Level of Service of Signalized Intersections
CIVL 4162/6162
Lesson Objective

• Quantify LOS for signalized intersections
• Apply HCM method to determine LOS
• Determine delays at signalized intersections
Saturation Flow Rate

- Saturation flow rate is given by

\[ s = \frac{3,600}{h} \]

where

- \( s \): saturation flow rate (vehicles per hour of green per lane, often used acronym of vphgpl)
- \( h \): saturation headway, seconds/vehicle
Saturation Flow Rate (2)

- On an average every vehicle consumes “h” seconds of green time to enter the intersection
- If every vehicle consumes “h” seconds of green time and if the signal were always green then “s” vehicles could enter the intersection
- Saturated flow rate can be multiplied with number of lanes to get saturation flow per lane group
Saturation Flow Rate

• The saturation flow rate for any lane group based on prevailing traffic parameters

\[ s_i = s_0 N \sum_{i} f_i \]

• Where
  - \( s_i \): Saturation flow rate of lane group \( i \)
  - \( s_0 \): Saturation flow rate under base conditions
  - \( N \): Number of lanes in the lane group
  - \( f_i \): Multiplicative adjustment factor for each prevailing condition
Saturation Flow Rate

\[ s = s_o N f_w f_H f_g f_p f_{tb} f_a f_{LU} f_{RT} f_{LT} f_{Rpb} f_{Lpb} \]

where:  
\( s = \) saturation flow rate under prevailing conditions, veh/hg 
\( s_o = \) saturation flow rate under ideal conditions, default value = 1,900 pc/hg/ln  
\( N = \) number of lanes in the lane group 
\( f_i = \) adjustment factor for prevailing condition “i” 

where: 
\( w = \) lane width 
\( HV = \) heavy vehicles 
\( g = \) grade 
\( p = \) parking 
\( bb = \) local bus blockage 
\( a = \) area type 
\( LU = \) lane utilization 
\( RT = \) right turn 
\( LT = \) left turn 
\( Rpb = \) ped/bike interference with right turns 
\( Lpb = \) ped/bike interference with left turns
Concept of Lane Group
Capacity of a Lane Group

- Capacity of a lane group
  
  $c_i = s_i \left( \frac{g_i}{C} \right)$

  - All notations previously defined
  - $C$ is cycle length in sec.
Volume to Capacity Ratio

\[ X_i = \frac{v_i}{c_i} = \frac{(v/s)_{ci}}{(g/C)_{ci}} \]

- \( X_i \): Degree of saturation (\( v/c \)) for lane group \( i \)
- \( v_i \): Demand flow rate for lane group \( i \)
- \( c_i \): Capacity for lane group \( i \)
- \( \left( \frac{v}{s} \right)_i \): Flow ratio for lane group \( i \)
- \( \left( \frac{g}{C} \right)_i \): Green ratio for lane group \( i \)
Critical v/c ratio for the intersection

- Critical v/c ratio
  \[ X_c = \frac{\sum_{i} \left( \frac{v}{s} \right)_{ci}}{\sum_{i} \left( \frac{g}{C} \right)_i} \]

  - \( X_c \): Critical v/c ratio for the intersection
  - \( \left( \frac{v}{s} \right)_{ci} \): v/c ratio for critical lane group \( i \)

- Alternatively
  \[ X_c = \frac{\sum_{i} \left( \frac{v}{s} \right)_i}{\frac{C - L_i}{L}} \]
  \( L \): Total lost time in sec
Performance of Intersection

• \( X_c \leq 1.0 \)
  - Physical design, cycle length and phasing sufficient to handle critical demands
  - Does not mean that all lane groups operate at \( X_c < 1 \)

• \( X_c > 1 \)
  - Increase cycle length
  - Provide efficient phasing plans
  - Increase capacity of critical lanes
## LOS Criteria

<table>
<thead>
<tr>
<th>Level of Service</th>
<th>Control Delay (s/veh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>≤10</td>
</tr>
<tr>
<td>B</td>
<td>&gt;10–20</td>
</tr>
<tr>
<td>C</td>
<td>&gt;20–35</td>
</tr>
<tr>
<td>D</td>
<td>&gt;35–55</td>
</tr>
<tr>
<td>E</td>
<td>&gt;55–80</td>
</tr>
<tr>
<td>F</td>
<td>&gt;80 or v/c &gt;1.00</td>
</tr>
</tbody>
</table>

Delay

• The most common measure used to describe operational quality at a signalized intersection is delay.

