeway data set by inspection and by calculating the mean deviations.

The resulting flow-speed-density relationships for each of the models based on the best regression fit are superimposed on the freeway data set in Figure 10.7. First, the resulting flow-speed-density relationships are discussed and then the model results are compared with the parameter values that appear to represent the field measured data set.

At density levels below 20 vehicles per mile per lane, the Greenberg and Underwood models overestimate speed. In the density range 20 to 60, all models underestimate speed and flow, and this is particularly disconcerting because of their estimation of capacity. As densities increase from 60 to 90, all models appear to track the field data reasonably well. At densities over 90 vehicles per mile per lane, the Greenshields model begins to deviate from the field data and at a density of 125, speeds and flows are predicted to approach zero.

Table 10.1 compares and summarizes the flow, speed, and density parameter values for each model, with estimates based on the field measured data set. All model
predictions of maximum flow are less than the estimates from the data set. The Greenberg and Underwood models have the lowest predictions of maximum flow. In regard to free-flow speeds, the Greenberg and Underwood models predict much higher values
TABLE 10.1 Comparison of Flow Parameters for Single-Regime Models

<table>
<thead>
<tr>
<th>Flow Parameter</th>
<th>Data Set</th>
<th>Greenshields</th>
<th>Greenberg</th>
<th>Underwood</th>
<th>Northwestern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum flow, $q_m$</td>
<td>1800–2000</td>
<td>1800</td>
<td>1565</td>
<td>1590</td>
<td>1810</td>
</tr>
<tr>
<td>Free-flow speed, $u_f$</td>
<td>50–55</td>
<td>57</td>
<td>$\infty$</td>
<td>75</td>
<td>49</td>
</tr>
<tr>
<td>Optimum speed, $u_o$</td>
<td>28–38</td>
<td>29</td>
<td>23</td>
<td>28</td>
<td>30</td>
</tr>
<tr>
<td>Jam density, $k_j$</td>
<td>185–250</td>
<td>125</td>
<td>185</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Optimum density, $k_o$</td>
<td>48–65</td>
<td>62</td>
<td>68</td>
<td>57</td>
<td>61</td>
</tr>
<tr>
<td>Mean deviation</td>
<td>—</td>
<td>4.7</td>
<td>5.4</td>
<td>5.0</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Source: Reference 1.

than would be expected from the data set. The Greenberg model appears to underestimate optimum speed. The Greenshields model significantly underestimates jam density, while the Underwood and Northwestern models predict jam densities of infinity. The Greenberg model slightly overestimates the optimum density. Finally, the Northwestern model exhibited the lowest mean deviation and the Greenberg model had the largest mean deviation.

In summary, four single-regime models have been described and then applied to a freeway data set. Each model had deficiencies over some portion of the density range. The most disconcerting feature of these models is their inability to track faithfully the measured field data near capacity conditions. One can observe a discontinuity in the flow–speed–density relationships as depicted by measured field data in the vicinity of capacity conditions. This has lead several researchers to propose two-regime models with separate formulations for the free-flow and congested-flow regimes.

10.3.2 Multiregime Models

Edie first proposed the idea of two-regime models in 1961 because of reservations of using car-following based models under free-flow conditions and his observation of the poor performance of the Greenberg model under free-flow conditions [9]. More specifically, Edie proposed the use of the Underwood model for the free-flow regime and the Greenberg model for the congested-flow regime. The equations for these two models were given earlier in the chapter as equations (10.9) and (10.10).

Supporting the idea of the use of multiregime models, a Northwestern University research team proposed three additional model formulations [1]. The first was the use
of the Greenshields-type model for the free-flow regime and the congested-flow regime separately. The equation for the Greenshields model was shown as equation (10.1). The second proposed multiregime model suggested a constant-speed model for the free-flow regime and a Greenberg model for the congested-flow regime. The equation for the Greenberg model was shown as equation (10.9). The last proposed multiregime model suggested a three-regime model with the free-flow regime, transitional-flow regime, and congested-flow regime each being represented by the Greenshields formulation, as shown in equation (10.1).

The first difficulty in multiregime models is determining the breakpoint between regimes. The Northwestern researchers applied the work of Quandt on likelihood functions to identify the breakpoints between regimes for all four multiregime models [13, 14] using the earlier presented freeway data set. Then using regression analysis, the best model was selected for each regime. The resulting equations and breakpoints for the four multiregime models are presented in the Table 10.2.

