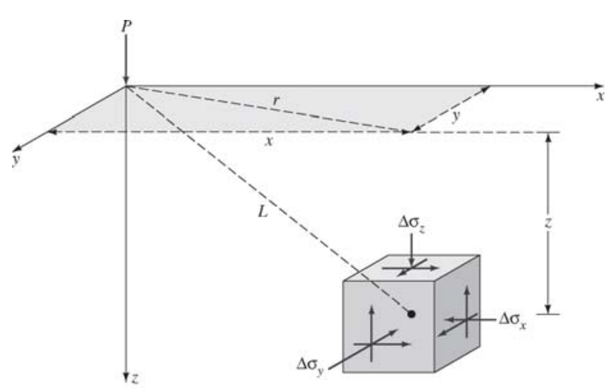
Vertical Stress Increases

Chapter 8

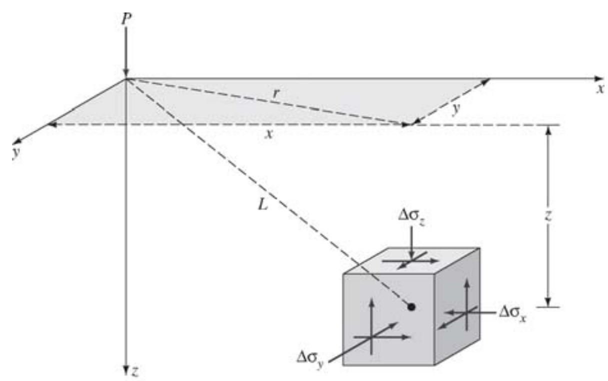
Point Load

$$\Delta\sigma_x = \frac{P}{2\pi}\left\{\frac{3x^2z}{L^5} - (1 - 2\mu_S)\left[\frac{x^2 - y^2}{Lr^2(L + z)} + \frac{y^2z}{L^3r^2}\right]\right\}$$



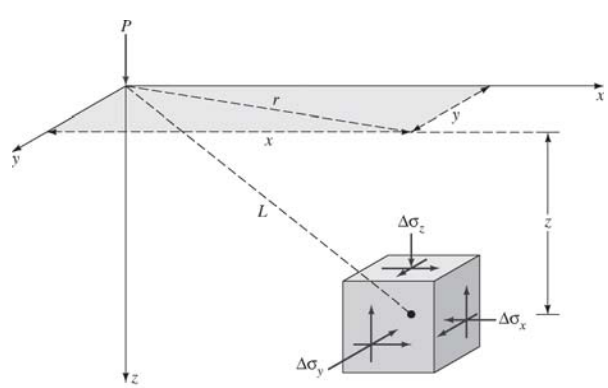
Point Load

$$\Delta\sigma_y = \frac{P}{2\pi}\left\{\frac{3y^2z}{L^5} - (1-2\mu_S)\left[\frac{y^2-x^2}{Lr^2(L+z)} + \frac{x^2z}{L^3r^2}\right]\right\}$$



Point Load

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}} = \frac{3P}{2\pi}\frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$



Point Load

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}} = \frac{3P}{2\pi}\frac{1/z^2}{[(r/z)^2+1]^{5/2}} = \frac{P}{z^2}I_1$$

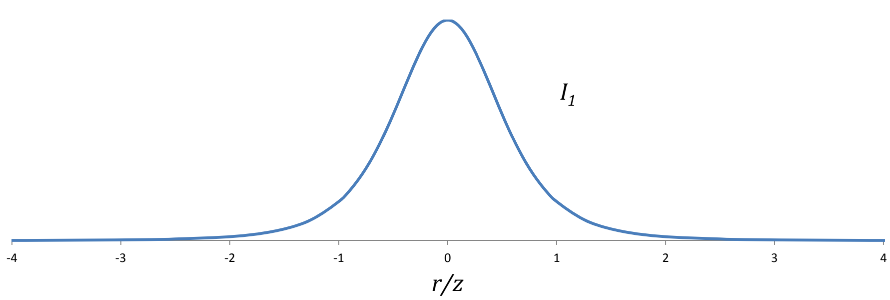
$$I_1 = \frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}$$