• Delay refers to the amount of time consumed in travelling the intersection
  - The difference between the arrival time and departure time
Estimating Delay

• Delay for each lane group

\[ d = d_1 + d_2 + d_3 \]

• Where

- \( d \): Average control delay per vehicle (sec/veh)
- \( d_1 \): Average uniform delay per vehicle (sec/veh)
- \( d_2 \): Average incremental delay per vehicle (sec/veh)
- \( d_3 \): Additional delay per vehicle due to preexisting queue (sec/veh)
Uniform Delay ($d_1$)

- Will define this few slides later

\[ d_1 = \frac{0.5C \left[ 1 - \left( \frac{g}{C} \right) \right]^2}{1 - \left[ \min(1, X) \cdot \frac{g}{C} \right]} \]

- C: Cycle length, sec
- g: Effective green time for lane group, sec
- X: v/c ratio for lane group (max, 1.0)
Incremental Delay \( (d_2) \)

- Resulted from random arrivals

\[
d_2 = 900T + \left[ (X - 1) + \sqrt{(X - 1)^2 + \left( \frac{8kIX}{cT} \right)} \right]
\]

- **T**: Analysis time period
- **X**: v/c ratio for lane group
- **c**: Capacity of a lane group
- **k**: Adjustment factor for a type of controller
- **I**: Upstream filtering/metering adjustment factor
Initial Queue Delay \( (d_3) \)

- Delay because of initial queue

\[
d_3 = \frac{3600}{vT} \left( t \frac{Q_b + Q_e - Q_{eo}}{2} + \frac{Q_e^2}{2c} - \frac{Q_{eo}^2}{2c} - \frac{Q_b^2}{2c} \right)
\]

- \( Q_b \): Initial queue at beginning of analysis period
- \( Q_e = Q_b + t(v - c) \)
- \( T \): Analysis period, h
- \( t \): Duration of unmet demand
- \( v \): Demand flow rate during analysis period
- \( Q_{eo} \): Queue at the end of first saturation analysis period when \( v > c \) and \( Q_b = 0 \)
- \( Q_{eo} = T(v-c) \text{ if } v \geq c \)
- \( Q_{eo} = 0 \text{ if } v < c \),
Aggregating Delay

- Lane group delays can be aggregated

\[ d_A = \frac{\sum_i d_i v_i}{\sum_i v_i} \]

\[ d_I = \frac{\sum_A d_A v_A}{\sum_A v_A} \]

- \( d_i \): total control delay per vehicle, lane group \( i \), sec/veh
- \( d_A \): total control delay per vehicle, approach \( A \), sec/veh
- \( d_I \): total control delay per vehicle, for intersection \( I \) as a whole, s/veh
- \( v_A \): Demand flow rate, approach \( A \)
- \( v_i \): Demand flow rate, lane group \( i \)
Interpreting Results

- After completion of HCM analysis the traffic engineer has the following to review:
  - v/c ratio (X) for every lane group
  - Critical v/c ratio for the intersection as a whole
  - Delays and LOS for each lane group
  - Delays and LOS for each approach
  - Delay for the overall intersection
HV% = 10% in all mvts.
100 peds/h in all crosswalks.
Crosswalk widths = 10 ft
Grades: NB +3%; EB level.

PHF = 0.90.
No RTOR.
Arrival Types: NB = 3; EB = 5.
Area Type: Fringe Area

Signal Timing

<table>
<thead>
<tr>
<th>Phase</th>
<th>G (s)</th>
<th>y (s)</th>
<th>ar (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EB</td>
<td>26.4</td>
<td>3.6</td>
<td>2.0</td>
</tr>
<tr>
<td>NB</td>
<td>22.0</td>
<td>3.6</td>
<td>2.4</td>
</tr>
</tbody>
</table>

C = 60 s
HW#6

• Compute LOS for one signalized intersection in your project study area
Forms of Delay

- **Stopped time delay**
  - Defined as the time of a vehicle is stopped in queue while waiting to pass through the intersection
  - Average stop time delay is the average of all vehicles during a specified time period

- **Approach delay**
  - Included stopped delay
  - but adds the time loss due to deceleration from the approach speed to a stop and the time loss due to reacceleration back to desired speed
Forms of Delay (2)

- Time-in-queue delay
  - Defined as the total time from a vehicle joining an intersection queue to its discharge across the STOP line on departure

- Travel time delay
  - More of a conceptual value
  - Difference between driver’s expected travel time through the intersection and the actual time taken.
  - Difficult to obtain “desired” value, so this is a philosophical concept

- Control delay
  - Delay caused by the control device (either a traffic signal or stop sign)
  - Approximately = time-in-queue delay + acceleration-deceleration delay
Delay Measures

Average delay is measured as sec/vehicle

D1 = stopped-time delay
D2 = approach delay
D3 = travel time delay
Basic Theoretical Models of Delay

```
Cumulative Vehicles

Slope = v

Q(t)

W(i)

Veh i

Slope = s

Aggregate Delay (veh-secs)

Time (secs)

G

R

Time t

G
```
Delay Components

- Assuming no pre-existing queue vehicles arriving when the light is green continue through the intersection.
- When the light turn RED, vehicles arrive but do not depart.
- Thus departure curve is parallel to the x-axis during RED interval.
Delay Components (1)

• When the next effective GREEN begins, vehicles queued during RED intervals depart from the intersection

• Departure curve “catches up” with the arrival curve before the next RED interval begins
Delay Components (2)

- The total time that any vehicle “i” spends waiting in the queue, $W(i)$ is given by the horizontal time-scale difference between the time of arrival and the time of departure.

