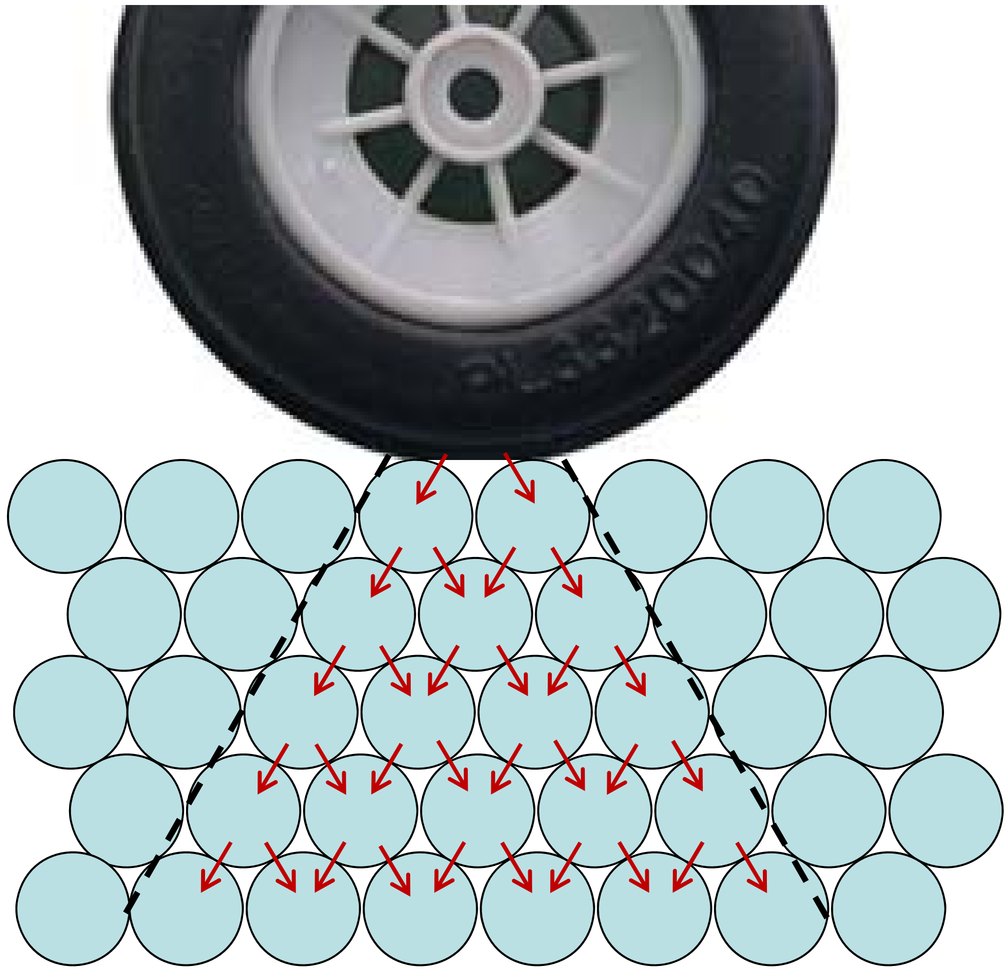
Stresses in Pavements

Elastic Halfspace Solutions

Flexible Pavements

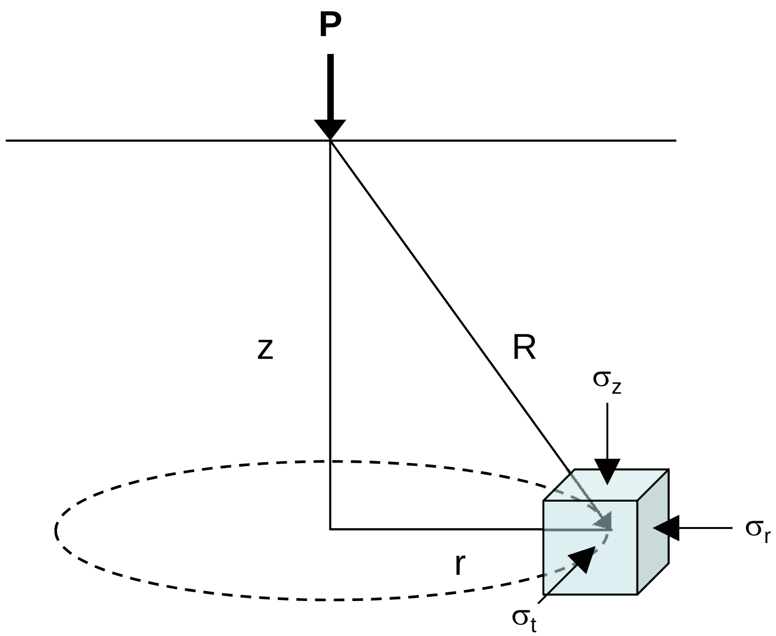


Stresses Due to Point Load

$$\sigma_z = \frac{P}{2\pi}\left[\frac{3z^3}{R^5}\right]$$

$$\sigma_r = \frac{P}{2\pi}\left[\frac{3r^2 z}{R^5} - \frac{1-2\mu}{R^2 + zR}\right]$$

$$\sigma_t = \frac{P}{2\pi}\left(1-2\mu\right)\left[\frac{z}{R^3} - \frac{1}{R^2 + zR}\right]$$



Example

$$\sigma_z = \frac{P}{2\pi}\left[\frac{3z^3}{R^5}\right]$$

$$\sigma_r = \frac{P}{2\pi}\left[\frac{3r^2z}{R^5} - \frac{1-2\mu}{R^2+zR}\right]$$

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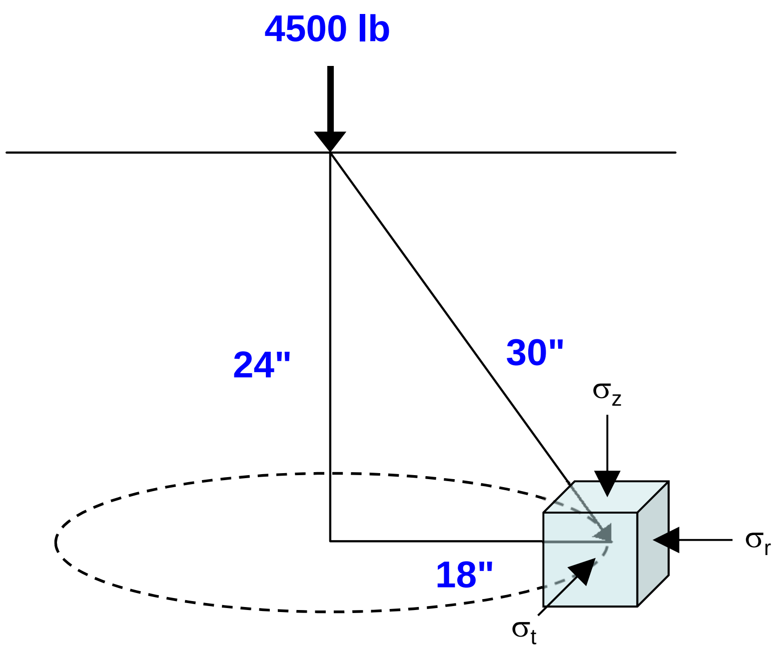


Assume μ = 0.35

Hooke's Law

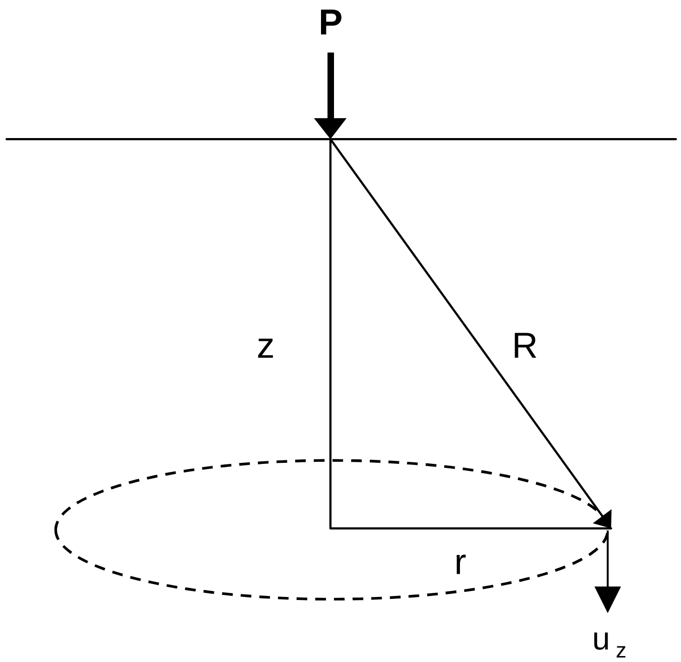
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu\left(\sigma_r + \sigma_t\right)\right]$$

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \mu\left(\sigma_z + \sigma_t\right)\right]$$

$$\varepsilon_t = \frac{1}{E}\left[\sigma_t - \mu\left(\sigma_r + \sigma_z\right)\right]$$

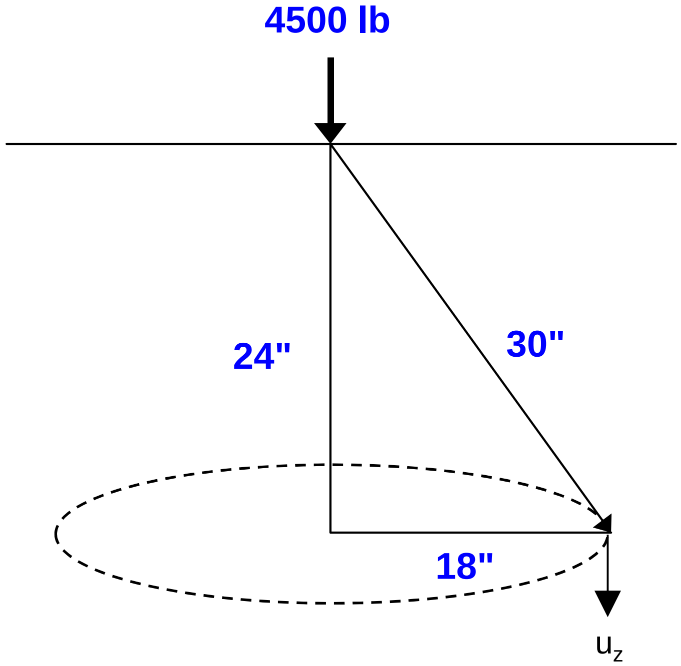
Deflections Due to Point Load

$$u_z = \int_{x=z}^{\infty} \varepsilon_z dx = \frac{P(1+\mu)}{2\pi E}\left[\frac{2(1-\mu)}{R} + \frac{z^2}{R^3}\right]$$



Example

$$u_z = \int_{x=z}^{\infty} \varepsilon_z dx = \frac{P(1+\mu)}{2\pi E}\left[\frac{2(1-\mu)}{R} + \frac{z^2}{R^3}\right]$$



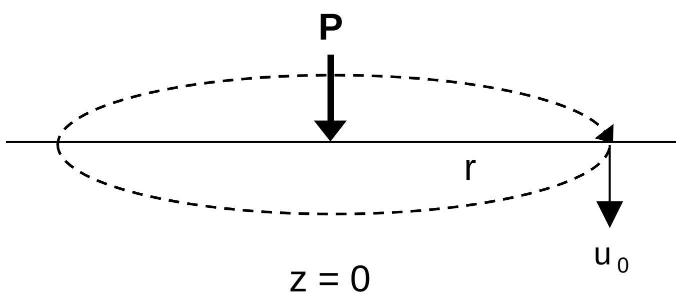
Assume μ = 0.35, E = 9600 psi

Surface Deflections

$$u_z = \frac{P(1+\mu)}{2\pi E}\left[\frac{2(1-\mu)}{R} + \frac{z^2}{R^3}\right]$$

Let z = 0, then R = r and

$$u_0 = \frac{P(1-\mu^2)}{\pi E r}$$



Example

$$u_z = \frac{P(1+\mu)}{2\pi E}\left[\frac{2(1-\mu)}{R} + \frac{z^2}{R^3}\right]$$

Let z = 0, then R = r and

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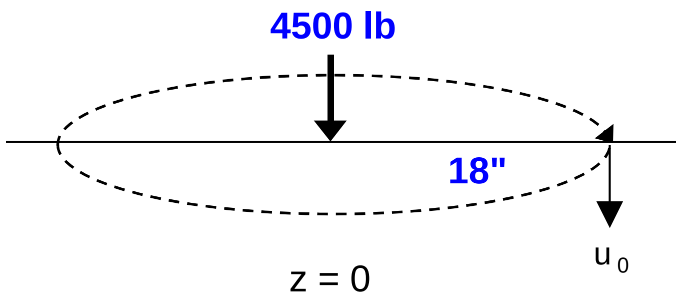


