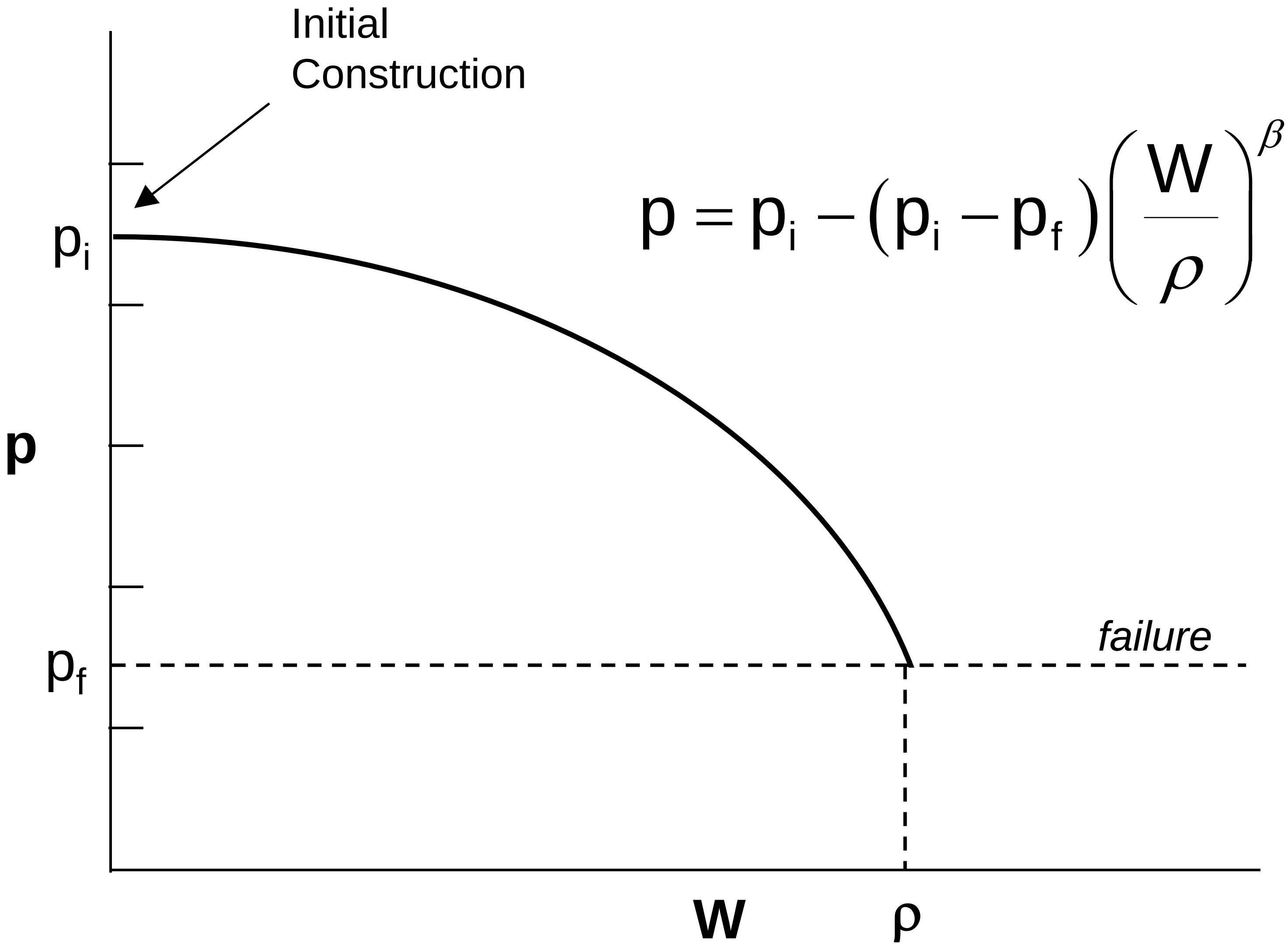
AASHTO Design Equation

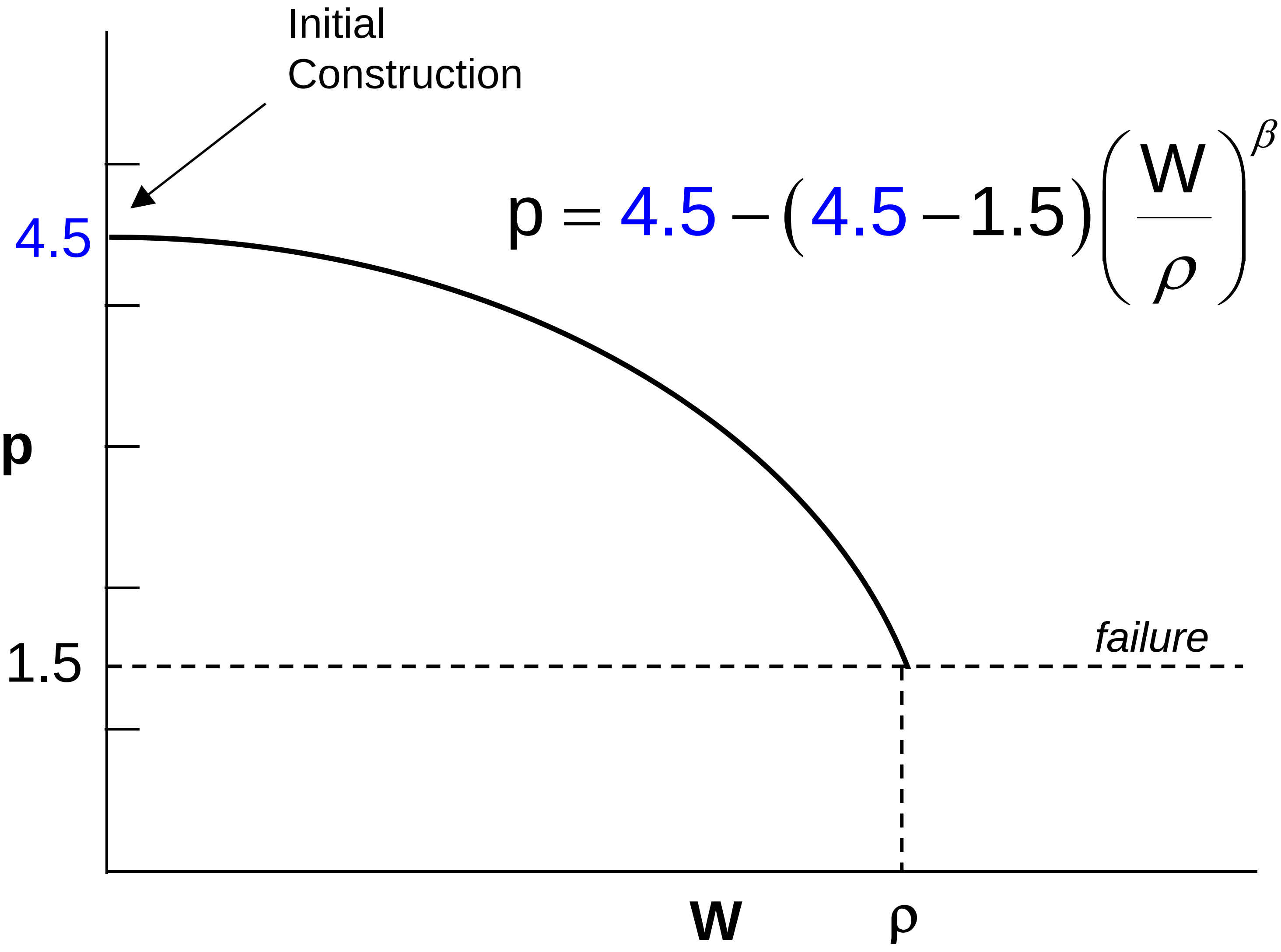
(Rigid Pavements)