Table 8.2 Variation of I_1 [Eq. (8.19)]

r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

Point Load

$$I_1 = \frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}$$



Example

A vertical point load of 25 kN is applied at the ground surface. What is the stress increase in the ground 2 m below the ground surface directly beneath the load?

$$\Delta\sigma = \frac{P}{z^2} I_1 = \frac{25\ kN}{(2\ m)^2}(0.4775) = 2.98\ kN/m^2$$

Table 8.2 Variation of I_1 [Eq. (8.19)]

r = 0, z = 2

r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

Example

A vertical point load of 25 kN is applied at the ground surface. What is the stress increase in the ground 2 m below the ground surface and 2 m to the side?

$$\Delta\sigma = \frac{P}{z^2} I_1 = \frac{25\ kN}{(2\ m)^2}(0.0844) = 0.5275\ kN/m^2$$

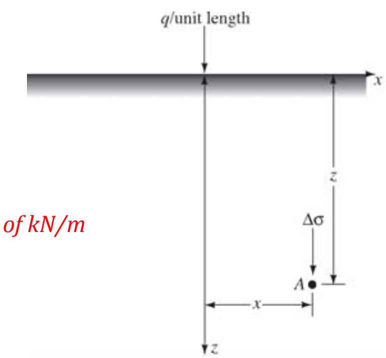
Table 8.2 Variation of I_1 [Eq. (8.19)]

r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

r = 2, z = 2

Line Load

$$\Delta\sigma = \frac{2qz^3}{\pi(x^2 + z^2)^2}$$



NOTE: q has units of kN/m

Line Load

$$\Delta\sigma = \frac{2qz^3}{\pi(x^2 + z^2)^2} = \frac{2q}{\pi z}\frac{1}{[(x/z)^2 + 1]^2} = \frac{q}{z}\frac{2/\pi}{[(x/z)^2 + 1]^2} = \frac{q}{z}I$$

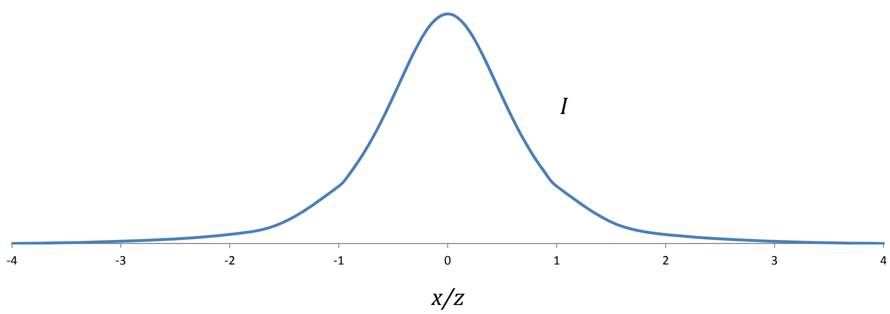
$$I = \frac{\Delta\sigma}{(q/z)} = \frac{2/\pi}{[(x/z)^2 + 1]^2}$$

Table 8.3 Variation of $\Delta\sigma/(q/z)$ with x/z [Eq. (8.22)]

x/z	$\frac{\Delta\sigma}{q/z}$	x/z	$\frac{\Delta\sigma}{q/z}$
0	0.637	0.7	0.287
0.1	0.624	0.8	0.237
0.2	0.589	0.9	0.194
0.3	0.536	1.0	0.159
0.4	0.473	1.5	0.060
0.5	0.407	2.0	0.025
0.6	0.344	3.0	0.006

Line Load

$$I = \frac{2/\pi}{[(x/z)^2 + 1]^2}$$



Example

A vertical line load of 8 kN/m is applied at the ground surface. What is the stress increase in the ground 4 m below the ground surface directly beneath the load?