Assume μ = 0.35, E = 9600 psi

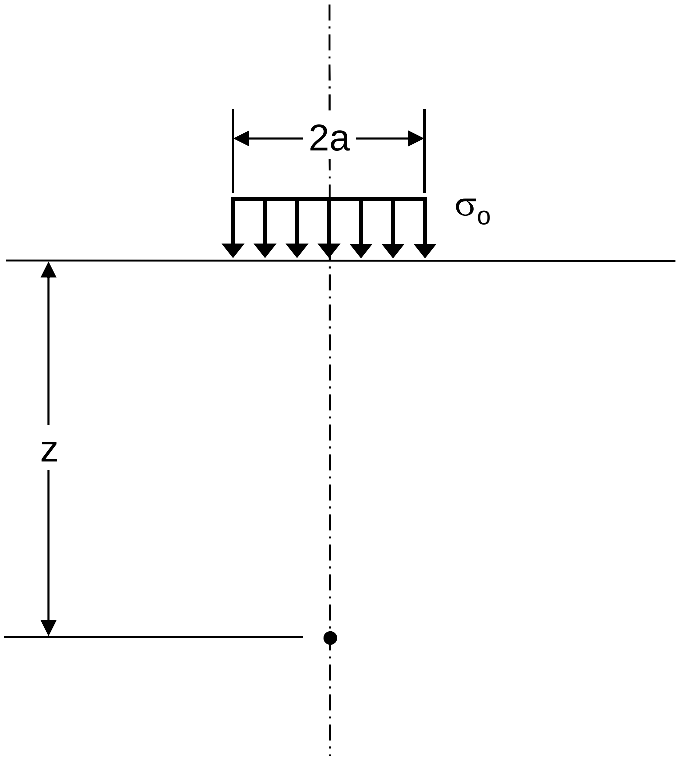
Stresses Due to Circular Load

Note: equations are only valid along load centerline

$$\sigma_z = \sigma_o \left\{ 1 - \frac{1}{\left[1 + (a/z)^2\right]^{3/2}} \right\}$$

$$\sigma_r = \frac{\sigma_o}{2} \left\{ 1 + 2\mu - \frac{2(1+\mu)}{\left[1 + (a/z)^2\right]^{1/2}} + \frac{1}{\left[1 + (a/z)^2\right]^{3/2}} \right\}$$

$$\sigma_t = \frac{\sigma_o}{2} \left\{ 1 + 2\mu - \frac{2(1+\mu)}{\left[1 + (a/z)^2\right]^{1/2}} + \frac{1}{\left[1 + (a/z)^2\right]^{3/2}} \right\}$$



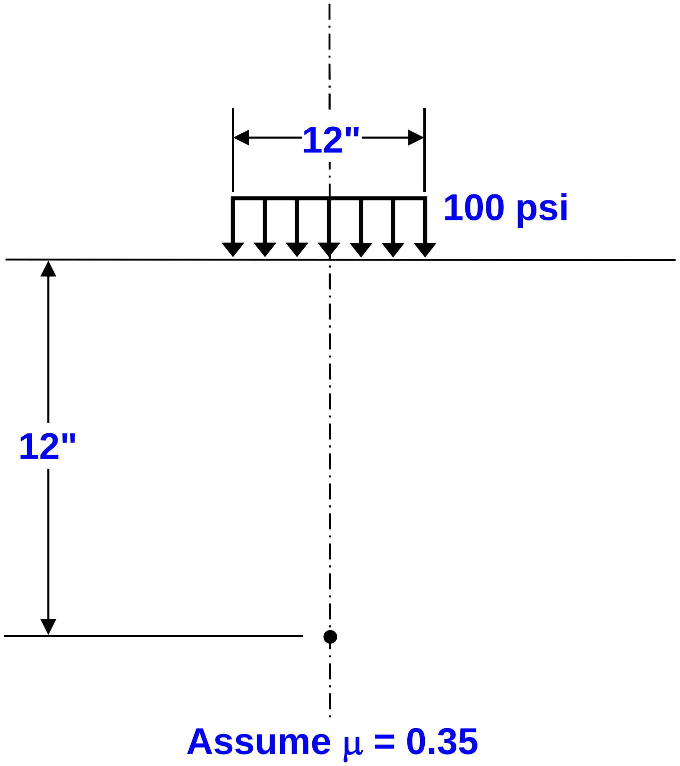
Example

Note: equations are only valid along load centerline

$$\sigma_z = \sigma_o \left\{ 1 - \frac{1}{\left[1 + (a/z)^2 \right]^{3/2}} \right\}$$

$$\sigma_r = \frac{\sigma_o}{2} \left\{ 1 + 2\mu - \frac{2(1+\mu)}{\left[1 + (a/z)^2 \right]^{1/2}} + \frac{1}{\left[1 + (a/z)^2 \right]^{3/2}} \right\}$$

$$\sigma_t = \frac{\sigma_o}{2} \left\{ 1 + 2\mu - \frac{2(1+\mu)}{\left[1 + (a/z)^2 \right]^{1/2}} + \frac{1}{\left[1 + (a/z)^2 \right]^{3/2}} \right\}$$



Hooke's Law

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu\left(\sigma_r + \sigma_t\right)\right]$$

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \mu\left(\sigma_z + \sigma_t\right)\right]$$

$$\varepsilon_t = \frac{1}{E}\left[\sigma_t - \mu\left(\sigma_r + \sigma_z\right)\right]$$

Deflections Due to Circular Load

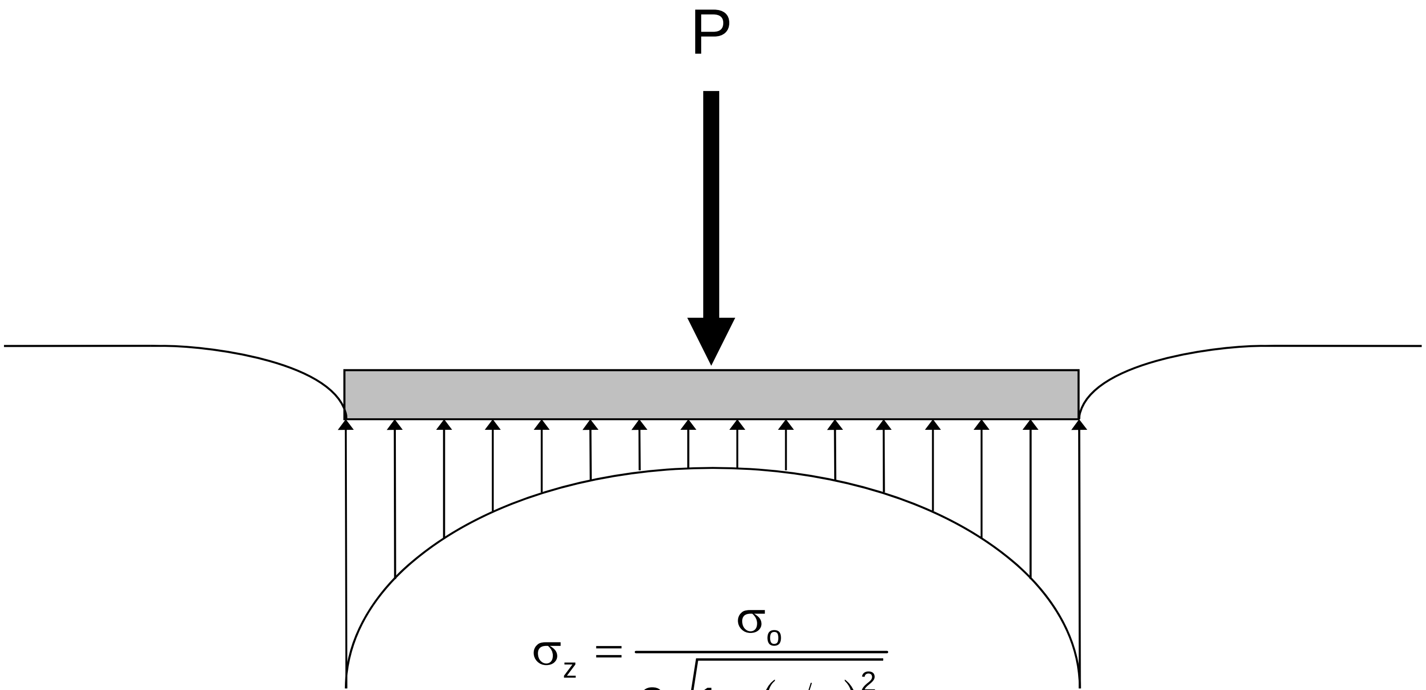
Note: equations are only valid along load centerline

$$d_z = \int_z^{\infty} \varepsilon_z dz = \frac{\sigma_o a(1+\mu)}{E}\left\{\frac{1}{\sqrt{1+(z/a)^2}} + (1-2\mu)\left[\sqrt{1+(z/a)^2} - (z/a)\right]\right\}$$

$$d_o = \int_0^{\infty} \varepsilon_z dz = \frac{2\sigma_o a(1-\mu^2)}{E} \quad @ \quad z = 0$$

Note: this is the deflection beneath a flexible (not rigid) circular plate

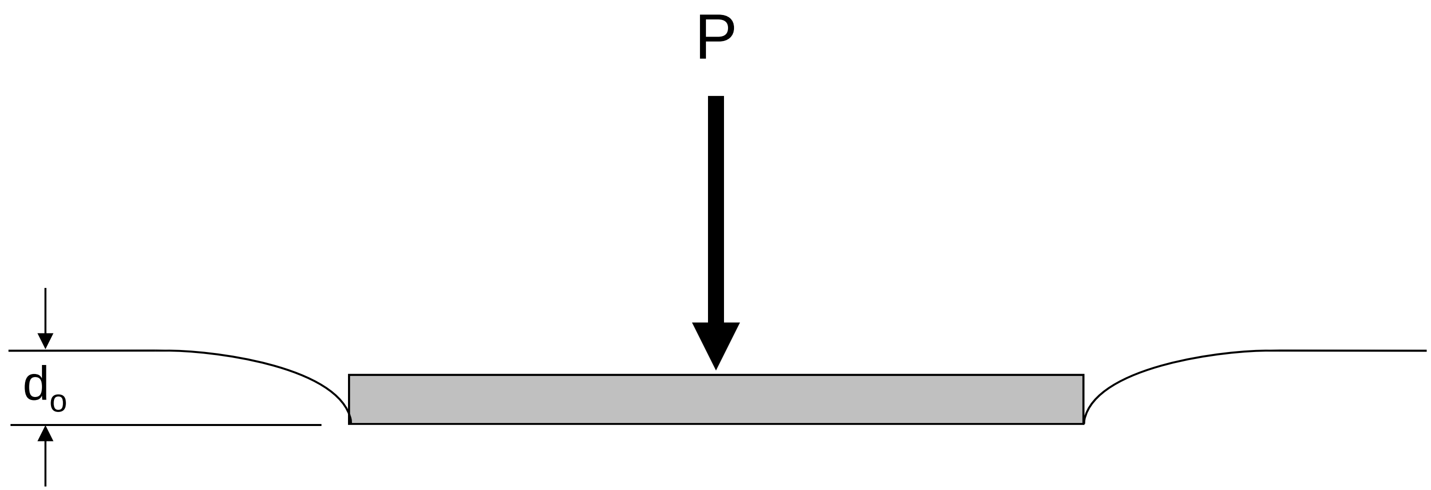
Rigid Loading



$$\sigma_z = \frac{\sigma_o}{2\sqrt{1-(r/a)^2}}$$

where $\sigma_o = \dfrac{P}{\pi a^2}$ is the average pressure on the plate

Rigid Loading



$$d_o = \frac{\pi \sigma_o a\left(1-\mu^2\right)}{2E}$$

Rigid vs. Flexible Loading

Flexible Plate

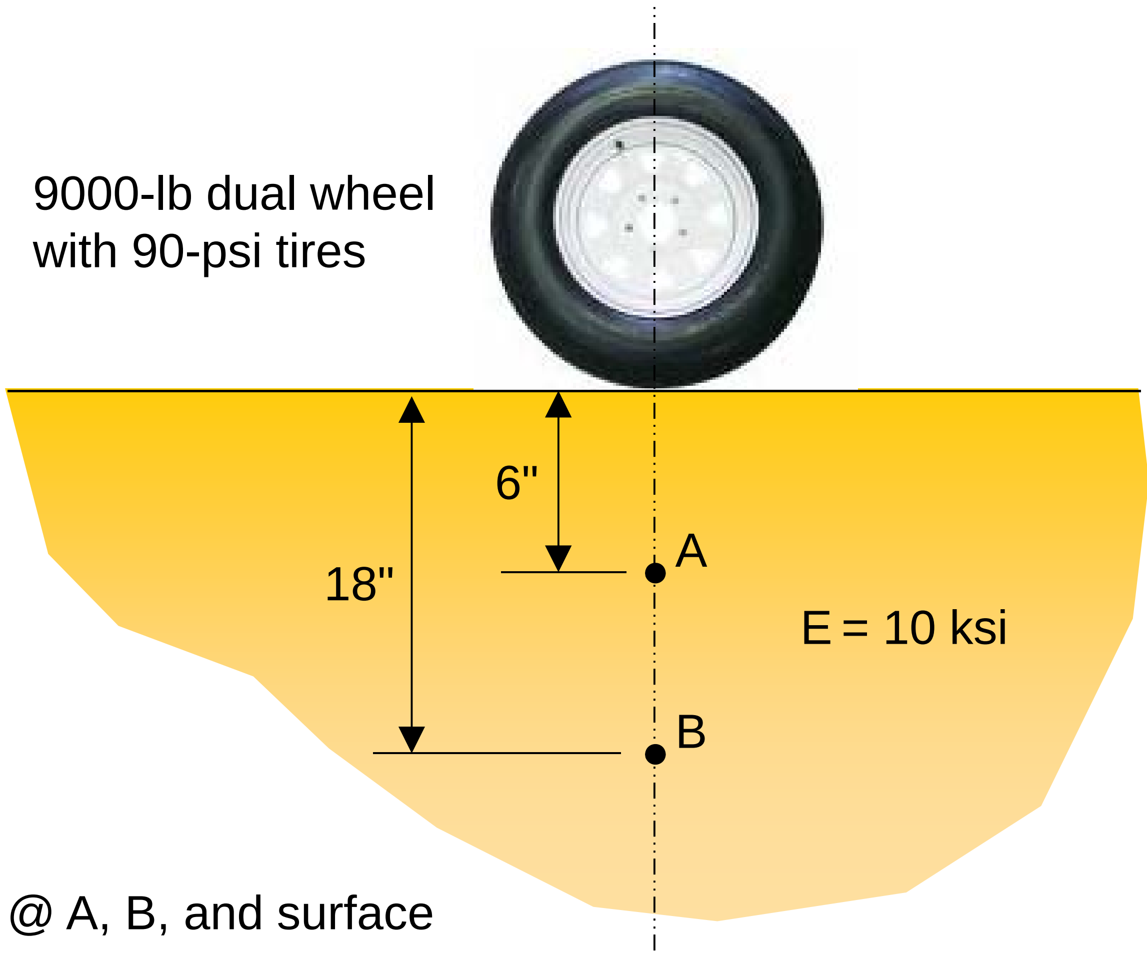
$$d_o = \frac{2\sigma_o a\left(1-\mu^2\right)}{E}$$