Pavement Performance



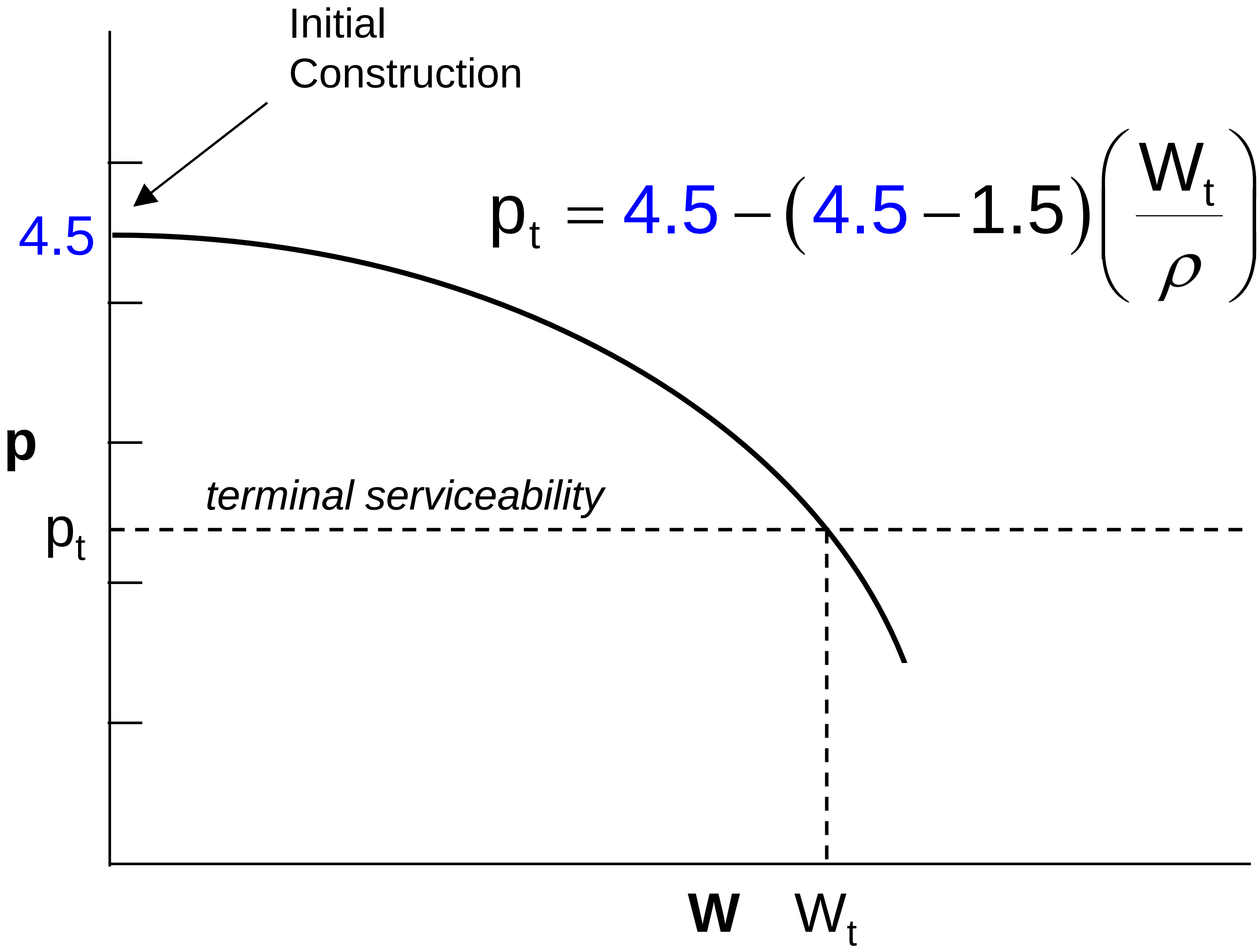
$$p = p_i - \left(p_i - p_f\right)\left(\frac{W}{\rho}\right)^{\beta}$$

Pavement Performance



$$p = 4.5 - (4.5 - 1.5)\left(\frac{W}{\rho}\right)^{\beta}$$

Pavement Performance



$$p_t = 4.5 - (4.5 - 1.5)\left(\frac{W_t}{\rho}\right)^{\beta}$$

Rigid Performance

$$\beta = 1.0 + \frac{3.63\left(\overset{18}{\cancel{L_1}} + \overset{1}{\cancel{L_2}}\right)^{5.20}}{\left(D+1\right)^{8.46} \underset{1}{\cancel{L_2}}^{3.52}}$$

$$\rho = \frac{10^{5.85}\left(D+1\right)^{7.35} \overset{1}{\cancel{L_2}}^{3.28}}{\left(\underset{18}{\cancel{L_1}} + \underset{1}{\cancel{L_2}}\right)^{4.62}}$$

Rigid Performance

$$\beta_{18} = 1.0 + \frac{1.62 \times 10^{7}}{\left(D+1\right)^{8.46}}$$

$$\rho_{18} = 0.875\left(D+1\right)^{7.35}$$

Rigid Performance

$$p_t = 4.5 - (4.5 - 1.5)\left(\frac{W_{18}}{\rho_{18}}\right)^{\beta_{18}}$$

$$\Downarrow$$

$$4.5 - p_t = (4.5 - 1.5)\left(\frac{W_{18}}{\rho_{18}}\right)^{\beta_{18}}$$

Rigid Performance

$$\frac{4.5 - \mathrm{p_t}}{4.5 - 1.5} = \left(\frac{\mathrm{W}_{18}}{\rho_{18}}\right)^{\beta_{18}}$$

$$\Downarrow$$

$$\underbrace{\log_{10}\left(\frac{4.5 - \mathrm{p_t}}{4.5 - 1.5}\right)}_{\mathrm{G_t}} = \beta_{18} \log_{10}\left(\frac{\mathrm{W}_{18}}{\rho_{18}}\right)$$

Rigid Performance

$$\log_{10}\left(\frac{4.5-\mathrm{p_t}}{4.5-1.5}\right)=\beta_{18}\left[\log_{10}\left(\mathrm{W}_{18}\right)-\log_{10}\left(\rho_{18}\right)\right]$$

$$\Downarrow$$

$$\frac{\log_{10}\left(\frac{4.5-\mathrm{p_t}}{4.5-1.5}\right)}{\beta_{18}}=\log_{10}\left(\mathrm{W}_{18}\right)-\log_{10}\left(\rho_{18}\right)$$

Rigid Performance

$$\log_{10}\left(\mathrm{W}_{18}\right)=\log_{10}\left(\rho_{18}\right)-\frac{\log_{10}\left(\frac{4.5-\mathrm{p}_{\mathrm{t}}}{4.5-1.5}\right)}{\beta_{18}}$$

$$\Downarrow$$

$$\log_{10}\left(\mathrm{W}_{18}\right)=\log_{10}\left(\rho_{18}\right)-\frac{\log_{10}\left(\frac{4.5-\mathrm{p}_{\mathrm{t}}}{4.5-1.5}\right)}{1.0+\frac{1.62\times10^{7}}{\left(\mathrm{D}+1\right)^{8.46}}}$$

Rigid Performance

$$\rho_{18} = 0.875\left(D+1\right)^{7.35}$$

$$\Downarrow$$

$$\log_{10}\left(\rho_{18}\right) = \overset{-0.06}{\cancel{\log_{10}\left(0.875\right)}} + 7.35\log_{10}\left(D+1\right)$$

Design Equation

$$\log_{10} W_{18} = 7.35 \log_{10}\left(D+1\right) - 0.06 + \frac{\log_{10}\left(\dfrac{4.5 - p_t}{4.5 - 1.5}\right)}{1.0 + \dfrac{1.62 \times 10^{7}}{\left(D+1\right)^{8.46}}}$$

Design Equation

$$\log_{10} W_{18} = 7.35 \log_{10}(D+1) - 0.06 + \frac{\log_{10}\left(\frac{\Delta PSI}{4.5-1.5}\right)}{1.0 + \frac{1.62\times 10^{7}}{(D+1)^{8.46}}}$$

1966

Design Equation

$$\log_{10} W_{18} = 7.35 \log_{10}\left(D+1\right) - 0.06 + \frac{\log_{10}\left(\frac{\Delta PSI}{4.5-1.5}\right)}{1.0 + \frac{1.62\times 10^{7}}{\left(D+1\right)^{8.46}}}$$

Concrete Tensile Strength (psi)

1972

$$+\left(4.22-0.32p_{t}\right)\log_{10}\left[\left(\frac{f_{t}}{690}\right)\frac{\left(D^{0.75}-1.132\right)}{\left(D^{0.75}-\frac{18.42}{\sqrt[4]{E_{c}/k}}\right)}\right]$$

Concrete elastic modulus (psi)

Effective modulus of subgrade reaction (psi/in)

Design Equation

$$\log_{10} W_{18} = Z_R S_0 + 7.35\log_{10}(D+1) - 0.06 + \frac{\log_{10}\left(\frac{\Delta PSI}{4.5-1.5}\right)}{1.0+\frac{1.62\times 10^{7}}{(D+1)^{8.46}}}$$

Concrete MOR (psi)

Drainage Coefficient

1993

$$+(4.22-0.32p_t)\log_{10}\left[\left(\frac{S'_c\cdot C_d}{215.63\cdot J}\right)\frac{\left(D^{0.75}-1.132\right)}{\left(D^{0.75}-\frac{18.42}{\sqrt[4]{E_c/k}}\right)}\right]$$

Joint Load Transfer Coefficient

Drainage Coefficient, C_d

Quality	Percentage of Time Material Approaches Saturation			
	< 1%	**1-5%**	**5-25%**	**> 25%**
Excellent	1.25-1.20	1.20-1.15	1.15-1.10	1.10
Good	1.20-1.15	1.15-1.10	1.10-1.00	1.00
Fair	1.15-1.10	1.10-1.00	1.00-0.90	0.90
Poor	1.10-1.00	1.00-0.90	0.90-0.80	0.80
Very Poor	1.00-0.90	0.90-0.80	0.80-0.70	0.70

Drainage Quality

Quality	**Water Removed Within**
Excellent	2 hours
Good	1 day
Fair	1 week
Poor	1 month
Very Poor	No drainage

Load Transfer Coefficient, J

	Asphalt Shoulders		Tied PCC Shoulders	
	Dowels	No Dowels	Dowels	No Dowels
JPCP	3.2	3.8 - 4.4	2.5 - 3.1	3.6 - 4.2
JRCP				
CRCP	2.9 - 3.2		2.3 - 2.9	

NOTE: Use higher *J* values when you have (a) low *k* values, (b) lots of trucks, (c) high concrete thermal coefficients, or (d) large variations in temperature.