$$\Delta\sigma = \frac{q}{z} I = \frac{8\ kN/m}{4\ m}(0.637) = 1.274\ kN/m^2$$

Table 8.3 Variation of $\Delta\sigma/(q/z)$ with x/z [Eq. (8.22)]

x = 0, z = 4

x/z	$\frac{\Delta\sigma}{q/z}$	x/z	$\frac{\Delta\sigma}{q/z}$
0	0.637	0.7	0.287
0.1	0.624	0.8	0.237
0.2	0.589	0.9	0.194
0.3	0.536	1.0	0.159
0.4	0.473	1.5	0.060
0.5	0.407	2.0	0.025
0.6	0.344	3.0	0.006

Example

A vertical line load of 8 kN/m is applied at the ground surface. What is the stress increase in the ground 4 m below the ground surface and 2 m off to the side?

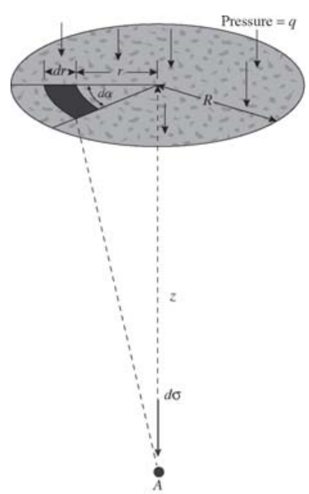
$$\Delta\sigma = \frac{q}{z} I = \frac{8\ kN/m}{4\ m}(0.407) = 0.814\ kN/m^2$$

Table 8.3 Variation of $\Delta\sigma/(q/z)$ with x/z [Eq. (8.22)]

x/z	$\frac{\Delta\sigma}{q/z}$	x/z	$\frac{\Delta\sigma}{q/z}$
0	0.637	0.7	0.287
0.1	0.624	0.8	0.237
0.2	0.589	0.9	0.194
0.3	0.536	1.0	0.159
0.4	0.473	1.5	0.060
0.5	0.407	2.0	0.025
0.6	0.344	3.0	0.006

x = 2, z = 4

Circular Loaded Area



$$\Delta\sigma = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\} = qI$$

NOTE: q has units of kN/m² or kPa

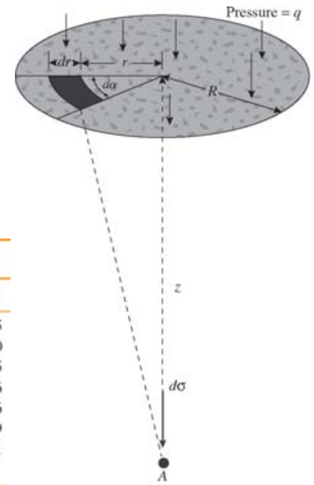
Circular Loaded Area

$$\Delta\sigma = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\} = qI$$

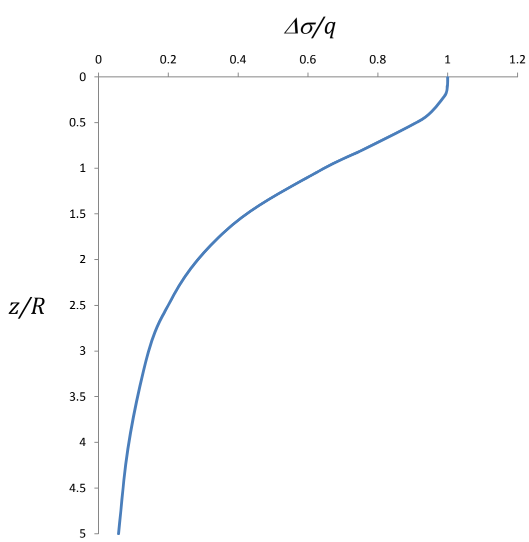
$$I = \frac{\Delta\sigma}{q} = 1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}$$

Table 8.4 Variation of $\Delta\sigma/q$ with z/R [Eq. (8.24)]

z/R	$\Delta\sigma/q$	z/R	$\Delta\sigma/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.1	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		



Circular Loaded Area



Example

A uniform load of 80 kPa is applied at the ground surface over a circular area with a diameter of 8 m. What is the stress increase directly beneath the center of the loaded area at a depth of 8 m?