Rigid Plate

$$d_o = \frac{\pi\sigma_o a\left(1-\mu^2\right)}{2E}$$

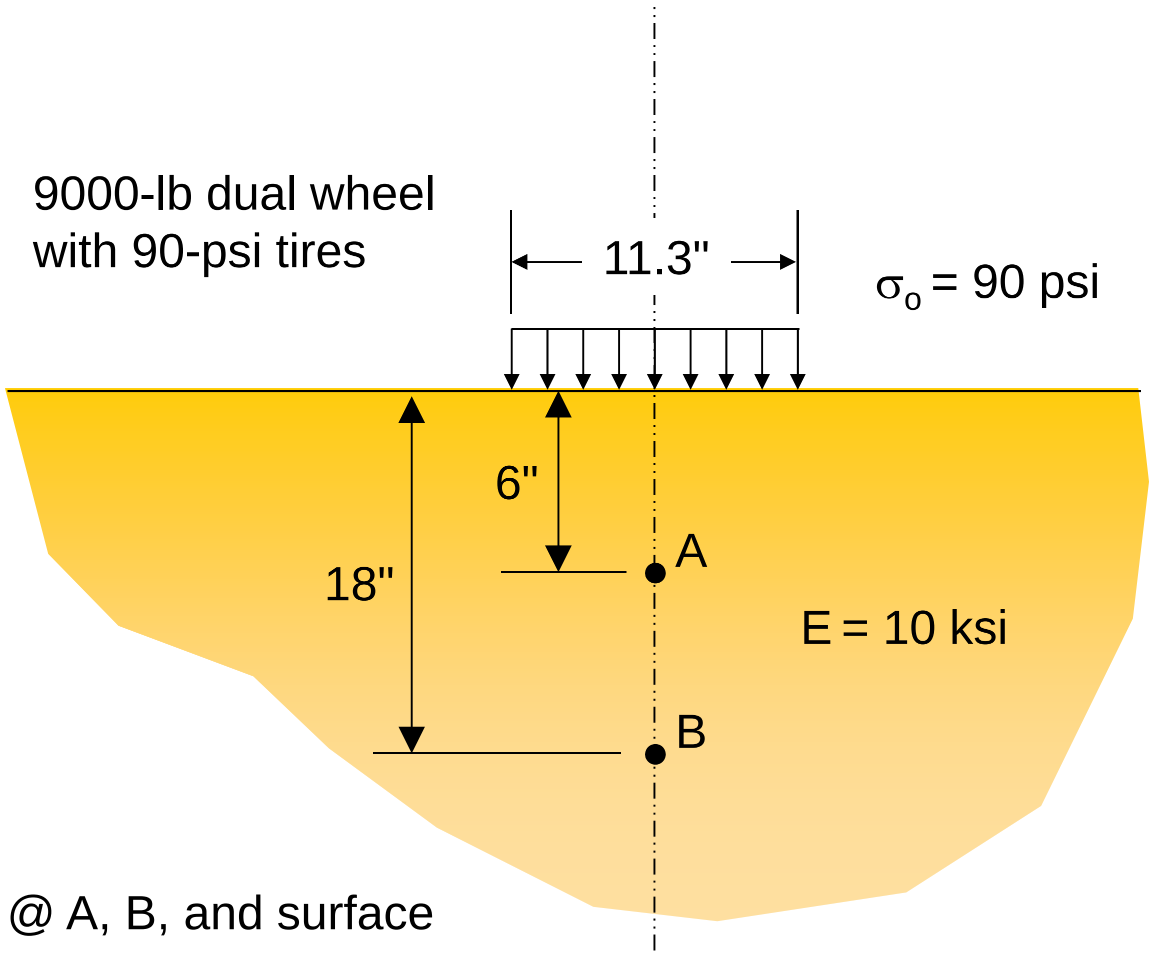
$$\frac{d_o^{\,flexible}}{d_o^{\,rigid}} = \frac{2\,\dfrac{\sigma_o a\left(1-\mu^2\right)}{E}}{\dfrac{\pi}{2}\,\dfrac{\sigma_o a\left(1-\mu^2\right)}{E}} = \frac{4}{\pi} = 1.27 \quad \Rightarrow \quad \frac{d_o^{\,rigid}}{d_o^{\,flexible}} = \frac{\pi}{4} = 0.79$$

Boussinesq Example



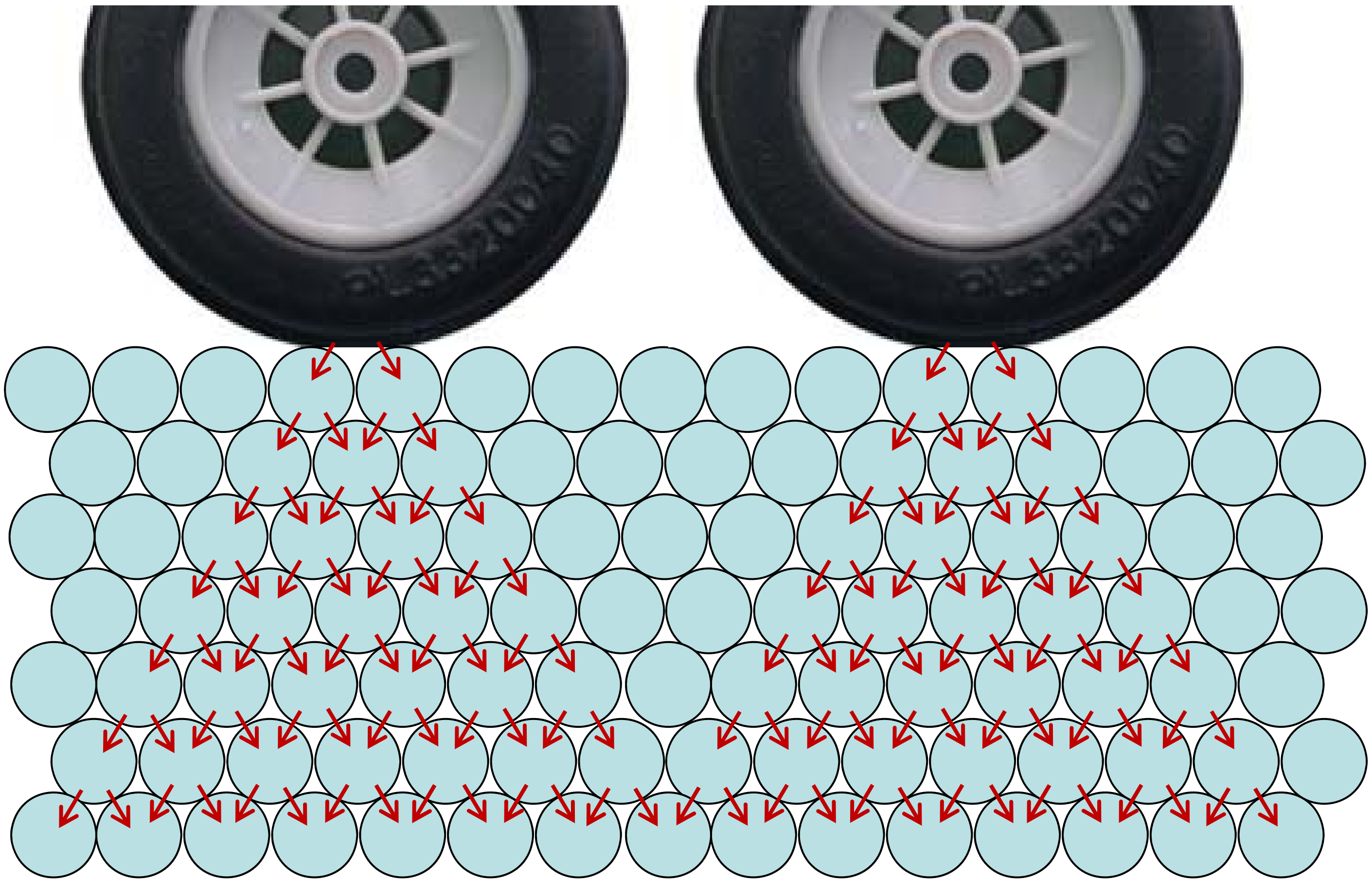
Find σ_z, d_z @ A, B, and surface

Boussinesq Example

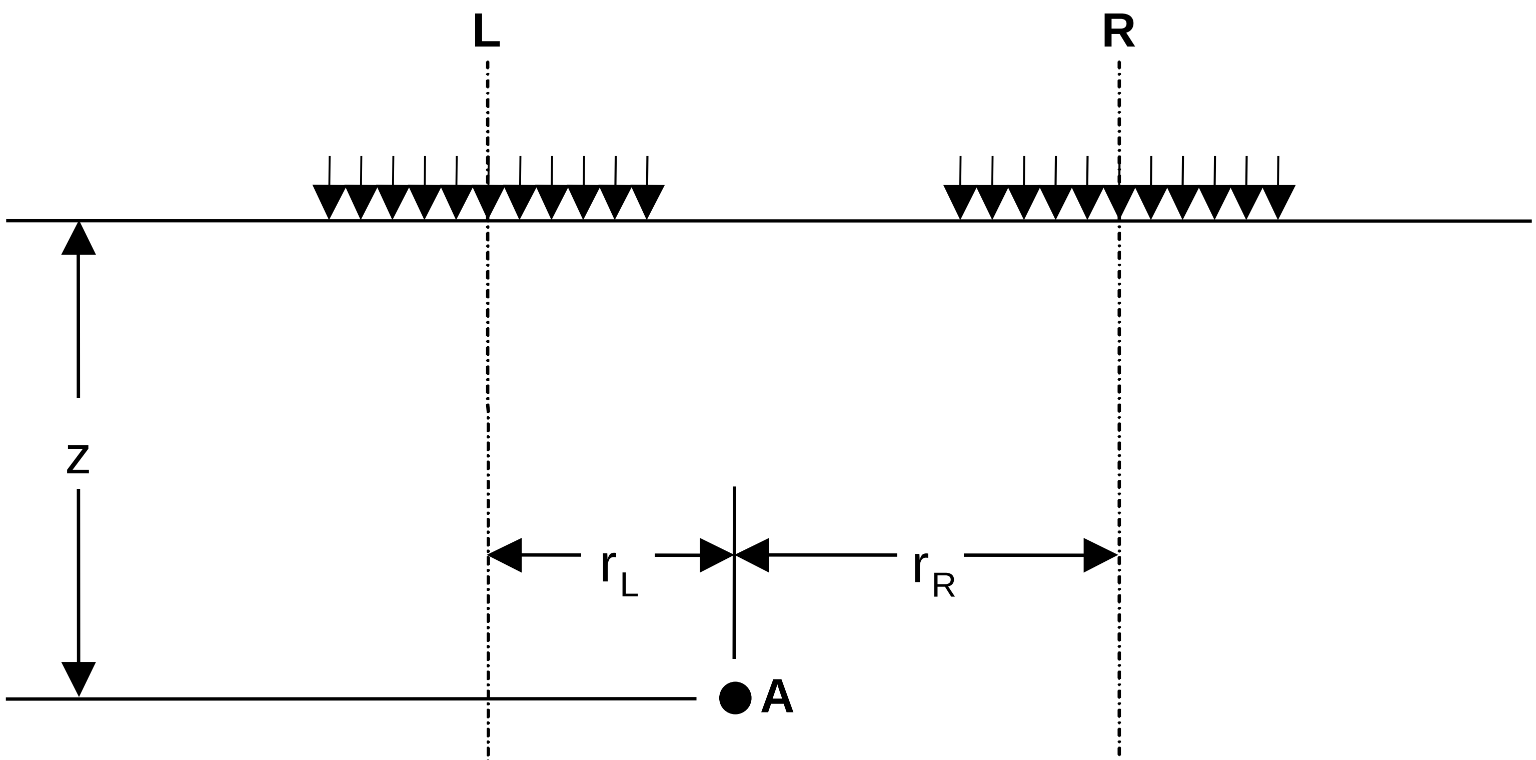


Find σ_z, d_z @ A, B, and surface

Multiple Wheel Loads



Multiple Wheel Loads



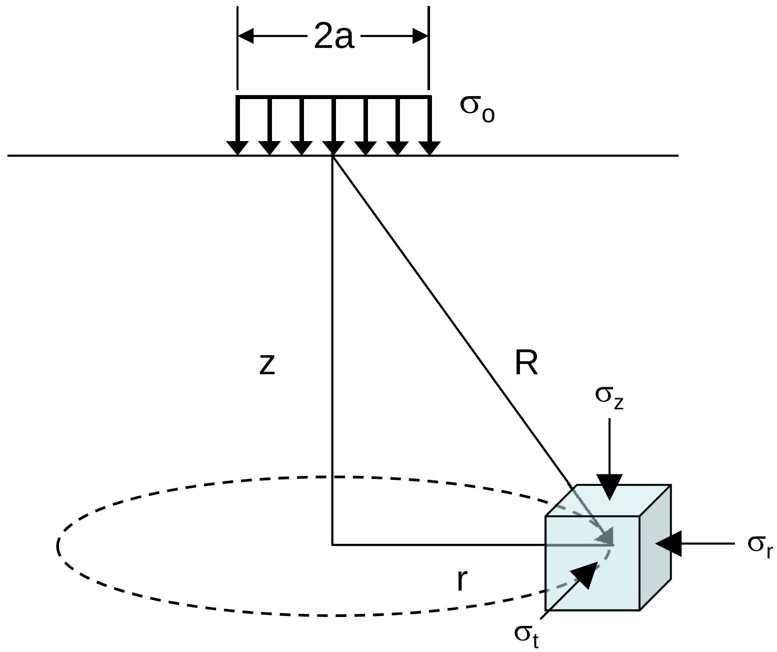
$$\sigma_A = \sigma_A^{\,L}\left(r_L, z\right) + \sigma_A^{\,R}\left(r_R, z\right)$$