Recommended Reliability

Functional Classification	**Urban**	**Rural**
Interstate and Other Freeways	85 – 99.9%	80 – 99.9%
Principal Arterials	80 – 99%	75 – 99%
Collectors	80 – 95%	75 – 95%
Local	50 – 80%	50 – 80%

Reliability, Z_R

Reliability (%)	Z_R
50	-0.000
60	-0.253
70	-0.524
75	-0.674
80	-0.841
85	-1.037
90	-1.282
91	-1.340
92	-1.405

Reliability (%)	Z_R
93	-1.476
94	-1.555
95	-1.645
96	-1.751
97	-1.881
98	-2.054
99	-2.327
99.9	-3.090
99.99	-3.750

Standard Deviation, S_o

Source	**Flexible**	**Rigid**
AASHO Road Test S_n	0.35	0.25
AASHO Road Test S_o	0.45	0.35
Typical Range for S_o	0.40 – 0.50	0.30 – 0.40

Typical Concrete Properties

$$S'_c = 8.4\sqrt{f'_c} \quad (psi)$$

$$f_t = 6.7\sqrt{f'_c} \quad (psi)$$

$$E_c = 57,000\sqrt{f'_c} \quad (psi)$$

Textbook Example

$k = 72\ psi/in$

$E_c = 5\times10^6\ psi$

$S'_c = 650\ psi$

$J = 3.2$

$C_d = 1.0$

$S_o = 0.29$

$R = 95\%$

$p_i = 4.2$

$p_t = 2.5$

$W_{18} = 5.1\times10^6$

Reliability, Z_R

Reliability (%)	Z_R
50	-0.000
60	-0.253
70	-0.524
75	-0.674
80	-0.841
85	-1.037
90	-1.282
91	-1.340
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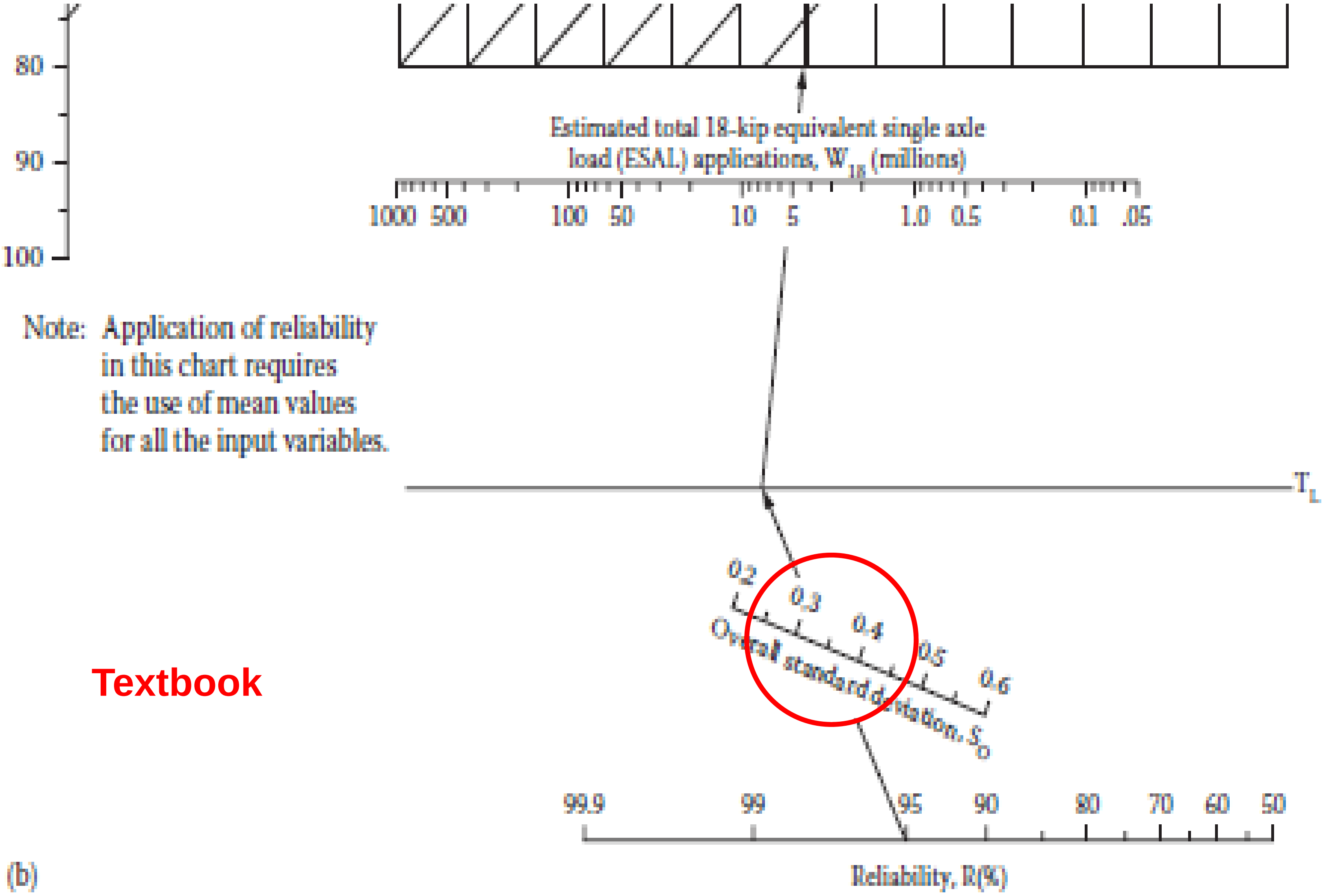
Reliability (%)	Z_R
93	-1.476
94	-1.555
95	-1.645
96	-1.751
97	-1.881
98	-2.054
99	-2.327
99.9	-3.090
99.99	-3.750

Design Equation

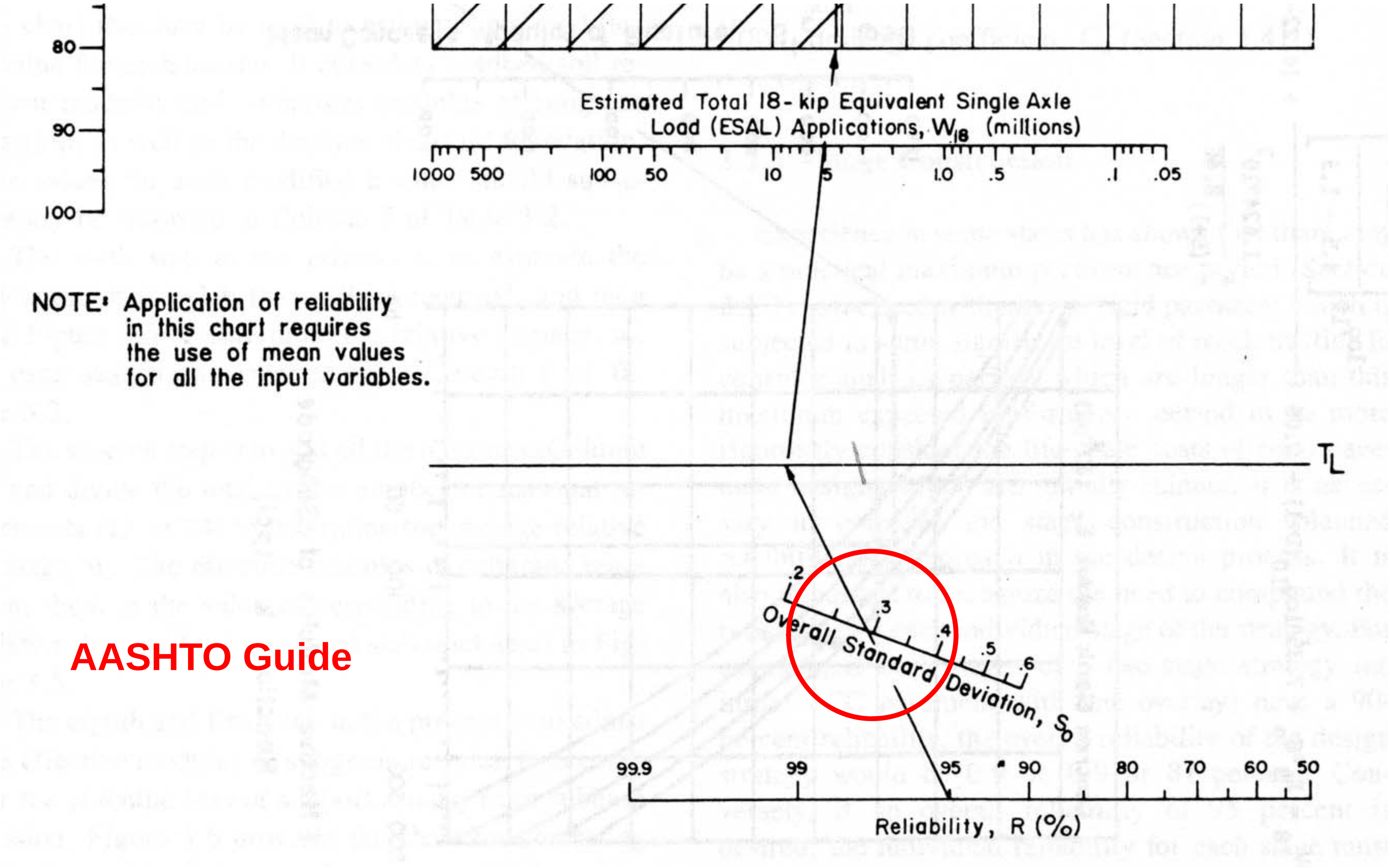
$$\log_{10} W_{18} = \overset{-1.645}{\cancel{Z_R}}\,\overset{0.29}{\cancel{S_0}} + 7.35\log_{10}(D+1) - 0.06 + \frac{\log_{10}\left(\dfrac{\overset{4.2-2.5=1.7}{\cancel{\Delta PSI}}}{4.5-1.5}\right)}{1.0+\dfrac{1.62\times10^{7}}{(D+1)^{8.46}}}$$

