Rigid Pavement Mechanics

Dowel Bars
Allowable Bearing Stress

ACI Empirical Equation

\[ f_b = \left( \frac{4 - d}{3} \right) f_c' \]

- \( f_b \) = allowable bearing stress
- \( f_c' \) = concrete compressive strength
- \( d \) = dowel bar diameter (in)
Bar Deformation Under Load
Bar Deformation Under Load

Freberg (1940)

\[ y_o = \frac{P_c \left( 2 + \beta z \right)}{4 \beta^3 E_d I_d} \]

- \( P_c \) = load on the critical dowel bar
- \( \beta \) = relative dowel/concrete stiffness
- \( z \) = maximum joint opening (0.25 in)
Actual Bearing Stress

Freberg (1940)

$$\beta = \frac{4\sqrt[4]{Kd}}{4E_d I_d}$$

$$I_d = \frac{\pi d^4}{64}$$

$E_d = \text{elastic modulus of the dowel}$

$I_d = \text{moment of inertia of the dowel}$

$d = \text{bar diameter}$
Bearing Stress Under Load

Freberg (1940)

\[ \sigma_b = K y_o = \frac{K P_c \left(2 + \beta z\right)_t}{4 \beta^3 E_d I_d} \]

\( K = \text{modulus of dowel support} \)

\( (300 - 1500 \ \text{ksi/in}) \)
Modulus of Dowel Support

Freberg (1940)

\[
\sigma_b \propto \frac{K}{4\beta^3 E_d I_d} = \frac{K}{4EI} \left( \frac{Kd}{4EI} \right)^3 = \left( d^{\frac{4}{3}} \right) \frac{1}{4} \sqrt[4]{\left( \frac{K}{4EI} \right)^3} = \frac{4}{4EI} \sqrt{K}
\]

\[
\sigma_b \propto 4\sqrt[4]{K}
\]
Dowel Spacing

\[ P_t \]

\[ P_c \]

1.8\( \ell \)
Load Transfer

Assumes 100% efficiency
Load Transfer

Assumes 100% efficiency
Load Transfer

Assumes 90% efficiency

\[ P \]

\[ 0.55P \]

\[ 0.45P \]
Load Transfer

Assumes 90% efficiency
Example

Find the required dowel bar size for contraction joints in a 10" concrete slab resting on a subgrade material with a 200 psi/in modulus of subgrade reaction. The design load is a 9,000-lb dual wheel load centered 6" from the slab edge. Assume a 12" dowel spacing.
Example

\[ \ell = \sqrt[4]{\frac{E_ch^3}{12(1-\nu^2)k}} = \sqrt[4]{\frac{3,000,000 \text{ psi}}{12(0.9775)(200 \text{ psi/in})}} = 33.6 \text{ in} \]

\[ \Downarrow \]

\[ 1.8\ell \approx 60 \text{ in} \]
Example

4050 lb

60"

1.0P_t  0.8P_t  0.6P_t  0.4P_t  0.2P_t

3.0P_t = 4050 lb
Example

Assumptions

d = 1 in (Trial 1)
K = 1500 ksi/in
E = 30,000 ksi
z = 0.25 in
Multiple Wheel Loads

4050# 84" 4050#
Dowel Spacing (Updated)

\[ P_t \]

\[ P_c \]

\[ 1.0 \ell \]
Dowel Spacing (Updated)

84"
Often, dowel bars are sized and spaced using heuristics.

AASHTO originally recommended dowel diameters $\frac{1}{8}$ of the slab thickness, which is 1" for an 8" slab, 1.25" for a 10" slab, and 1.5" for a 12" slab.

PCA recommends using 1.25" dowel bars for slabs up to 10" thick and 1.5" bars for slabs over 10" thick. A common size is 1.5" by 18" or 20" long. The most common spacing is 12" center-to-center, but alternatives exist.
Alternative Dowel Spacing
Alternative Dowel Spacing