Foster & Ahlvin

Charles R. Foster

Richard G. Ahlvin

Foster & Ahlvin



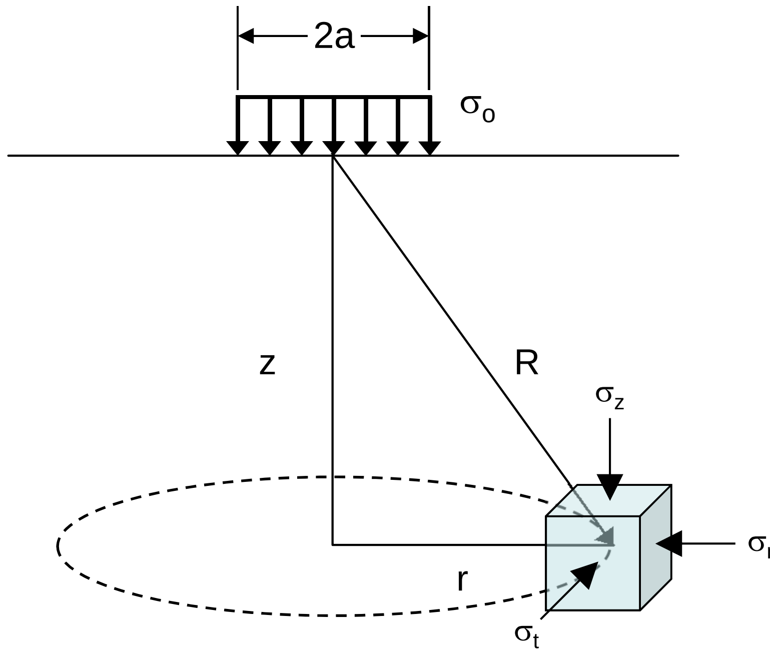
$\mu = 0.5$ (i.e., incompressible)

Ahlvin & Ulery

Richard G. Ahlvin

H. H. Ulery

Ahlvin & Ulery



$\mu = \text{anything}$

Ahlvin and Ulery (1962)

$$\sigma_z = p[\mathbf{A} + \mathbf{B}]$$

$$\sigma_r = p[2\mu\mathbf{A} + \mathbf{C} + (1-2\mu)\mathbf{F}]$$

E = function

$$\sigma_t = p[2\mu\mathbf{A} - \mathbf{D} + (1-2\mu)\mathbf{E}]$$

E = modulus

$$\Delta_z = \frac{pa(1+\mu)}{E}\left[\frac{z}{a}\mathbf{A} + (1-\mu)\mathbf{H}\right]$$ (deflection)

TABLE 3.1 **(TABLE 2.1 in 2nd Edition)**
Example Values of Coefficients

r/a	z/a	A	B	C	D	E	F	G	H	
0	0	1	0	0	0	0.5	0.5	0	2	
	0.1	0.9005	0.09852	−0.04926	0.04296	0.45025	0.45025	0	1.80998	
	0.5	0.55279	0.35777	−0.17889	0.17889	0.27639	0.27639	0	1.23607	
	1	0.29289	0.35355	−0.17678	0.17678	0.14645	0.14645	0	0.82843	
	2	0.10557	0.17889	−0.08944	0.08944	0.05279	0.05279	0	0.47214	0.18889
	5	0.01942	0.03772	−0.01886	0.01886	0.00971	0.00971	0	0.19805	
0.2	0	1	0	0	0	0.5	0.5	0	1.97987	
	0.1	0.89748	0.10140	−0.05142	0.04998	0.44949	0.44794	0.00315	1.79018	
	0.5	0.54403	0.35752	−0.17835	0.17917	0.27407	0.26997	0.04429	1.22176	
	1	0.28763	0.34553	−0.17050	0.17503	0.14483	0.14280	0.05266	0.85005	0.82005
	2	0.10453	0.18144	−0.08491	0.09080	0.05105	0.05348	0.02102	0.47022	
	5	0.01938	0.03760	−0.01810	0.01950	0.00927	0.01011	0.00214	0.19785	
1	0	0.5	0	0	0	0.5	0	0.31831	1.27319	
	0.1	0.43015	0.05388	0.02247	0.07635	0.39198	0.03817	0.31405	1.18107	
	0.5	0.28156	0.13591	0.00483	0.14074	0.21119	0.07037	0.26216	0.90298	
	1	0.17868	0.15355	−0.02843	0.12513	0.11611	0.06256	0.18198	0.67769	
	2	0.08269	0.11331	−0.04144	0.07187	0.04675	0.03593	0.07738	0.43202	
	5	0.01835	0.03384	−0.01568	0.01816	0.00929	0.00905	0.00992	0.19455	
2	0	0	0	0	0	0.12500	−0.12500	0	0.51671	
	0.1	0.00856	−0.00845	0.01536	0.00691	0.11806	−0.10950	0.00159	0.51627	
	0.5	0.03701	−0.02651	0.05690	0.03039	0.09180	−0.05479	0.03033	0.49728	
	1	0.05185	−0.01005	0.05429	0.04456	0.06552	−0.01367	0.06434	0.45122	
	2	0.04496	0.02836	0.01267	0.04103	0.03454	0.01043	0.06275	0.35054	
	5	0.01573	0.02474	−0.00939	0.01535	0.00873	0.00700	0.01551	0.18450	

Table A

									z/a								
r/a	0	0.1	0.2	0.3	0.1	0.5	0.6	0.7	0.8	0.9	1	1.2	1.5	2	2.5	3	4
0	1	0.90050	0.80388	0.71265	0.62861	0.55279	0.48550	0.42651	0.37531	0.33104	0.29289	0.23178	0.16795	0.10557	0.07152	0.05132	0.02986
0.2	1	0.89718	0.79821	0.70518	0.62015	0.54103	0.47691	0.41874	0.36832	0.32492	0.28763	0.22795	0.16552	0.10453	0.07098	0.05101	0.02976
0.4	1	0.88679	0.77884	0.68316	0.59241	0.51622	0.45078	0.39491	0.34729	0.30669	0.27005	0.21662	0.15877	0.10140	0.06947	0.05022	0.02907
0.6	1	0.86126	0.73483	0.62690	0.53767	0.46448	0.40427	0.35428	0.31243	0.27707	0.24697	0.19890	0.14804	0.09647	0.06698	0.04886	0.02832
0.8	1	0.78712	0.63014	0.52081	0.41329	0.38390	0.33676	0.29833	0.26581	0.23832	0.21468	0.17626	0.13436	0.09011	0.06373	0.04707	0.02802
1	0.5	0.43015	0.38269	0.34375	0.31048	0.28156	0.25588	0.21727	0.21297	0.19488	0.17868	0.15101	0.11892	0.08269	0.05974	0.04487	0.02749
1.2	0	0.09645	0.15433	0.17964	0.18700	0.18556	0.17952	0.17124	0.16206	0.15253	0.14329	0.12570	0.10296	0.07471	0.05555	0.04241	0.02651
1.5	0	0.02787	0.05251	0.07199	0.08593	0.09499	0.10010	0.10228	0.10236	0.10094	0.09849	0.09192	0.08048	0.06275	0.04880	0.03839	0.02490
2	0	0.00856	0.01680	0.02440	0.03118	0.03701		0.04558			0.05185	0.05260	0.05116	0.04496	0.03787	0.03150	0.02193
3	0	0.00211	0.00419	0.00622		0.01013					0.01742	0.01935	0.02142	0.02221	0.02143	0.01980	0.01592
4	0	0.00084	0.00167	0.00250		0.00407					0.00761	0.00871	0.01013	0.01160	0.01221	0.01220	0.01109
5	0	0.00042	0.00083			0.00209					0.00393	0.00459	0.00548	0.00659	0.00732	0.00770	0.00768
6	0		0.00048			0.00118					0.00226	0.00269	0.00325	0.00399	0.00463	0.00505	0.00536
8	0		0.00020			0.00053					0.00097	0.00115	0.00141	0.00180	0.00214	0.00242	0.00282
10	0					0.00025					0.00050		0.00073	0.00094	0.00115	0.00132	0.00160
12	0					0.00014					0.00029		0.00043	0.00056	0.00068	0.00079	0.00099
14	0					0.00009					0.00018		0.00027	0.00036	0.00013	0.00051	0.00065