1993

$$+\left(4.22-0.32\overset{2.5}{\cancel{p_t}}\right)\log_{10}\left[\left(\frac{\overset{650}{\cancel{S_c'}}\cdot\overset{1.0}{\cancel{C_d}}}{215.63\cdot\underset{3.2}{\cancel{J}}}\right)\frac{\left(D^{0.75}-1.132\right)}{\left(D^{0.75}-\dfrac{18.42}{\sqrt[4]{\underset{5\times10^{6}}{\cancel{E_c}}/\underset{72}{\cancel{k}}}}\right)}\right]$$


80
90
100
Estimated total 18-kip equivalent single axle load (ESAL) applications, W_{18} (millions)
1000
500
100
50
10
5
1.0
0.5
0.1
.05
Note: Application of reliability in this chart requires the use of mean values for all the input variables.
T_L
0.2
0.3
0.4
0.5
0.6
Overall standard deviation, S_O
99.9
99
95
90
80
70
60
50
Reliability, R(%)
(b)

Textbook



NOTE: Application of reliability in this chart requires the use of mean values for all the input variables.

AASHTO Guide

Factors Affecting k_{eff}

D = *slab thickness (in)*

M_R = *subgrade modulus of rupture (psi)*

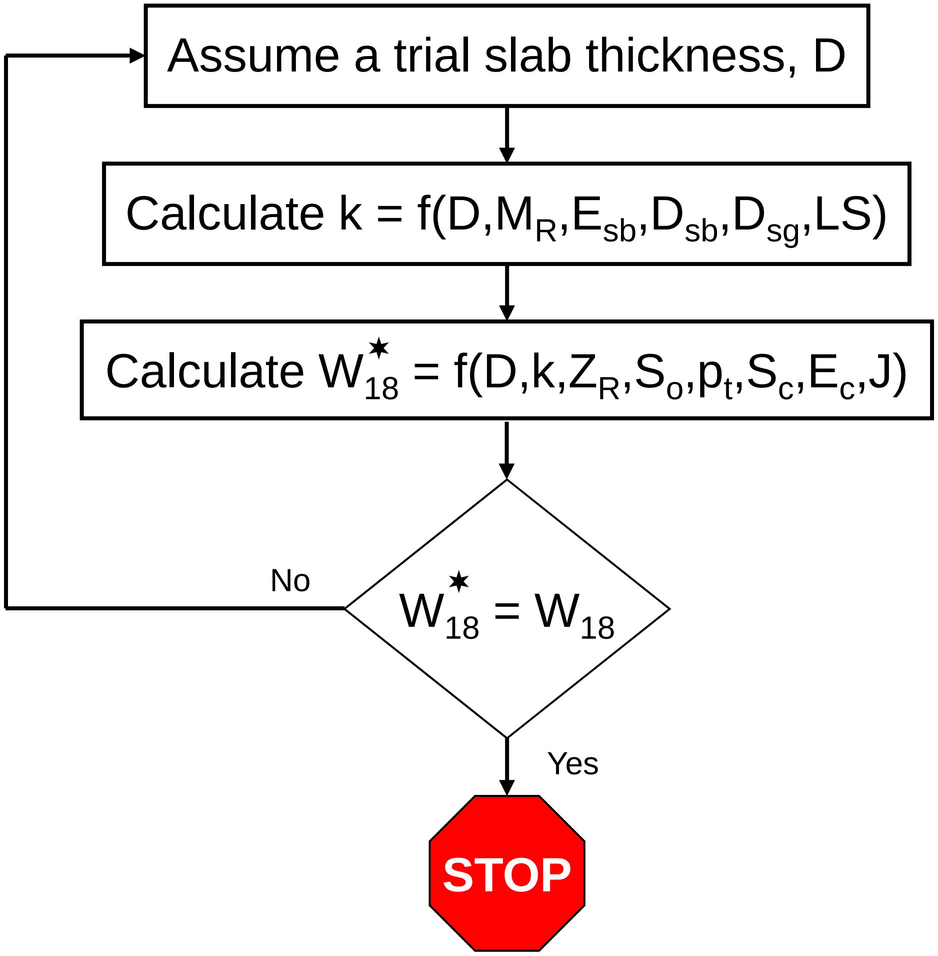
E_{sb} = *subbase elastic modulus (psi)*