- The total number of vehicles queued at any time $t$, $Q(t)$, is the vertical scale difference between the number of vehicles that have arrived and the number of vehicles that have departed.
Delay Components (3)

• The average delay for all vehicles passing through the signal is the area between the arrival and departure curve (vehicles x time)
Delay Scenario (1)
Delay Scenario (2)

Cumulative Vehicles, $i$

arrival function, $a(t)$

departure function, $d(t)$

(b) Individual Cycle Failures Within a Stable Operation
Delay Scenario (3)

Cumulative Vehicles, $i$

arrival function, $a(t)$
slope = $s$

capacity function, $c(t)$
slope = $c$

departure function, $d(t)$

Time, $t$

(c) Demand Exceeds Capacity for a Significant Period
Arrival Patterns

- Uniform Arrivals
  (a)

- Random Arrivals
  (b)

- Reality = Platoon arrivals—No theoretical solution available
  (c)
Components of Delay

- **Uniform Delay**
  - Delay based on an assumption of uniform arrivals and stable flow with no individual cycle failures

- **Random Delay**
  - Additional delay above and beyond uniform delay because flow is randomly distributed rather than uniform at isolated intersection

- **Overflow Delay**
  - Additional delay that occurs when the capacity of an individual phase or series of phases is less than the demand or arrival flow rate
Webster’s Uniform Delay

Cumulative Vehicles

\[ R = C \left[1 - \frac{g}{C}\right] \]

\[ \text{Slope} = v \]

\[ \text{Slope} = s \]

Aggregate Delay (veh-secs)

Time (secs)

G R G
Uniform Delay
Derivation of Uniform Delay
Random Delay

- The uniform delay model assumes that arrivals are uniform and that no signal phases fail, i.e.
- Arrival flow is less than capacity during every signal cycle of the analysis period
- At isolated intersections, vehicle arrivals are more likely to be random
- A number of stochastic models have derived
- Such models assume that inter-vehicle arrival times are distributed according to Poisson distribution with underlying average arrival rate of $v$ vehicles per unit time
Random Delay

- Such models account for both the underlying randomness of arrivals and the fact that some individual cycles could fail because of randomness.
- The additional delay is referred as “overflow delay”, but it does not address $v/c>1.0$.
- The most frequently used random delay as per Webster’s formulation as below.
Total Delay
Overflow Delay

• Oversaturation is used to describe extended time periods during which arriving vehicles exceed capacity of the intersection approach to discharge vehicles.

• In such cases queues grow and overflow delay, in addition to uniform delay accrues.

• Because overflow delay accounts for the failure of an extended series of phases, it encompasses a portion of random delay as well.
Delay in Oversaturated Period

Cumulative Vehicles

Overflow Delay

Uniform Delay

Slope = v

Slope = c

Slope = s

Time
Uniform Delay when \( X=1 \)

- Uniform delay when \( v/c = 1.0 \) is referred as \( UD_0 \)
Overflow Delay

Cumulative Vehicles

\begin{align*}
&\text{Slope } = v \\
&\text{Slope } = c
\end{align*}

\begin{align*}
&vT \\
&cT
\end{align*}

Time
Overflow Delay for a Time Period
Overflow Delay Between Two Time Periods

Cumulative Vehicles

\[ \nu T_2 \]

\[ \nu T_1 \]

\[ c T_2 \]

\[ c T_1 \]

Slope = \( \nu \)

Slope = \( c \)

Time

\( T_1 \)

\( T_2 \)
Inconsistencies in Random and Overflow Delay (1)

- The inconsistency occurs when $v/c$ is in the vicinity of 1.0
Inconsistencies in Random and Overflow Delay (2)

• If the v/c ratio is below 1.0, then a random delay model is being used
  - Because there is no overflow delay in this case

• As X approaches 1.0, random delay increases asymptotically
Inconsistencies in Random and Overflow Delay (3)

- When v/c ratio is greater than 1.0, then overflow delay model is applied.

- However, when X=1, \( OD=0 \)
- But increases uniformly with increasing values of X thereafter.
Inconsistencies in Random and Overflow Delay (4)

- Neither model is accurate in the vicinity of $X=1$

- In terms of practical terms, most studies confirms that the uniform delay is a sufficient predictive tool (except the issue with platooned arrivals) when the $v/c$ ratio is 0.85 or less.

- In this range the true value of random delay is miniscule and there is no overflow delay.
Inconsistencies in Random and Overflow Delay (5)

• Similarly, the simple theoretical overflow delay is a reasonable predictor when $v/c \geq 1.15$

• The problem is that the most interesting case fall in the intermediate range
  - $0.85 < v < 1.15$

• For which neither model is adequate

• Much of the recent work in delay modeling attempts to bridge this gap
Example-1

- An intersection approach has an flow rate of 1000 veh/hr, a saturation flow rate of 2,800 veh/hr/gr, a cycle length of 90 sec, and g/C ratio of 0.55. What average delay per vehicle is expected under these condition?