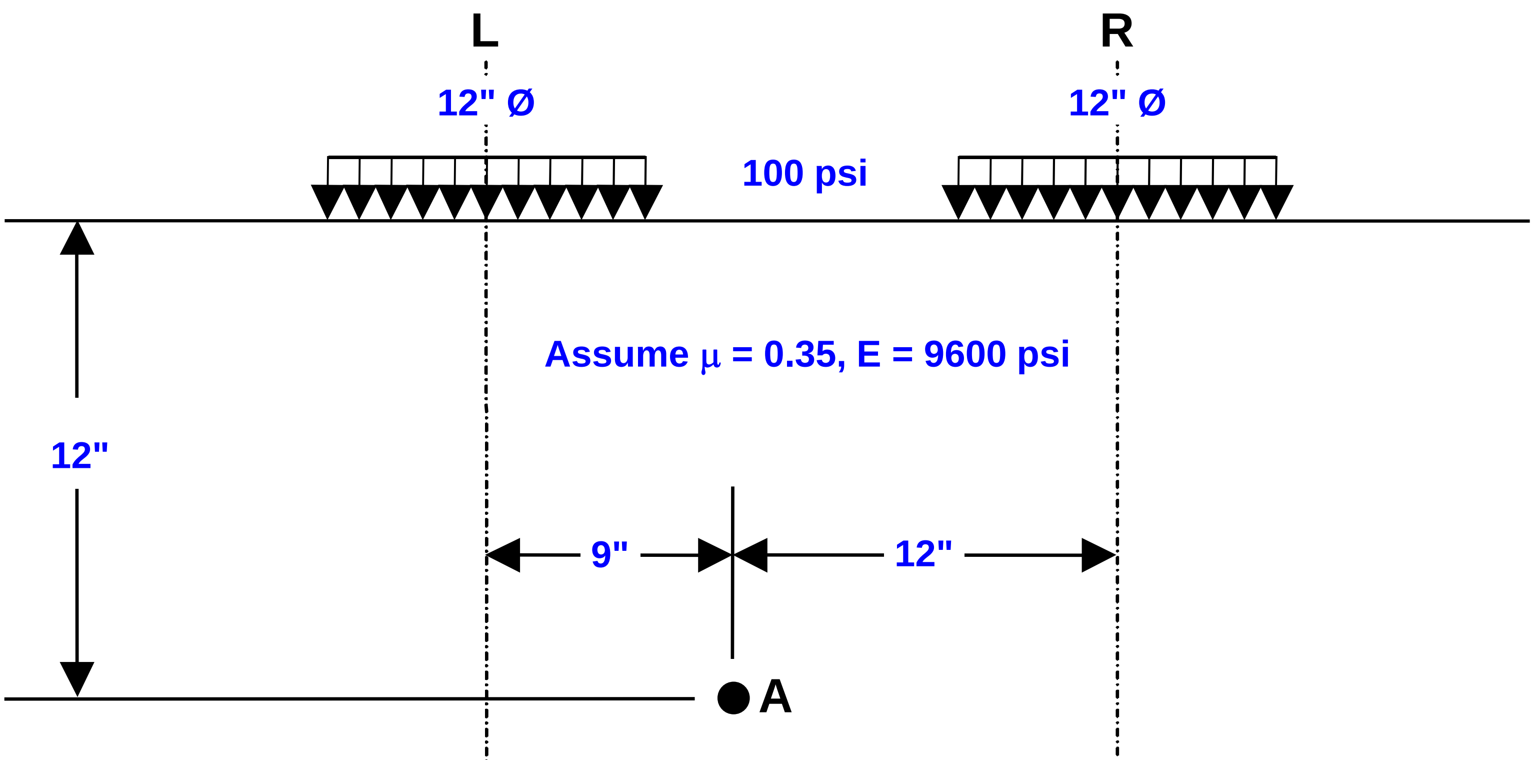
Table B

r/a \ z/a	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.5	2	2.5	3	4
0	0	0.09852	0.18857	0.26362	0.32016	0.35777	0.37831	0.38487	0.38091	0.36962	0.35355	0.31485	0.25602	0.18889	0.12807	0.09487	0.05707
0.2	0	0.10140	0.19306	0.26787	0.32259	0.35752	0.37531	0.37962	0.37408	0.36275	0.34553	0.30730	0.25025	0.18144	0.12633	0.09394	0.05666
0.4	0	0.11138	0.20772	0.28018	0.32748	0.35323	0.36308	0.36072	0.35133	0.33734	0.32075	0.28481	0.23338	0.16644	0.12126	0.09099	0.05562
0.6	0	0.13424	0.23524	0.29483	0.32273	0.33106	0.32822	0.31929	0.30699	0.29299	0.27819	0.24836	0.20691	0.15198	0.11327	0.08635	0.05383
0.8	0	0.18796	0.25983	0.27257	0.26925	0.26236	0.25411	0.24638	0.23779	0.22891	0.21978	0.20113	0.17368	0.13375	0.10298	0.08033	0.05145
1	0	0.05388	0.08513	0.10757	0.12404	0.13591	0.14440	0.14986	0.15292	0.15404	0.15355	0.14915	0.13732	0.11331	0.09130	0.07325	0.04773
1.2	0	-0.07899	-0.07759	-0.04316	-0.00766	0.02165	0.04457	0.06209	0.07530	0.08507	0.09210	0.10002	0.10193	0.09254	0.07869	0.06551	0.04532
1.5	0	-0.02672	-0.04448	-0.04999	-0.04535	-0.03455	-0.02101	-0.00702	0.00614	0.01795	0.02814	0.04378	0.05745	0.06371	0.06022	0.05354	0.03995
2	0	-0.00845	-0.01593	-0.02166	-0.02522	-0.02651		-0.02329			-0.01005	0.00023	0.01385	0.02836	0.03429	0.03511	0.03066
3	0	-0.00210	-0.00412	-0.00599		-0.00991					-0.01115	-0.00995	-0.00669	0.00028	0.00661	0.01112	0.01515
4	0	-0.00081	-0.00166	-0.00245		-0.00388					-0.00608	-0.00632	-0.00600	-0.00410	-0.00130	0.00157	0.00595
5	0	-0.00042	-0.00083			-0.00199					-0.00344	-0.00378	-0.00401	-0.00371	-0.00271	-0.00134	0.00155
6	0		-0.00024			-0.00116					-0.00210	-0.00236	-0.00265	-0.00278	-0.00250	-0.00192	-0.00029
8	0		-0.00010			-0.00049					-0.00092	-0.00107	-0.00126	-0.00148	-0.00156	-0.00151	-0.00109
10	0					-0.00025					-0.00048		-0.00068	-0.00084	-0.00094	-0.00099	-0.00091
12	0					-0.00014					-0.00023		-0.00040	-0.00050	-0.00059	-0.00065	-0.00068
14	0					-0.00009					-0.00018		-0.00026	-0.00033	-0.00039	-0.00046	-0.00050

Table H

r/a \ z/a	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.5	2	2.5	3	4
0	2	1.80998	1.63961	1.48806	1.35407	1.23607	1.13238	1.04131	0.96125	0.89072	0.82843	0.72410	0.60555	0.47214	0.38151	0.32457	0.24620
0.2	1.97987	1.79018	1.62068	1.47044	1.33802	1.22176	1.11998	1.03037	0.95175	0.88251	0.82005	0.71882	0.60233	0.47022	0.38403	0.32403	0.24588
0.4	1.91751	1.72886	1.56242	1.40979	1.28963	1.17894	1.08350	0.99794	0.92386	0.85856	0.80465	0.70370	0.59246	0.46512	0.38098	0.32184	0.24820
0.6	1.80575	1.61961	1.46001	1.32442	1.20822	1.10830	1.02154	0.94049	0.87928	0.82616	0.76809	0.67937	0.57633	0.45656	0.37608	0.31887	0.25128
0.8	1.62553	1.44711	1.30614	1.19210	1.09555	1.01312	0.94120	0.87742	0.82136	0.77950	0.72507	0.64814	0.55559	0.44502	0.36940	0.31461	0.24168
1	1.27319	1.18107	1.09996	1.02740	0.96202	0.90298	0.84917	0.80030	0.75571	0.71495	0.67769	0.61107	0.53138	0.43202	0.36155	0.30969	0.23932
1.2	0.93676	0.92670	0.90098	0.86136	0.83042	0.79308	0.75653	0.72143	0.68809	0.65677	0.62701	0.57329	0.50496	0.41702	0.35243	0.30301	0.23668
1.5	0.71185	0.70888	0.70074	0.68823	0.67238	0.65429	0.63169	0.61442	0.59398	0.57361	0.55364	0.51552	0.46379	0.39242	0.33698	0.29364	0.23164
2	0.51671	0.51627	0.51382	0.50966	0.50112	0.49728		0.48061			0.45122	0.43013	0.39872	0.35054	0.30913	0.27453	0.22188
3	0.33815	0.33794	0.33726	0.33638		0.33293					0.31877	0.31162	0.29945	0.27740	0.25550	0.23487	0.19908
4	0.252	0.25184	0.25162	0.25124		0.24996					0.24386	0.24070	0.23195	0.22418	0.21208	0.19977	0.17640
5	0.20045	0.20081	0.20071			0.19982					0.19673	0.19520	0.19053	0.18618	0.17898	0.17154	0.15596
6	0.16626		0.16688			0.16668					0.16516	0.16369	0.16199	0.15846	0.15395	0.14919	0.13864
8	0.12576		0.12512			0.12493					0.12394	0.12350	0.12281	0.12124	0.11928	0.11694	0.11172
10	0.09918					0.09996					0.09952		0.09876	0.09792	0.09700	0.09558	0.09300
12	0.08346					0.08295					0.08292		0.08270	0.08196	0.08115	0.08061	0.07864
14	0.07023					0.07123					0.07104		0.07064	0.07020	0.06960	0.06097	0.06848