D_{sb} = *subbase thickness (in)*

D_{sg} = *subgrade thickness (in)*

LS = *loss of support factor*

Solution Procedure



Assume a trial slab thickness, D

Calculate $k = f(D,M_R,E_{sb},D_{sb},D_{sg},LS)$

Calculate $W^{\star}_{18} = f(D,k,Z_R,S_o,p_t,S_c,E_c,J)$

No

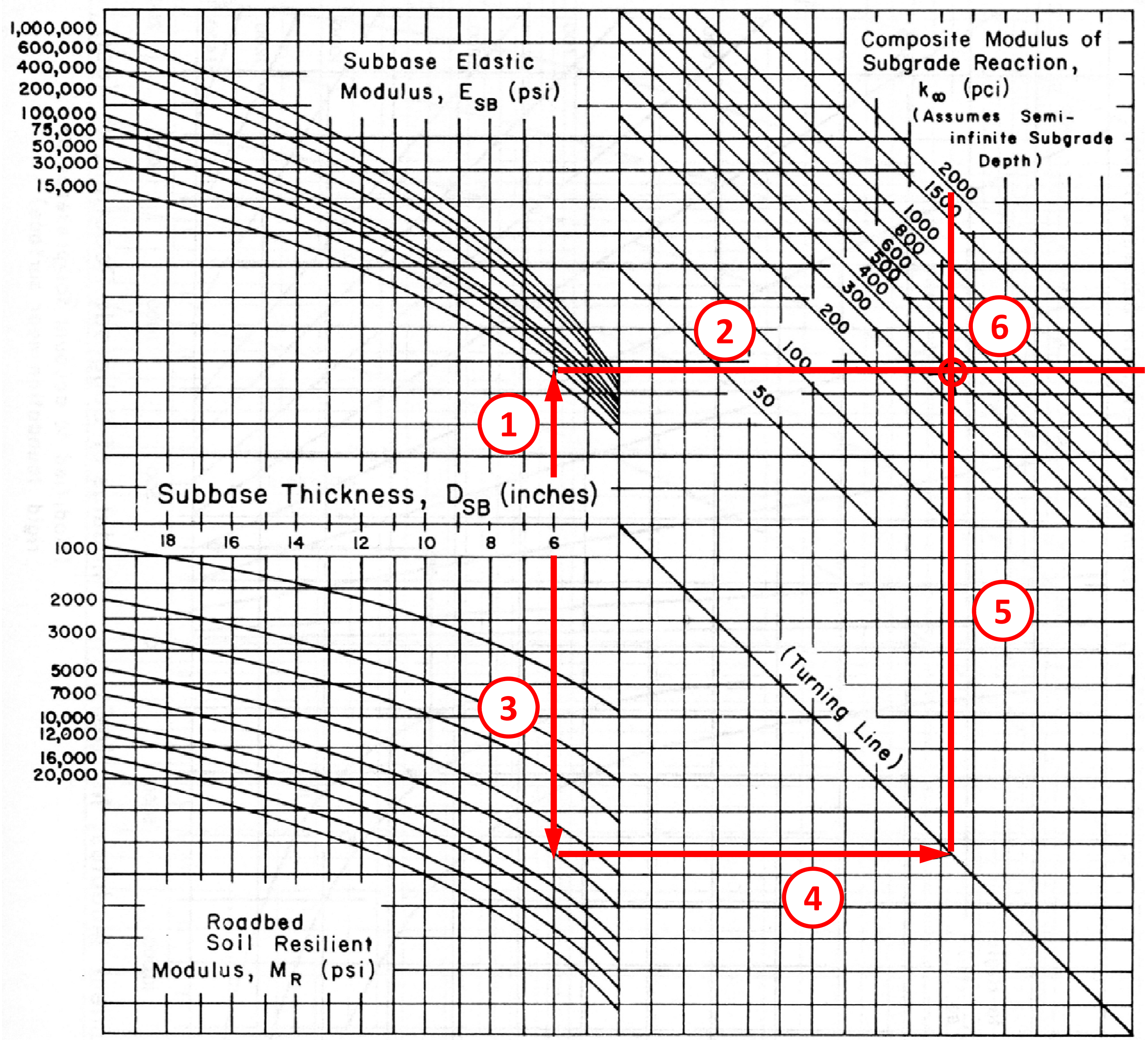
$W^{\star}_{18} = W_{18}$

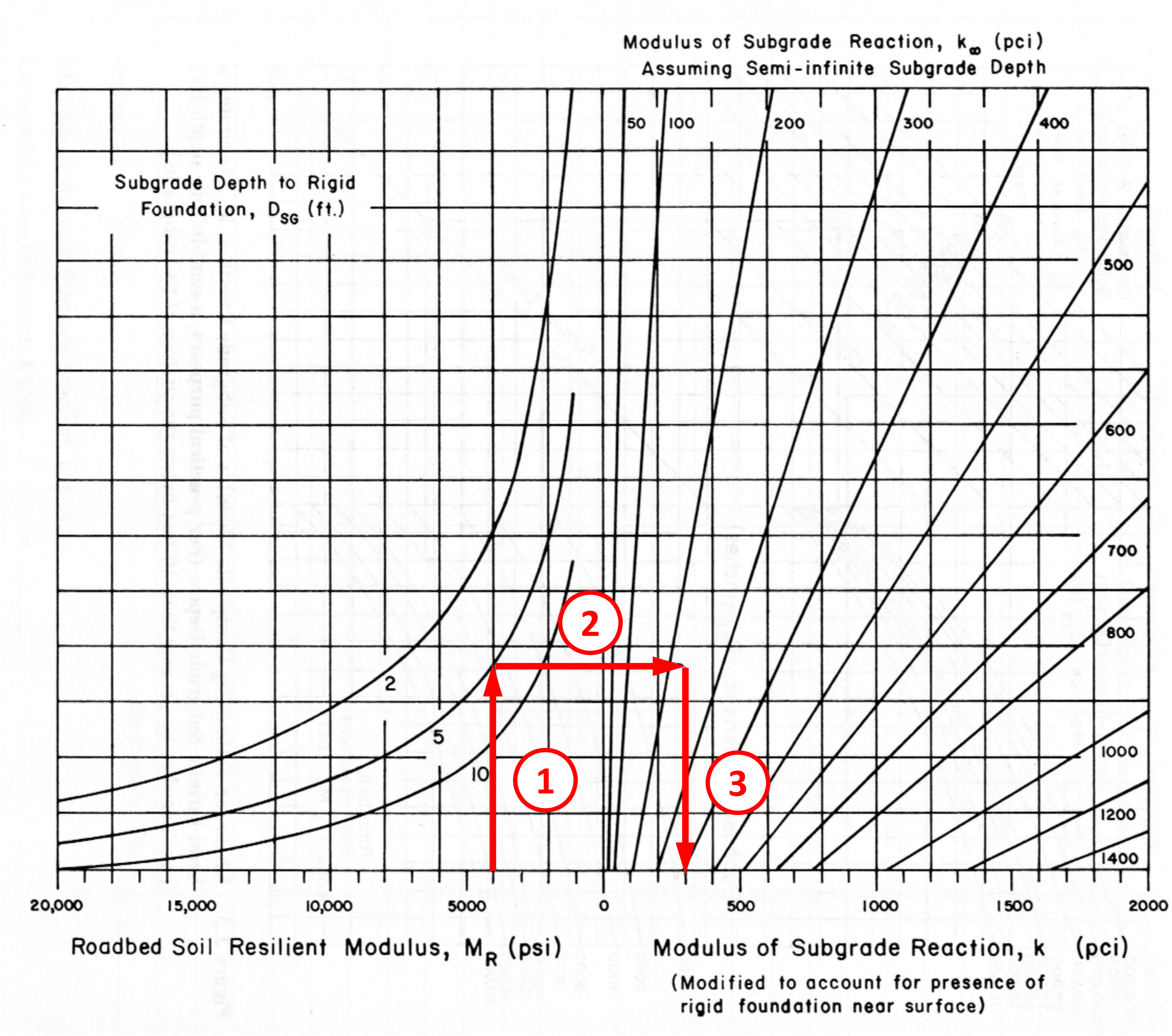
Yes

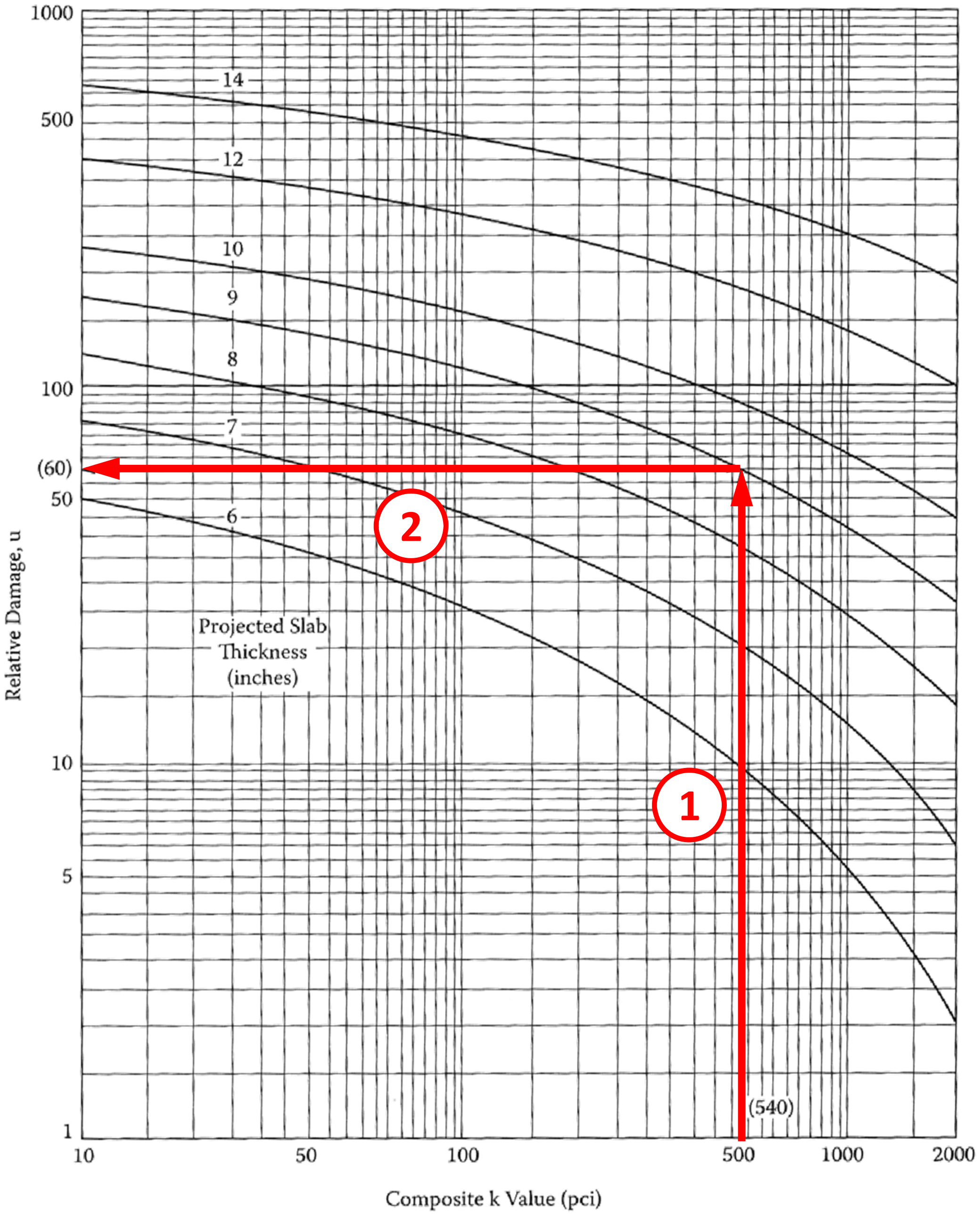
STOP

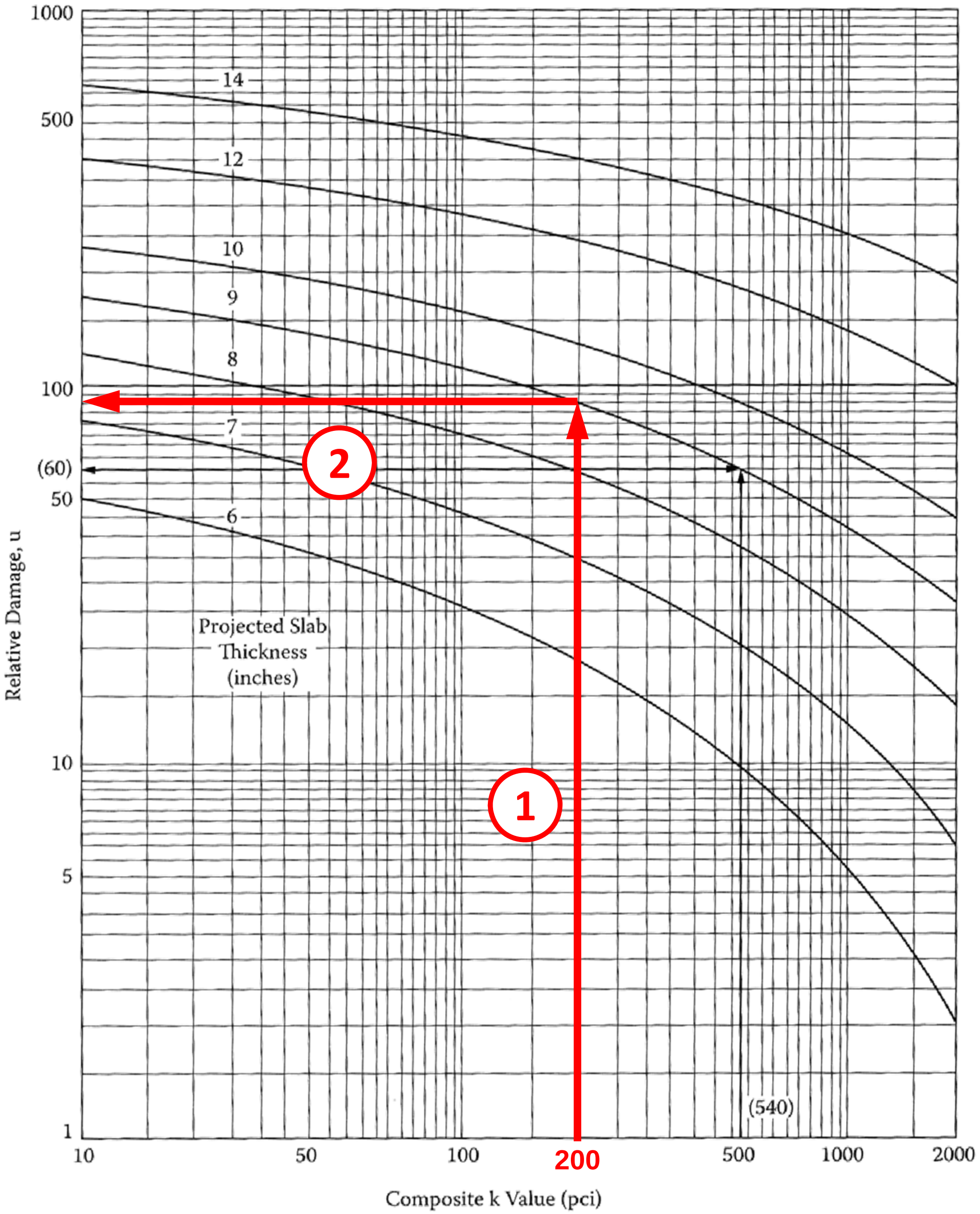
Calculating k_{eff}

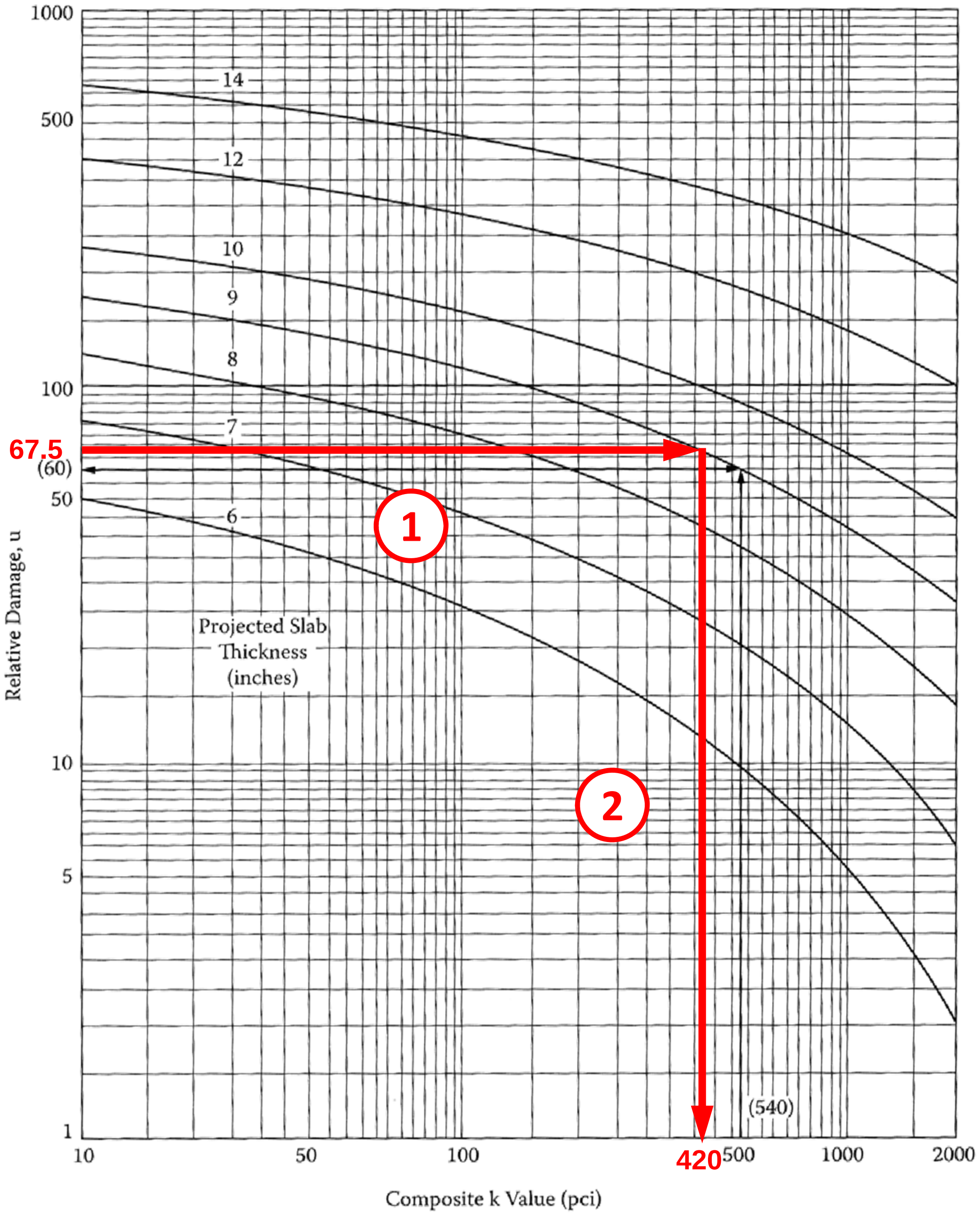
1. Determine seasonal values for M_R and E_{SB}
2. Calculate composite k values assuming no bedrock
3. Adjust k values for shallow bedrock (if needed)
4. For a given slab thickness, D
    a) Convert the k values to damage factors (u_f)
    b) Calculate an average damage factor (u_{avg})
    c) Convert u_{avg} into a year-round k_{eff} value
    d) Adjust k_{eff} for loss of support over time


1,000,000
600,000
400,000
200,000
100,000
75,000
50,000
30,000
15,000
Subbase Elastic Modulus, E_{SB} (psi)
Composite Modulus of Subgrade Reaction, k_{∞} (pci)
(Assumes Semi-infinite Subgrade Depth)
2000
1500
1000
800
600
500
400
300
200
100
50
Subbase Thickness, D_{SB} (inches)
18
16
14
12
10
8
6
1000
2000
3000
5000
7000
10,000
12,000
16,000
20,000
(Turning Line)
Roadbed Soil Resilient Modulus, M_R (psi)
1
2
3
4
5
6