<table>
<thead>
<tr>
<th>Multiregime Model</th>
<th>Free-Flow Regime</th>
<th>Transitional-Flow Regime</th>
<th>Congested-Flow Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edie model</td>
<td>( u = 54.9e^{-k/10.9} ) (( k \leq 50 ))</td>
<td>—</td>
<td>( u = 26.8 \ln \left( \frac{162.5}{k} \right) ) (( k \geq 50 ))</td>
</tr>
<tr>
<td>Two-regime linear model</td>
<td>( u = 60.9 - 0.515k ) (( k \leq 65 ))</td>
<td>—</td>
<td>( u = 40 - 0.265k ) (( k \geq 65 ))</td>
</tr>
<tr>
<td>Modified Greenberg model</td>
<td>( u = 48 ) (( k \leq 35 ))</td>
<td>—</td>
<td>( u = 32 \ln \left( \frac{145.5}{k} \right) ) (( k \geq 35 ))</td>
</tr>
<tr>
<td>Three-regime linear model</td>
<td>( u = 50 - 0.098k ) (( k \leq 40 ))</td>
<td>( u = 81.4 - 0.913k ) (( 40 \leq k \leq 65 ))</td>
<td>( u = 40.0 - 0.265k ) (( k \geq 65 ))</td>
</tr>
</tbody>
</table>

Source: Reference 1.

To study the attributes of these multiregime models and to provide an opportunity to compare model characteristics, the equations and breakpoints shown in Table 10.2 are superimposed on the freeway data set in Figure 10.8. The multiregime models all track the freeway data set in a very reasonable manner and much better than any of the single-regime models. Table 10.3 compares and summarizes the flow, speed, and density parameter values for each model with estimates based on the field measured data set. In regard to maximum flow, the Edie model slightly overestimates while the other three models slightly underestimate. The linear two-regime slightly overestimates free-flow speed and the modified Greenberg slightly underestimates free-flow speed. The optimum speed is slightly overestimated by the Edie and three-regime linear model. All models underestimate jam density rather significantly. The three-regime linear model
Figure 10.8 Multiregime Models Superimposed on Freeway Data Set (From Reference 1)

underestimates optimum density. All multiregime models have smaller mean deviations than those of the single-regime models. The Edie and three-regime linear models have the smallest mean deviations.
### TABLE 10.3
Comparison of Flow Parameters for Multiregime Models

<table>
<thead>
<tr>
<th>Flow Parameter</th>
<th>Data Set</th>
<th>Two-Regime Linear</th>
<th>Modified Greenberg</th>
<th>Three-Regime Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum flow, $q_m$</td>
<td>1850–2000</td>
<td>2025</td>
<td>1800</td>
<td>1760</td>
</tr>
<tr>
<td>Free-flow speed, $u_f$</td>
<td>50–55</td>
<td>55</td>
<td>61</td>
<td>48</td>
</tr>
<tr>
<td>Optimum speed, $u_o$</td>
<td>28–38</td>
<td>40</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>Jam density, $k_j$</td>
<td>185–250</td>
<td>162</td>
<td>151</td>
<td>146</td>
</tr>
<tr>
<td>Optimum density, $k_o$</td>
<td>48–65</td>
<td>50</td>
<td>59</td>
<td>54</td>
</tr>
<tr>
<td>Mean deviation</td>
<td>—</td>
<td>3.6</td>
<td>4.2</td>
<td>3.9</td>
</tr>
</tbody>
</table>

*Source: Reference 1.*

In summary, the multiregime models provide a considerable improvement over single-regime models. However, the multiregime models and particularly the single-regime models had different strengths and weaknesses. Further, each model appeared discretely different rather than as exhibiting a continuous spectrum of a family of models. Two earlier points set the stage for Section 10.4. Recall that the General Motors and Port of New York researchers found that the Greenberg model could be derived from the third car-following model [7, 8]. Is it possible that other traffic stream models are related to other car-following models? Also, recall that Drew [11] and Pipes [12] introduced the idea of generalizing traffic stream models into families of models by inserting the parameter $n$. By varying $n$ a spectrum of traffic stream models could be formulated.

### 10.4 PROPOSED FAMILY OF MODELS

The sequential development of car-following theories was presented in Chapter 6 and the final generalized model is shown by the following expression [7, 8]:

$$
\bar{X}_{n+1}(t + \Delta t) = \frac{\alpha_{n,m}(\bar{X}_{n+1}(t + \Delta t))^{m}}{(\bar{X}_n(t) - \bar{X}_{n+1}(t))^{m}} [\bar{x}_n(t) - \bar{X}_{n+1}(t)]
$$  \hspace{1cm} (10.14)

It was also shown in Chapter 6 that the Greenberg macroscopic model (discussed earlier in this chapter) could be derived from the third car-following model, in which the $m$ and