Example

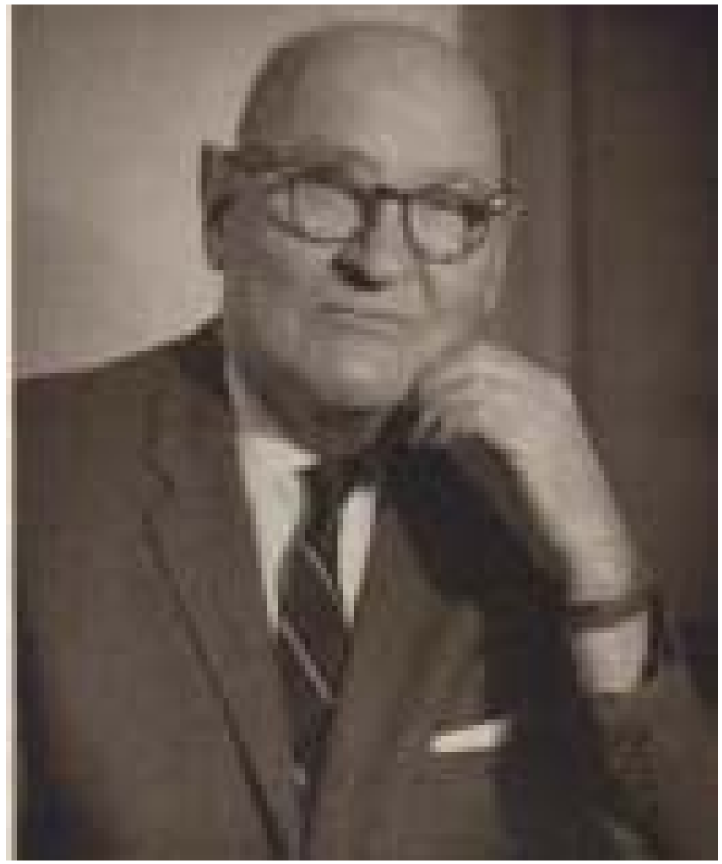


Find σ_z, Δ_z @ A

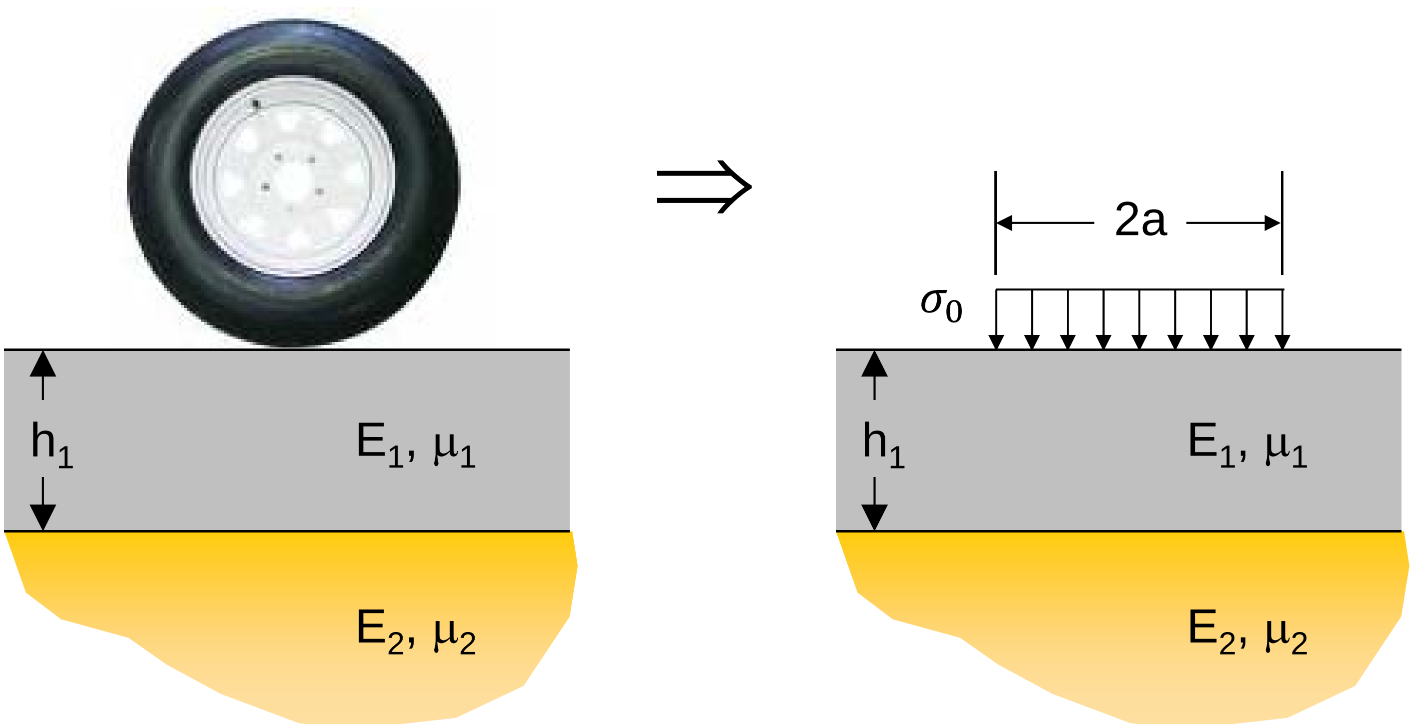
Stresses in Pavements

Layered Elastic Solutions

Donald Burmister

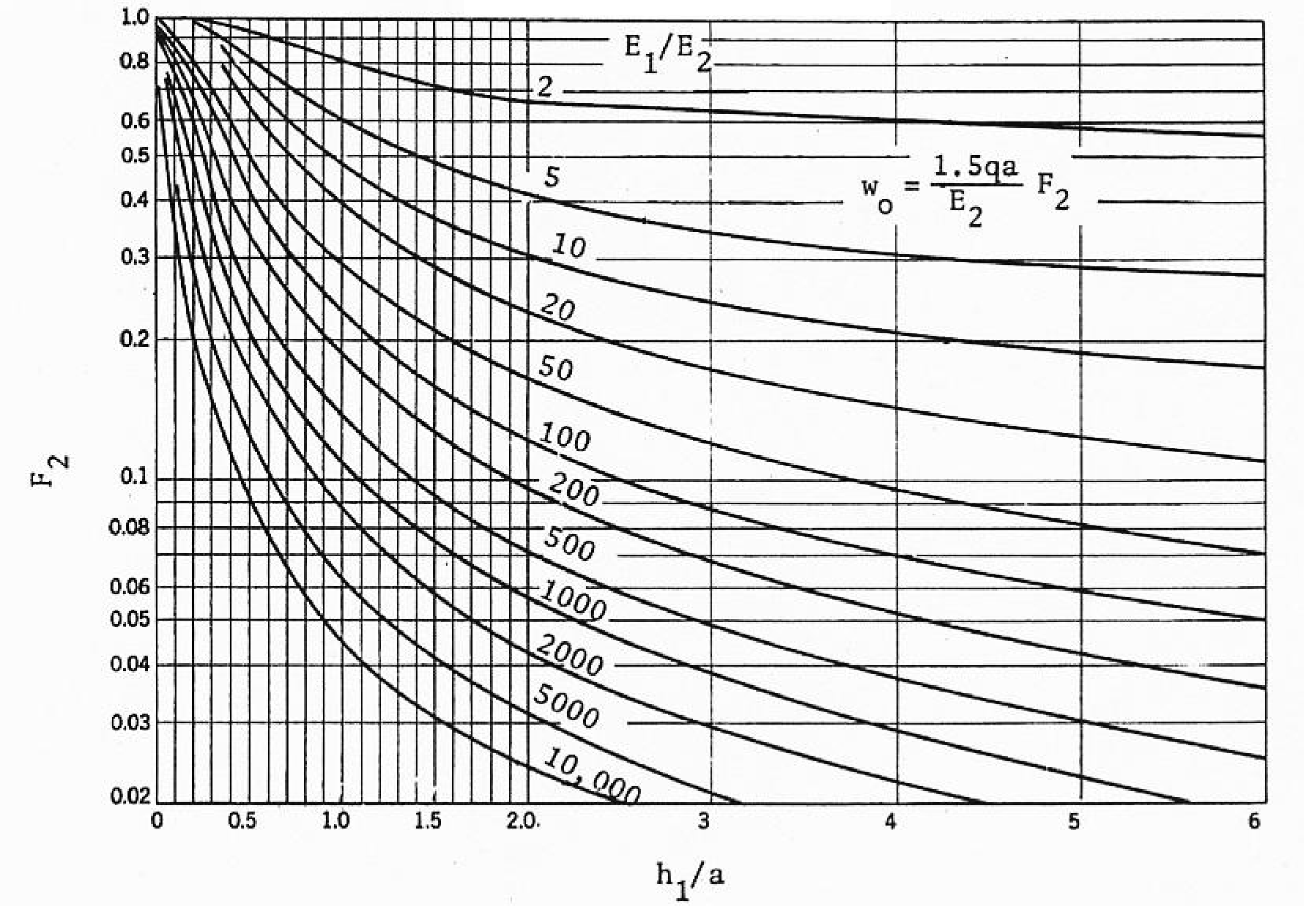


Burmister's Solution



Burmister's Solution

Surface deflection

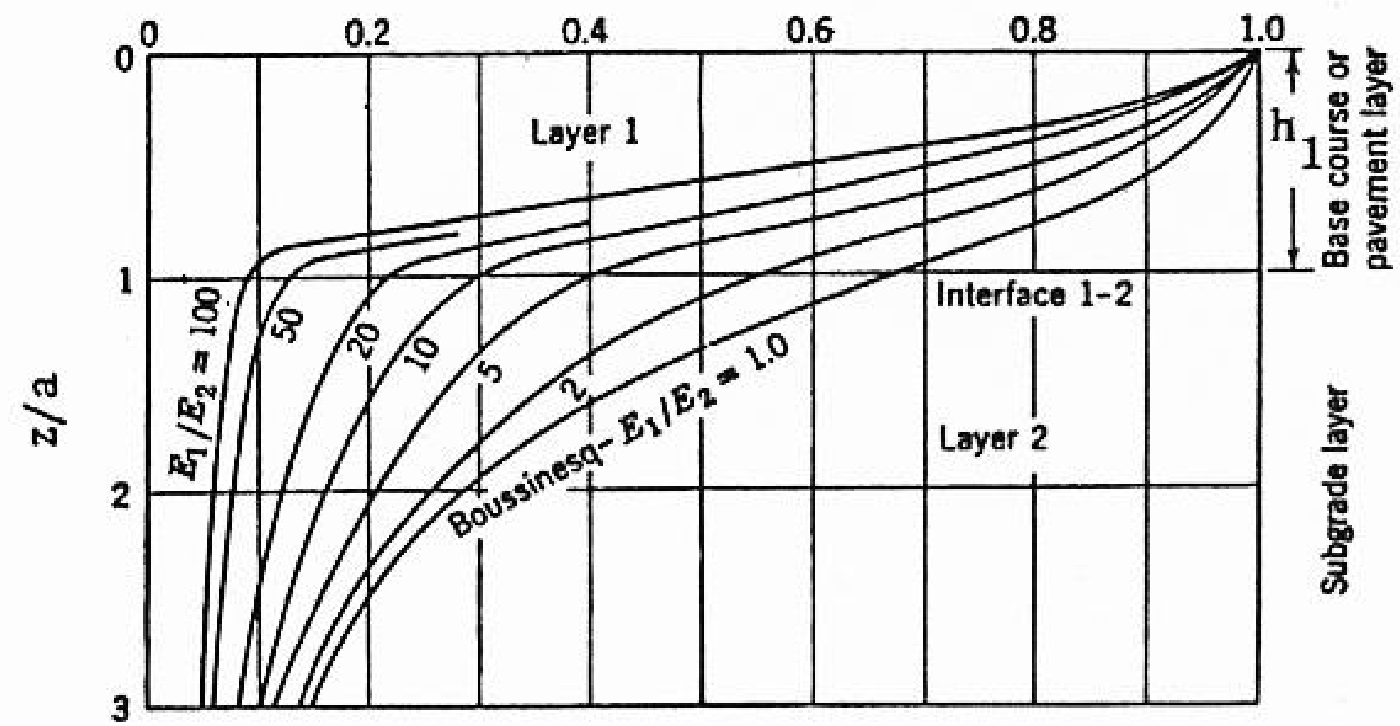


Note: incompressible media (i.e., μ = 0.5)