Modulus of Subgrade Reaction, k_{∞} (pci)
Assuming Semi-infinite Subgrade Depth
50
100
200
300
400
500
600
700
800
1000
1200
1400
Subgrade Depth to Rigid Foundation, D_{SG} (ft.)
2
5
10
①
②
③
20,000
15,000
10,000
5000
0
500
1000
1500
2000
Roadbed Soil Resilient Modulus, M_R (psi)
Modulus of Subgrade Reaction, k (pci)
(Modified to account for presence of rigid foundation near surface)


1000
500
100
(60)
50
10
5
1
14
12
10
9
8
7
6
Projected Slab Thickness (inches)
Relative Damage, u
10
50
100
500
1000
2000
(540)
Composite k Value (pci)
1
2


1000
500
100
(60)
50
10
5
1
14
12
10
9
8
7
6
Projected Slab Thickness (inches)
Relative Damage, u
10
50
100
200
500
1000
2000
(540)
Composite k Value (pci)
①
②


1000
500
100
50
10
5
1
67.5
(60)
14
12
10
9
8
7
6
Projected Slab Thickness (inches)
Relative Damage, u
①
②
(540)
10
50
100
420
500
1000
2000
Composite k Value (pci)

Convert k to Damage Factor

$$u_i = \left[D^{0.75} - \frac{18.42}{\sqrt[4]{E_c / k_i}} \right]^b$$

$$b = 4.22 - 0.32 \times p_t$$

Convert Damage Factor to k

$$k_{eff} = \frac{E_c}{\left[\dfrac{18.42}{D^{0.75} - \left(\bar{u}\right)^{1/b}}\right]^4}$$

$$b = 4.22 - 0.32 \times p_t$$

Simplified Version

$$\text{Let } E_c = 5\times10^6 \text{ psi and } p_t = 2.5$$

$$u_i = \left[D^{0.75} - 0.39\left(k_i\right)^{0.25} \right]^{3.42}$$

Equation in the textbook is wrong

$$k_{eff} = \left[\frac{D^{0.75} - \left(\bar{u}\right)^{0.292}}{0.39} \right]^{4}$$

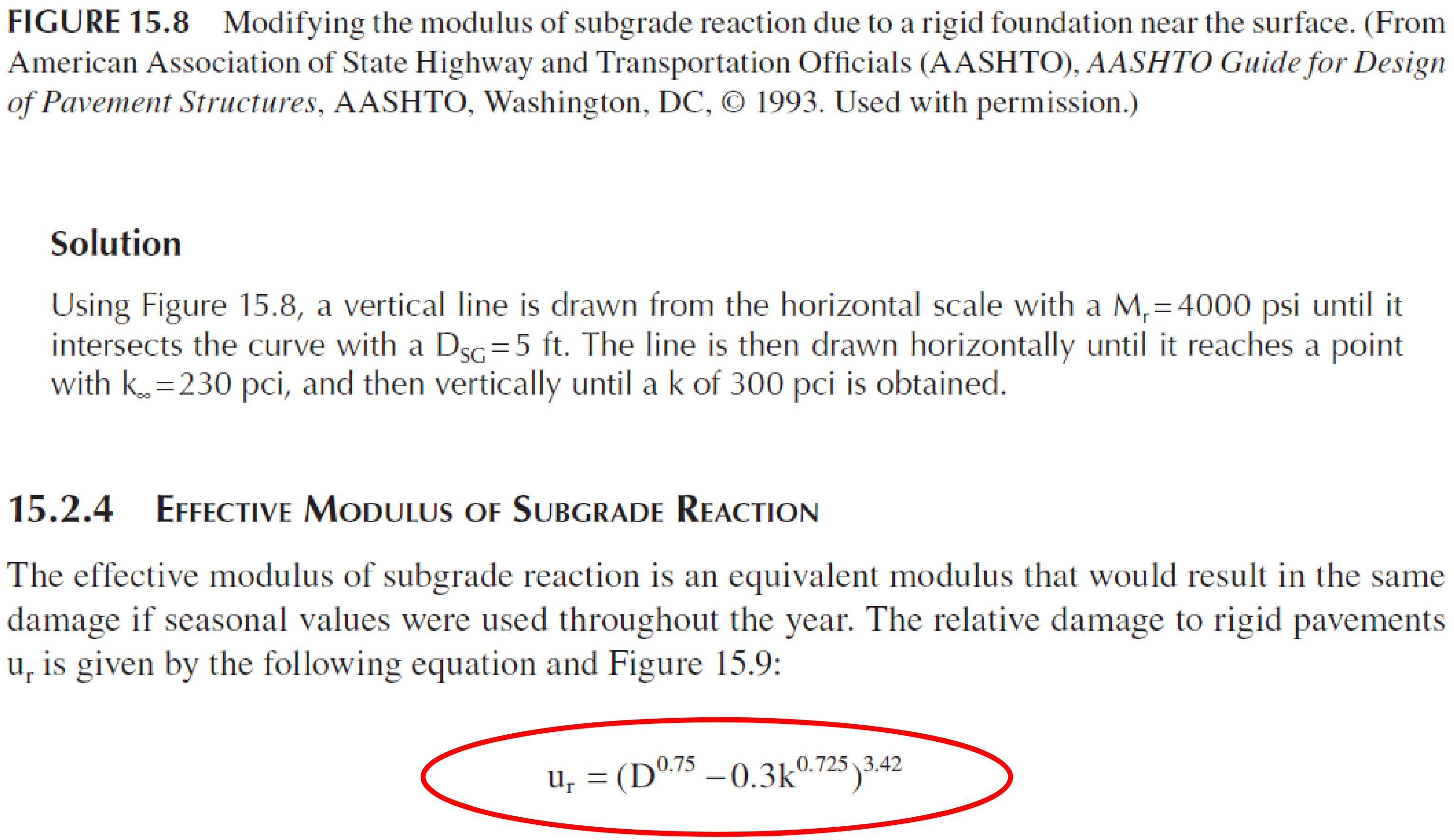


FIGURE 15.8 Modifying the modulus of subgrade reaction due to a rigid foundation near the surface. (From American Association of State Highway and Transportation Officials (AASHTO), *AASHTO Guide for Design of Pavement Structures*, AASHTO, Washington, DC, © 1993. Used with permission.)

Solution

Using Figure 15.8, a vertical line is drawn from the horizontal scale with a $M_r = 4000$ psi until it intersects the curve with a $D_{SG} = 5$ ft. The line is then drawn horizontally until it reaches a point with $k_\infty = 230$ pci, and then vertically until a k of 300 pci is obtained.

15.2.4 Effective Modulus of Subgrade Reaction

The effective modulus of subgrade reaction is an equivalent modulus that would result in the same damage if seasonal values were used throughout the year. The relative damage to rigid pavements u_r is given by the following equation and Figure 15.9:

$$u_r = (D^{0.75} - 0.3k^{0.725})^{3.42}$$

WRONG!

FIGURE 15.9 Estimating relative damage to rigid pavements. (From American Association of State Highway and Transportation Officials (AASHTO), *AASHTO Guide for Design of Pavement Structures*, AASHTO, Washington, DC, © 1993. Used with permission.)

Example 15.4

Given D = 9 in. and k = 540 pci, determine the u_r using the aforementioned equation and Figure 15.9.

Solution

WRONG!

From the equation $u_r = [9^{0.75} - 0.3(540)^{0.725}]^{3.42} = 60.3$, which is similar to the value obtained using Figure 15.9.

To account for a potential loss of support by foundation erosion or differential vertical soil movement, the effective modulus of subgrade reaction must be reduced by the loss of subgrade support (LS) factor as shown in Figure 15.10. Table 15.17 provides recommendations for LS values for different types of subbases and subgrades.

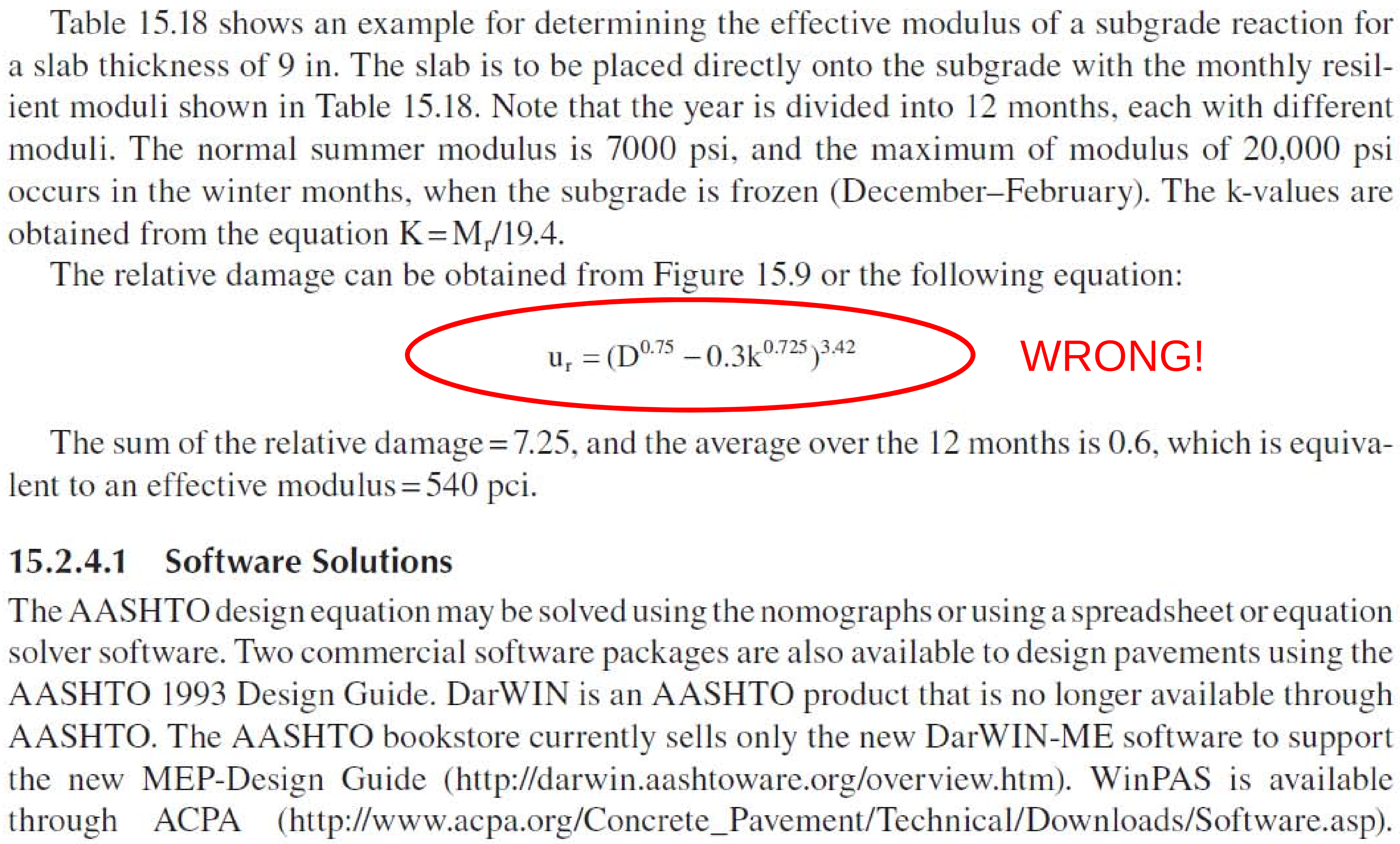


Table 15.18 shows an example for determining the effective modulus of a subgrade reaction for a slab thickness of 9 in. The slab is to be placed directly onto the subgrade with the monthly resilient moduli shown in Table 15.18. Note that the year is divided into 12 months, each with different moduli. The normal summer modulus is 7000 psi, and the maximum of modulus of 20,000 psi occurs in the winter months, when the subgrade is frozen (December–February). The k-values are obtained from the equation $K=M_r/19.4$.

The relative damage can be obtained from Figure 15.9 or the following equation:

$$u_r = (D^{0.75} - 0.3k^{0.725})^{3.42}$$

WRONG!

The sum of the relative damage = 7.25, and the average over the 12 months is 0.6, which is equivalent to an effective modulus = 540 pci.

15.2.4.1 Software Solutions

The AASHTO design equation may be solved using the nomographs or using a spreadsheet or equation solver software. Two commercial software packages are also available to design pavements using the AASHTO 1993 Design Guide. DarWIN is an AASHTO product that is no longer available through AASHTO. The AASHTO bookstore currently sells only the new DarWIN-ME software to support the new MEP-Design Guide (http://darwin.aashtoware.org/overview.htm). WinPAS is available through ACPA (http://www.acpa.org/Concrete_Pavement/Technical/Downloads/Software.asp).

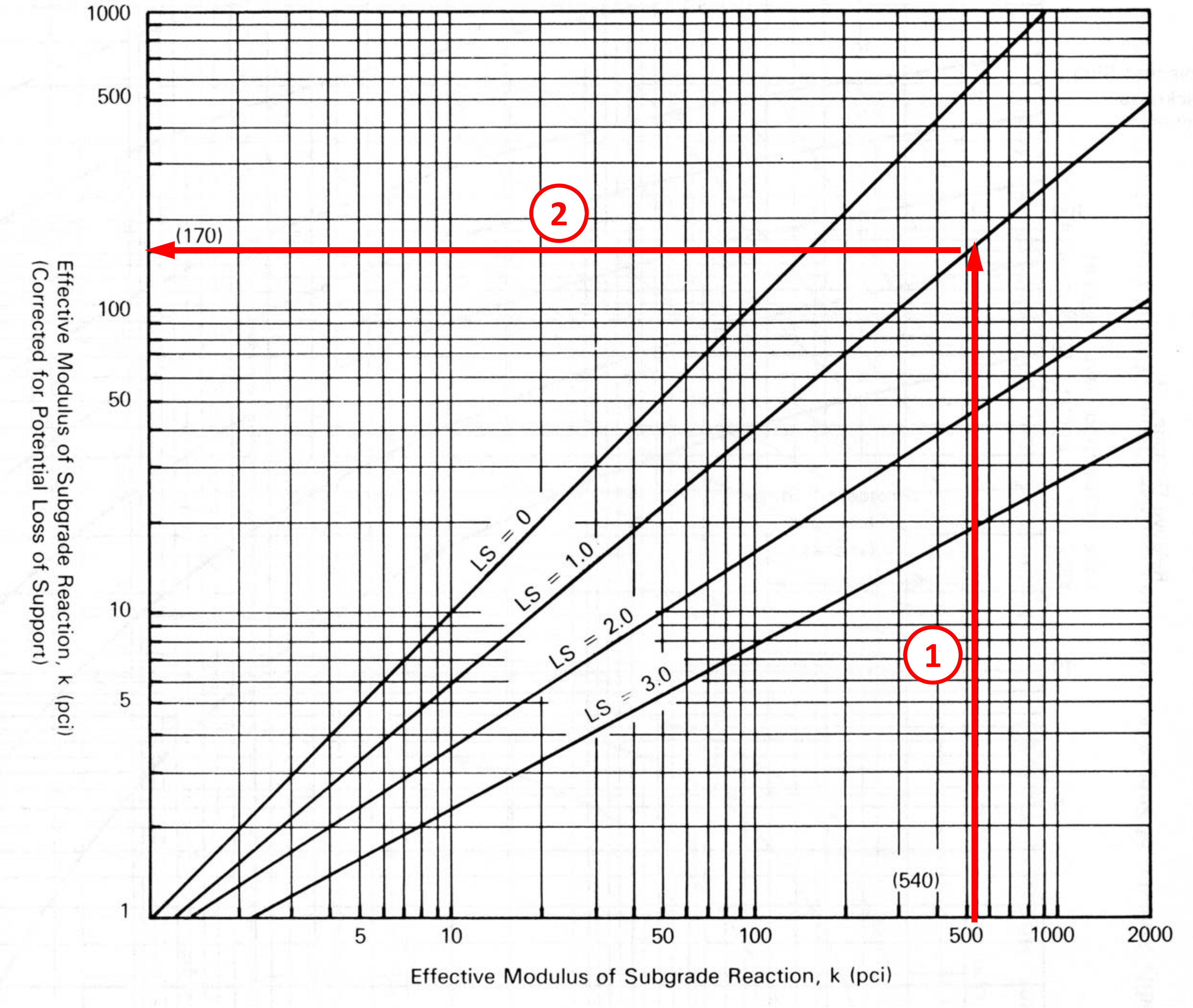
TABLE 15.17

Recommended Loss of Subgrade Support Factors

Type of Material	Loss of Support (LS)
Cement-treated granular base (E = 1,000,000–2,000,000 psi)	0.0–1.0
Cement aggregate mixtures (E = 500,000–1,000,000 psi)	0.0–1.0
Asphalt-treated base (E = 350,000–1,000,000 psi)	0.0–1.0
Bituminous stabilized mixtures (E = 40,000–300,000 psi)	0.0–1.0
Lime stabilized (E = 20,000–70,000 psi)	1.0–3.0
Unbound granular materials (E = 15,000–45,000 psi)	1.0–3.0
Fine-grained or natural subgrade materials (E = 3000–40,000 psi)	2.0–3.0

Source: Reprinted from Portland Cement Association (PCA), *Design and Control of Concrete Mixtures*, Engineering Bulletin 001, 14th edn., PCA, Skokie, IL, 2002.

Note: E in this table refers to the general symbol for the elastic or resilient modulus of the material.


1000
500
100
50
10
5
1
5
10
50
100
500
1000
2000
Effective Modulus of Subgrade Reaction, k (pci)
(Corrected for Potential Loss of Support)
Effective Modulus of Subgrade Reaction, k (pci)
LS = 0
LS = 1.0
LS = 2.0
LS = 3.0
(170)
(540)
2
1