Burmister's Solution

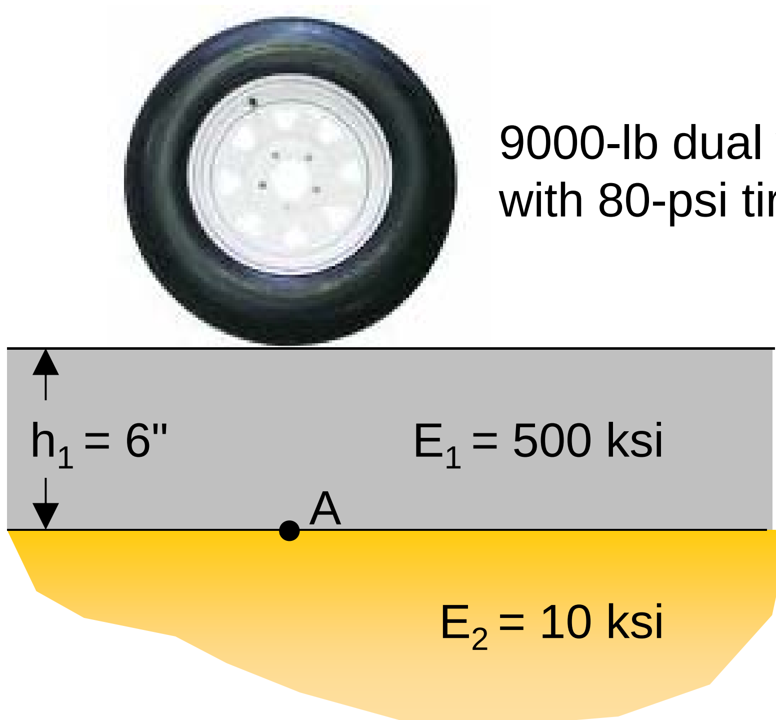
Vertical stress

$$I = \sigma_z / \sigma_0$$



Note: incompressible media and h_1 = a only

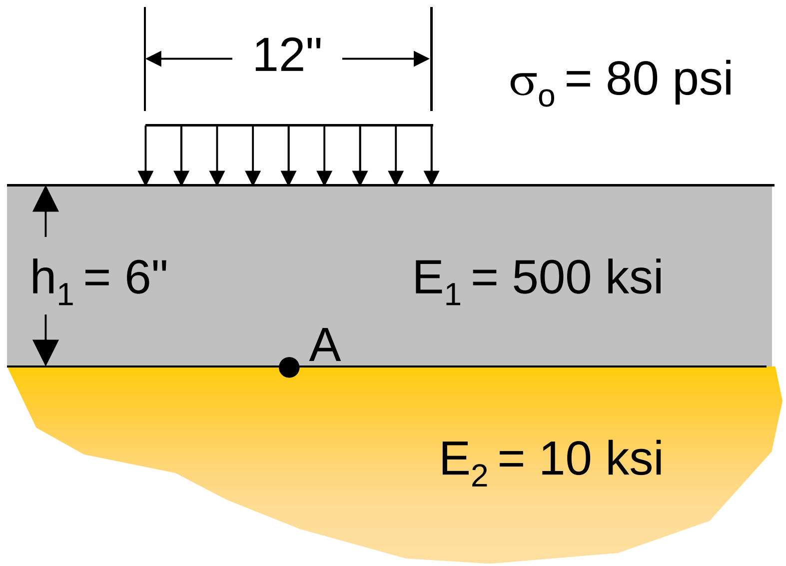
Burmister Example



9000-lb dual wheel
with 80-psi tires

Find w_0 and σ_z @ A

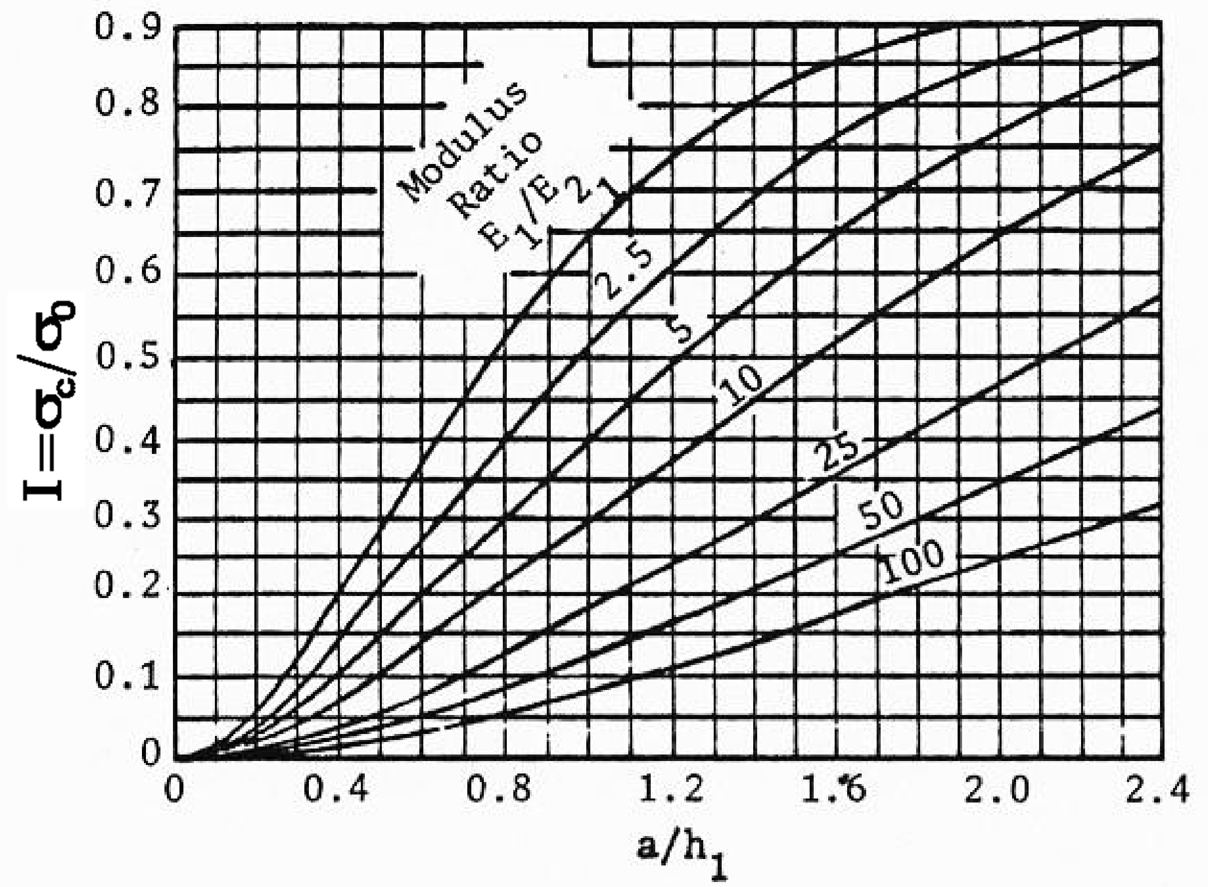
Burmister Example



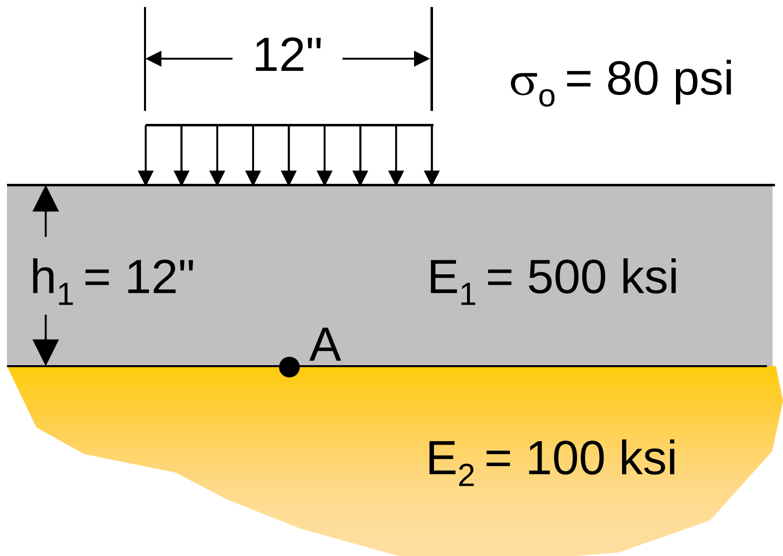
Find w_0 and σ_z @ A

Huang’s Solution

Interface Vertical Stress

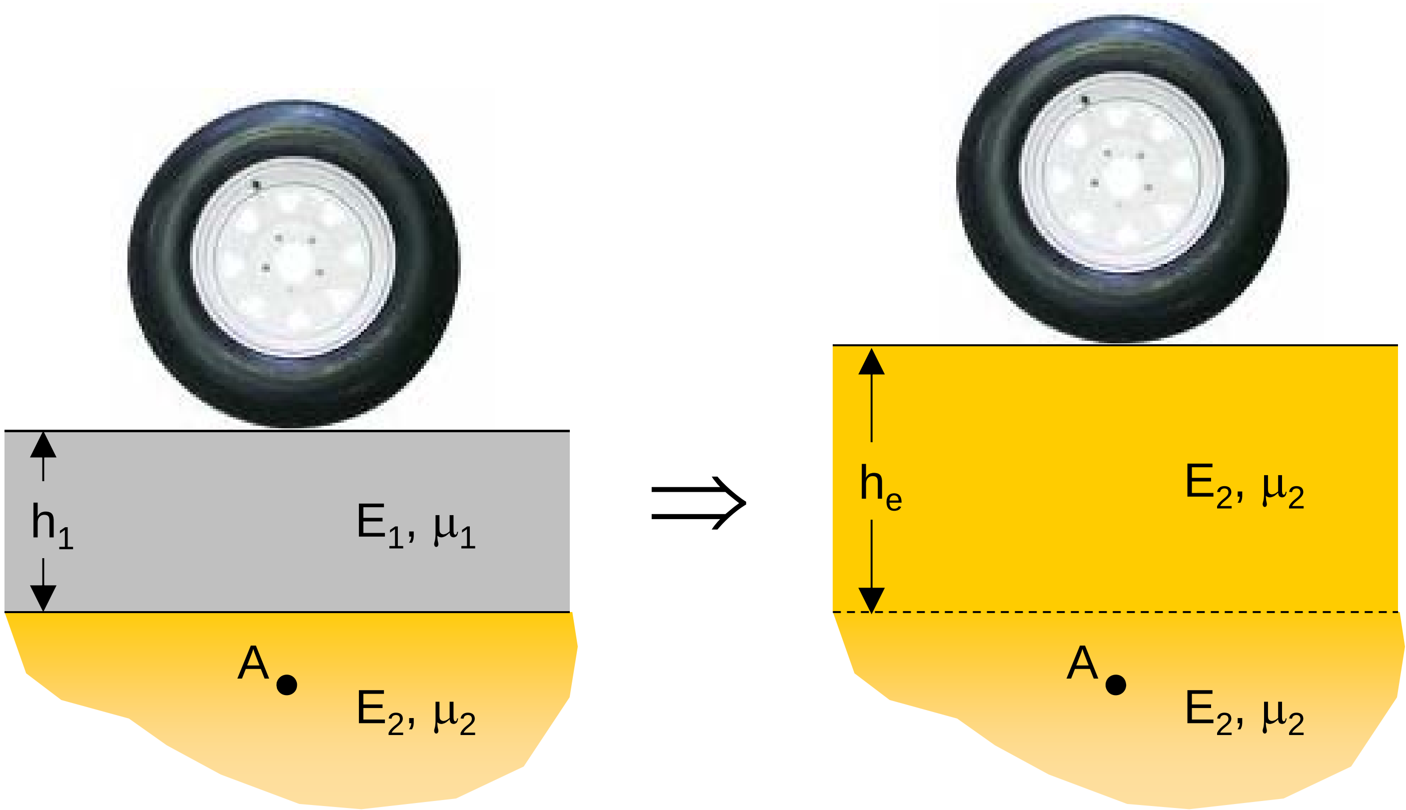


Huang Example



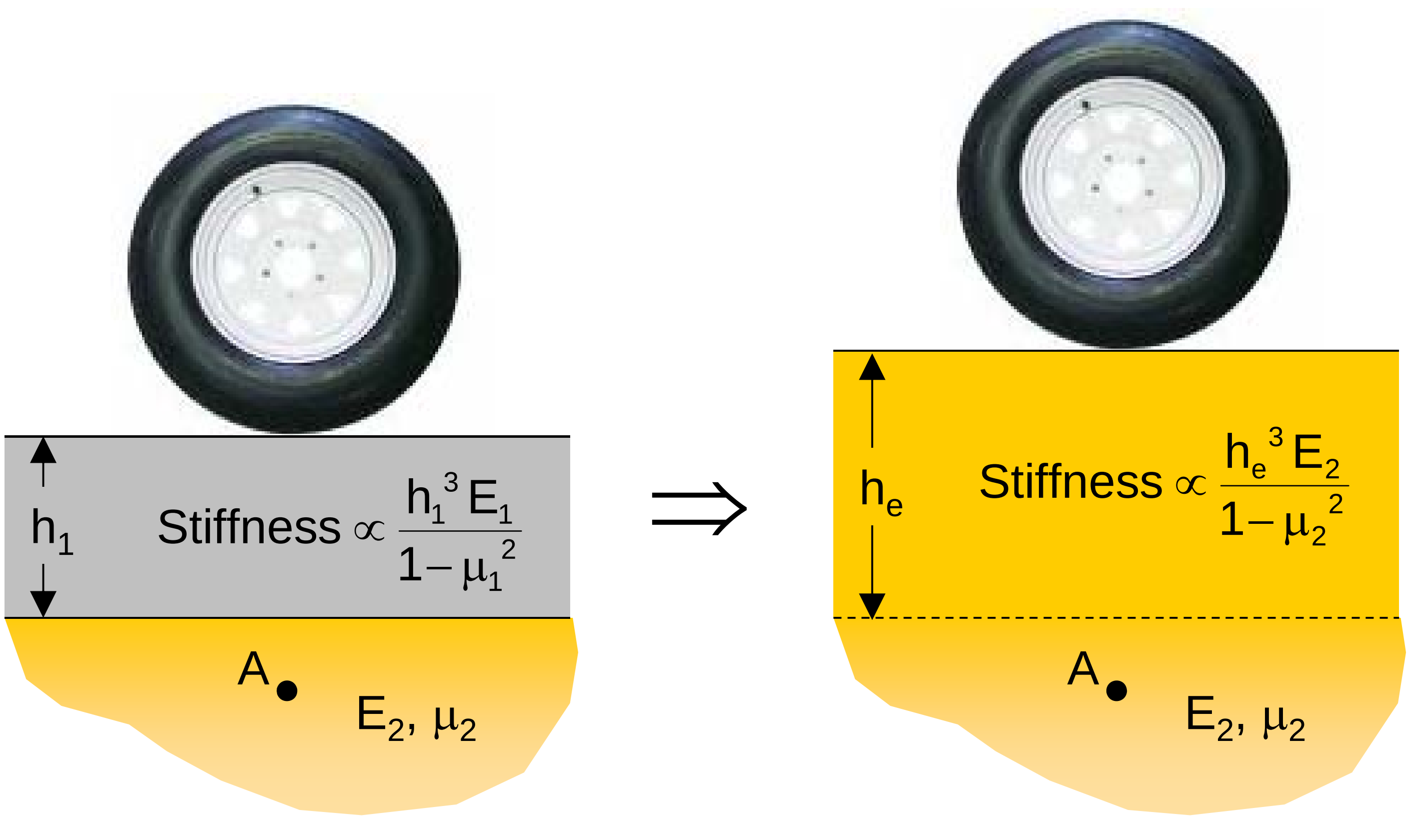
Find σ_c @ A

Odemark's Method



Note: only valid at or below the layer interface

Odemark's Method



Note: only valid at or below the layer interface

Odemark's Method

$$\frac{h_e^3 E_2}{1-\mu_2^2} = \frac{h_1^3 E_1}{1-\mu_1^2}$$

$$h_e = h_1 \sqrt[3]{\frac{E_1}{E_2} \frac{\left(1-\mu_2^2\right)}{\left(1-\mu_1^2\right)}}$$

Odemark's Method

$$h_e = h_1 \sqrt[3]{\frac{E_1}{E_2}}$$

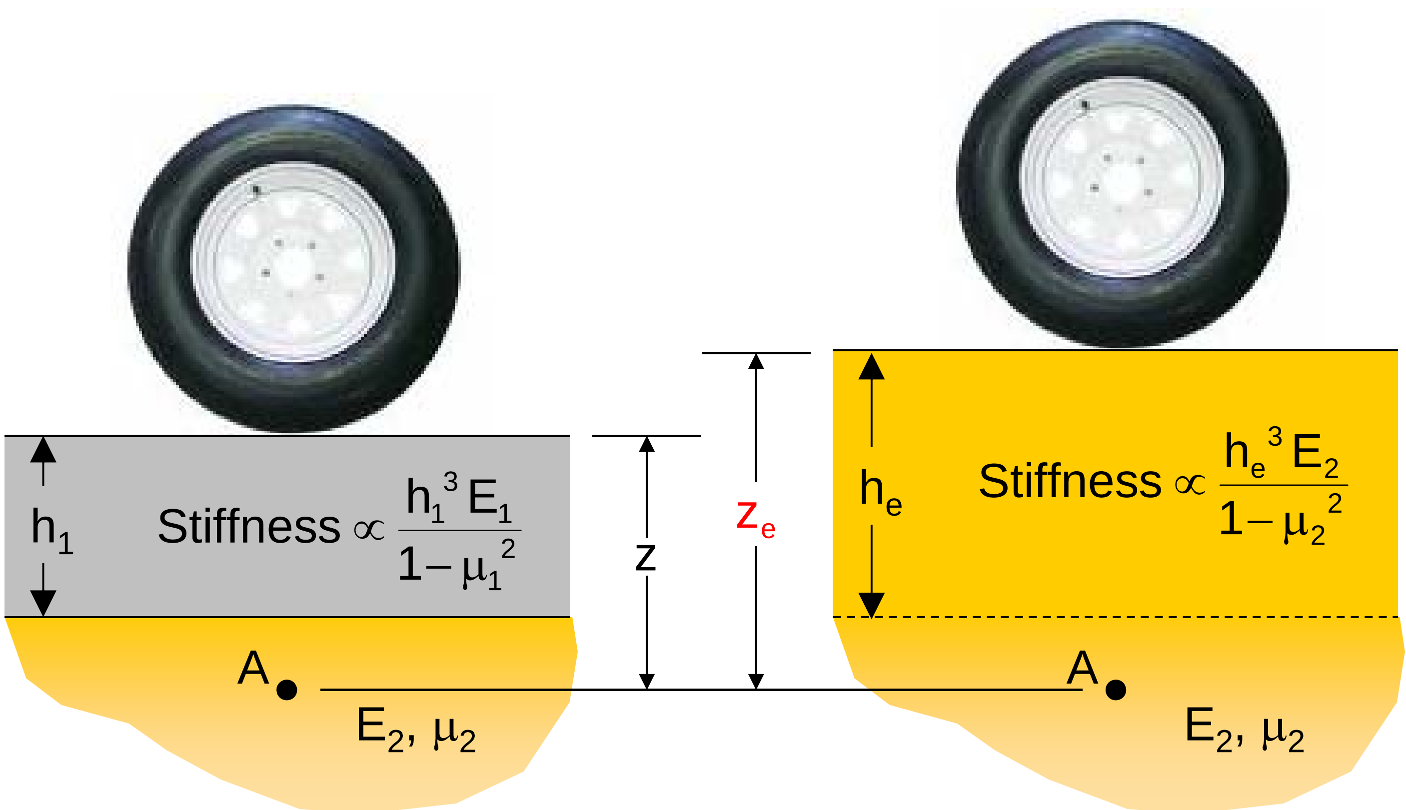
If we assume $\mu_1 \approx \mu_2$

Odemark's Method

$$h_e = f \times h_1 \sqrt[3]{\frac{E_1}{E_2}}$$

In a 2-layer pavement system, use f = 0.9 to convert the upper layer. In a multi-layer pavement system, use f = 1.0 to convert the uppermost layer and 0.8 to convert the rest of the layers.

Odemark's Method



Note: only valid at or below the layer interface

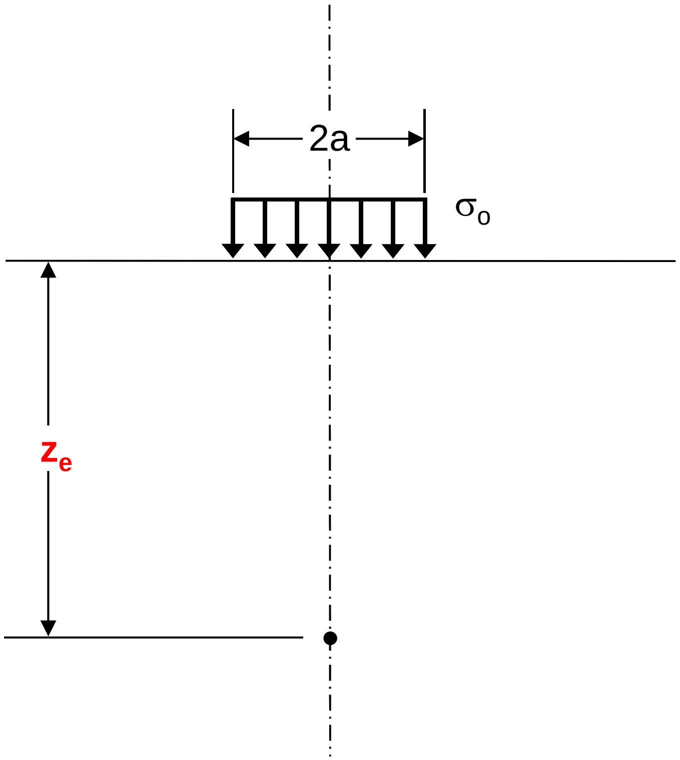
Stresses Due to Circular Load

Note: equations are only valid along load centerline

$$\sigma_z = \sigma_o \left\{ 1 - \left[\frac{z_e/a}{\sqrt{1+(z_e/a)^2}} \right]^3 \right\}$$

$$\sigma_r = \frac{\sigma_o}{2} \left\{ 1 + 2\mu - \frac{2(1+\mu)(z_e/a)}{\sqrt{1+(z_e/a)^2}} + \left[\frac{z_e/a}{\sqrt{1+(z_e/a)^2}} \right]^3 \right\}$$

$$\sigma_t = \frac{\sigma_o}{2} \left\{ 1 + 2\mu - \frac{2(1+\mu)(z_e/a)}{\sqrt{1+(z_e/a)^2}} + \left[\frac{z_e/a}{\sqrt{1+(z_e/a)^2}} \right]^3 \right\}$$



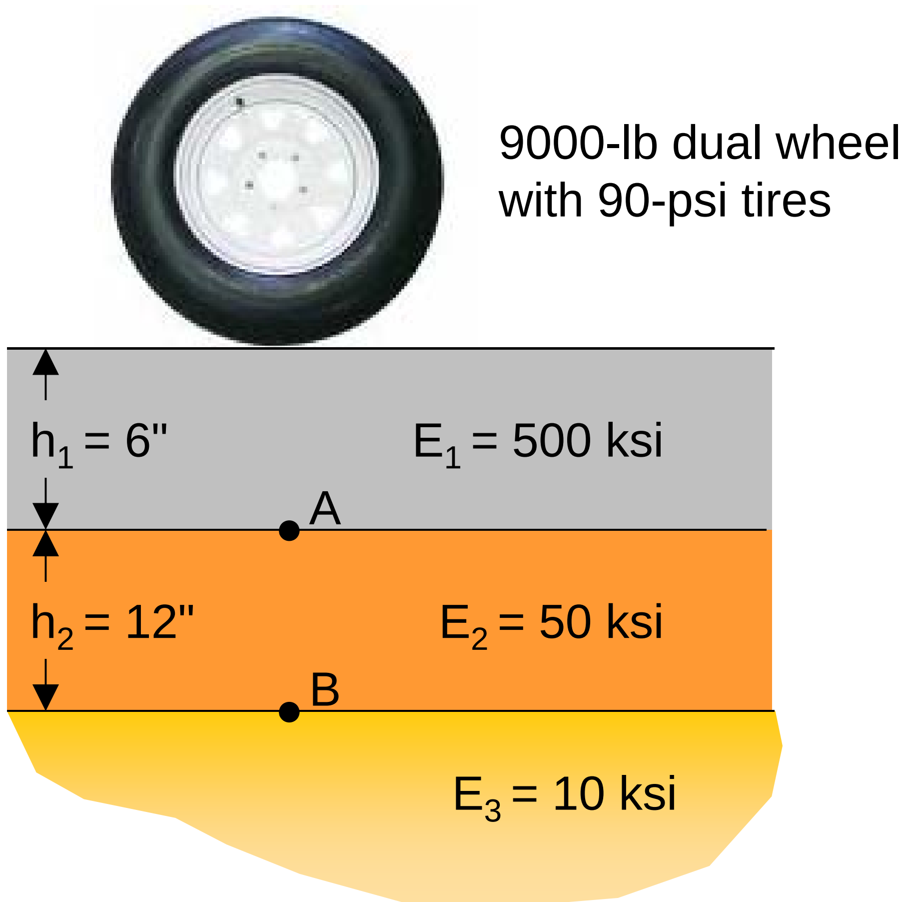
Deflections Due to Circular Load

Note: equations are only valid along load centerline

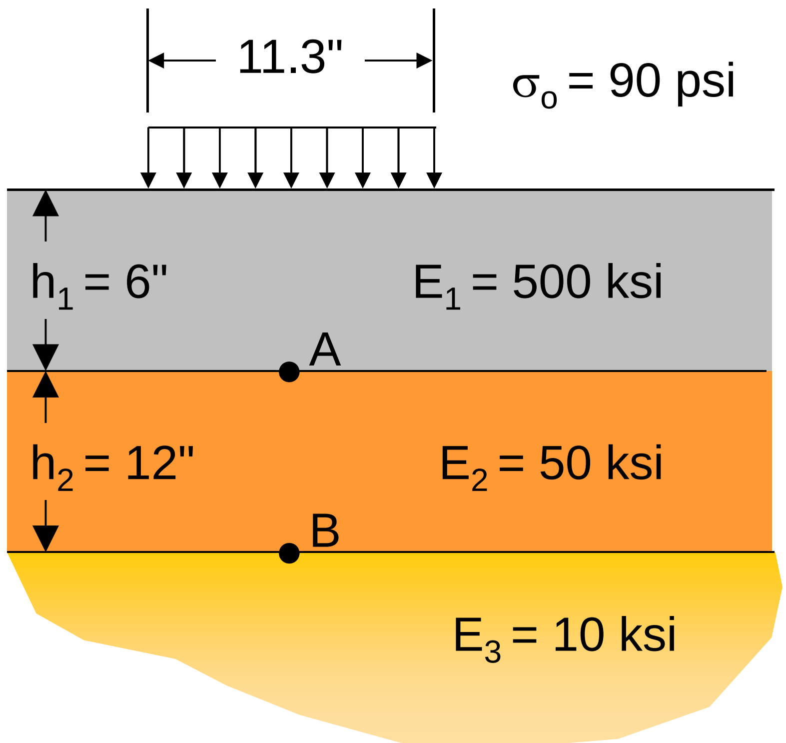
$$d_z = \int_{z_e}^{\infty} \varepsilon_z dz = \frac{\sigma_o a(1+\mu)}{E}\left\{\frac{1}{\sqrt{1+(z_e/a)^2}} + (1-2\mu)\left[\sqrt{1+(z_e/a)^2} - (z_e/a)\right]\right\}$$

$$d_o = \frac{2\sigma_o a(1-\mu^2)}{E} \quad @ \quad z = 0$$

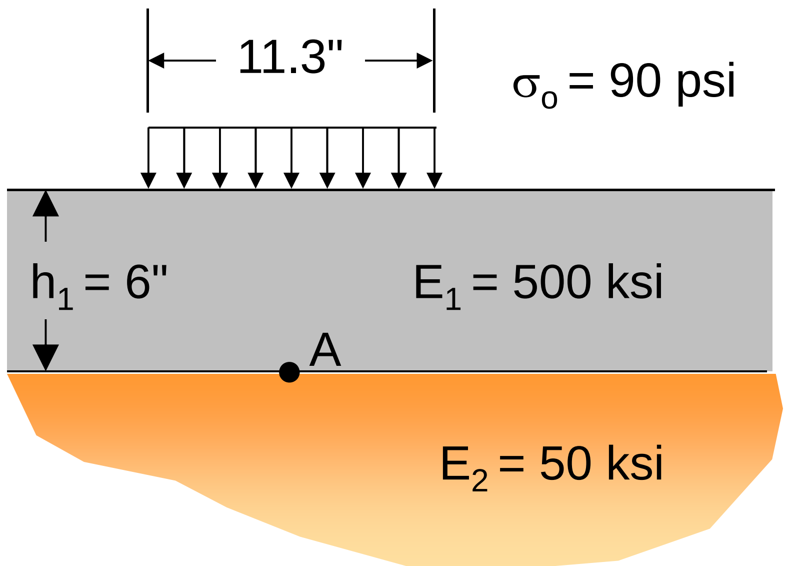
Odemark Example



Odemark Example



Odemark Example



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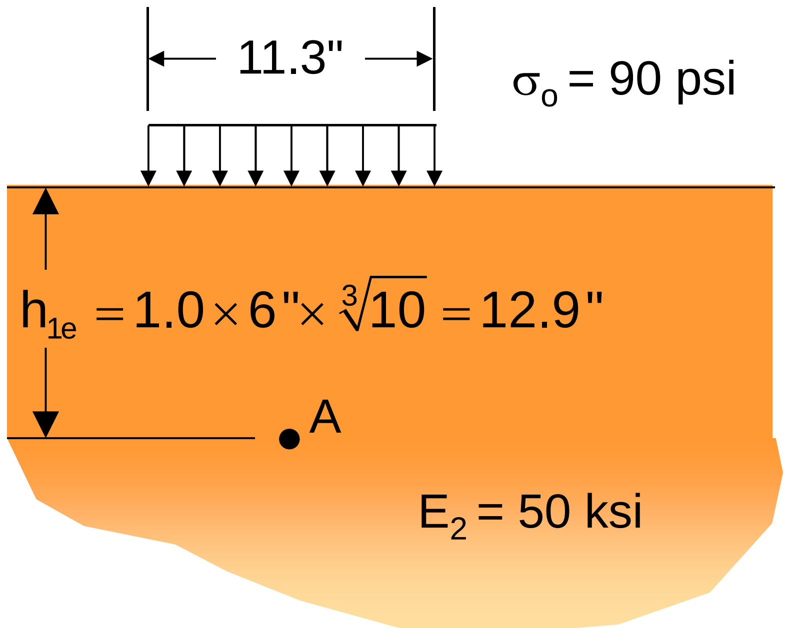

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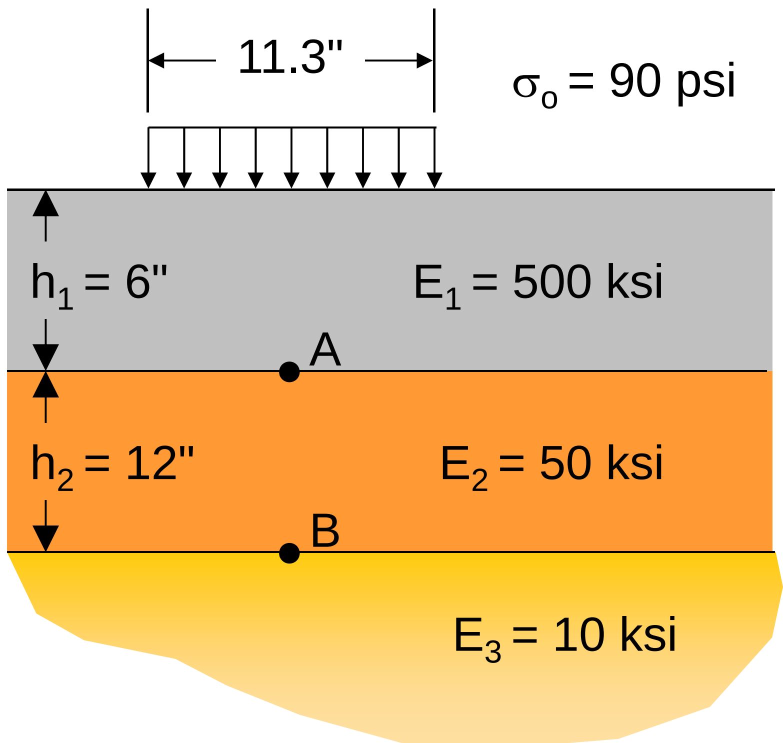

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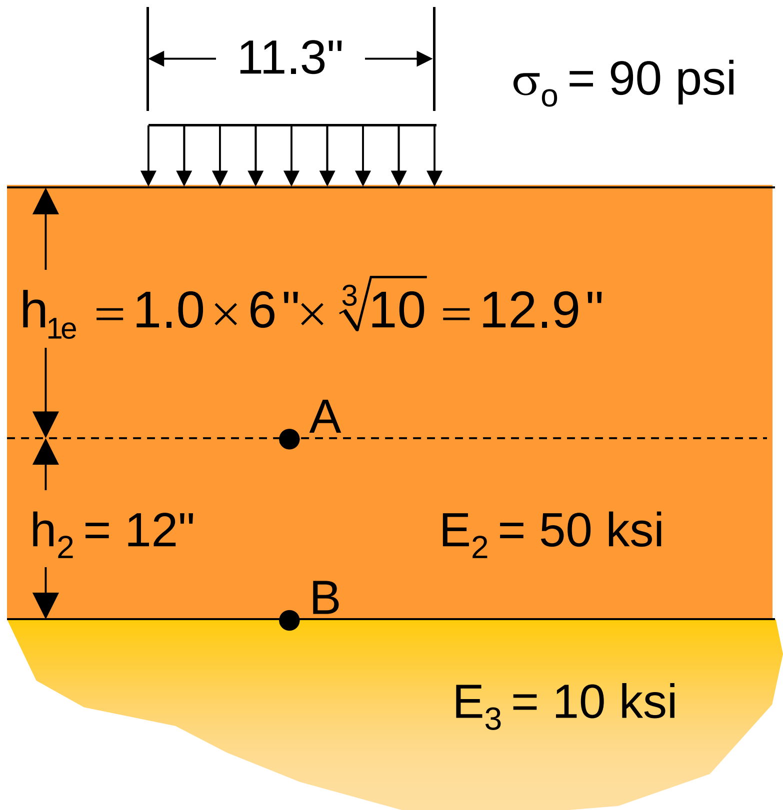
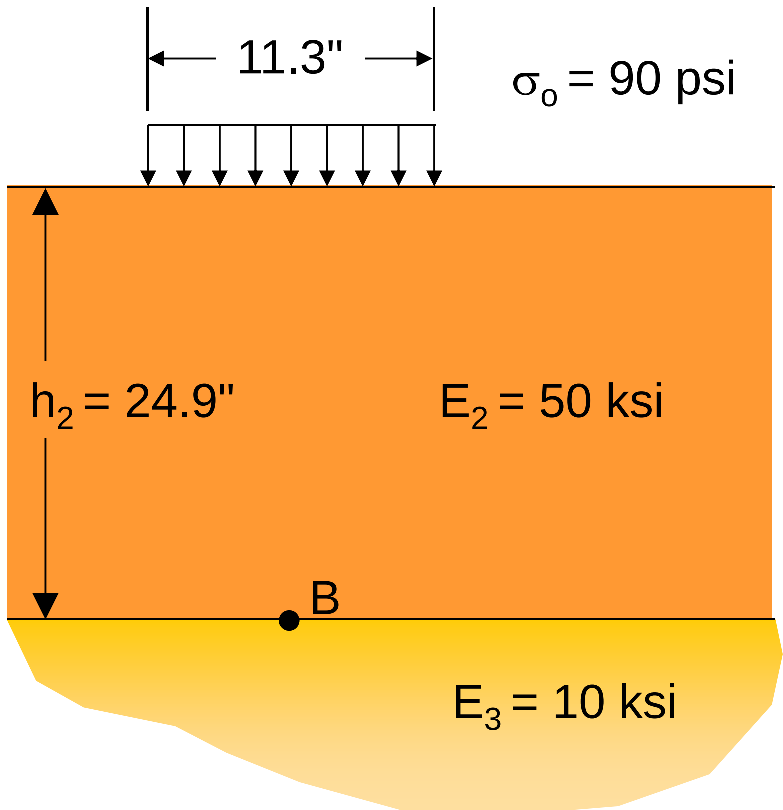


Odemark Example



Odemark Example