$$\Delta\sigma = qI = (80\ kPa)(0.2845) = 22.76\ kPa$$

Table 8.4 Variation of $\Delta\sigma/q$ with z/R [Eq. (8.24)]

z/R	$\Delta\sigma/q$	z/R	$\Delta\sigma/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.1	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

z = 8, R = 4

Circular Loaded Area

$$\frac{\Delta\sigma}{q} = I_2$$

Table 8.5 Variation of I_2 [Eq. (8.25)]

	r/R					
z/R	0	0.2	0.4	0.6	0.8	1.0
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.999	0.999	0.998	0.996	0.976	0.484
0.2	0.992	0.991	0.987	0.970	0.890	0.468
0.3	0.976	0.973	0.963	0.922	0.793	0.451
0.4	0.949	0.943	0.920	0.860	0.712	0.435
0.5	0.911	0.902	0.869	0.796	0.646	0.417
0.6	0.864	0.852	0.814	0.732	0.591	0.400
0.7	0.811	0.798	0.756	0.674	0.545	0.367
0.8	0.756	0.743	0.699	0.619	0.504	0.366
0.9	0.701	0.688	0.644	0.570	0.467	0.348
1.0	0.646	0.633	0.591	0.525	0.434	0.332
1.2	0.546	0.535	0.501	0.447	0.377	0.300
1.5	0.424	0.416	0.392	0.355	0.308	0.256
2.0	0.286	0.286	0.268	0.248	0.224	0.196
2.5	0.200	0.197	0.191	0.180	0.167	0.151
3.0	0.146	0.145	0.141	0.135	0.127	0.118
4.0	0.087	0.086	0.085	0.082	0.080	0.075

Example

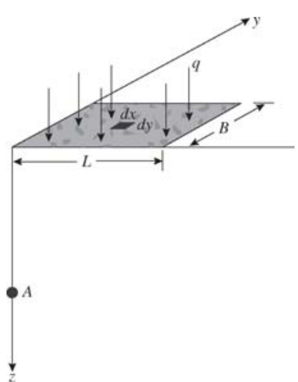
A uniform load of 80 kPa is applied at the ground surface over a circular area with a diameter of 8 m. What is the stress increase 2 m below the ground surface at a radial distance of 4 m from the load centerline?

$$\Delta\sigma = qI_2 = (80\ kPa)(0.417) = 33.36\ kPa$$

Table 8.5 Variation of I_2 [Eq. (8.25)]

	r/R					
z/R	0	0.2	0.4	0.6	0.8	1.0 (r = 4 = R)
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.999	0.999	0.998	0.996	0.976	0.484
0.2	0.992	0.991	0.987	0.970	0.890	0.468
0.3	0.976	0.973	0.963	0.922	0.793	0.451
0.4	0.949	0.943	0.920	0.860	0.712	0.435
z = 8, R = 4 — 0.5	0.911	0.902	0.869	0.796	0.646	0.417
0.6	0.864	0.852	0.814	0.732	0.591	0.400
0.7	0.811	0.798	0.756	0.674	0.545	0.367
0.8	0.756	0.743	0.699	0.619	0.504	0.366
0.9	0.701	0.688	0.644	0.570	0.467	0.348

Rectangular Loaded Area



$$\Delta\sigma = qI_3$$

$$I_3 = \frac{1}{4\pi}\left[\frac{2m'n'\sqrt{m'^2+n'^2+1}}{m'^2+n'^2+m'^2n'^2+1}\left(\frac{m'^2+n'^2+2}{m'^2+n'^2+1}\right) + \tan^{-1}\left(\frac{2m'n'\sqrt{m'^2+n'^2+1}}{m'^2+n'^2-m'^2n'^2+1}\right)\right]$$

$$m' = \frac{B}{z} \qquad n' = \frac{L}{z}$$

$$ATAN2(m'^2+n'^2-m'^2n'^2+1\,,2m'n'\sqrt{m'^2+n'^2+1})$$

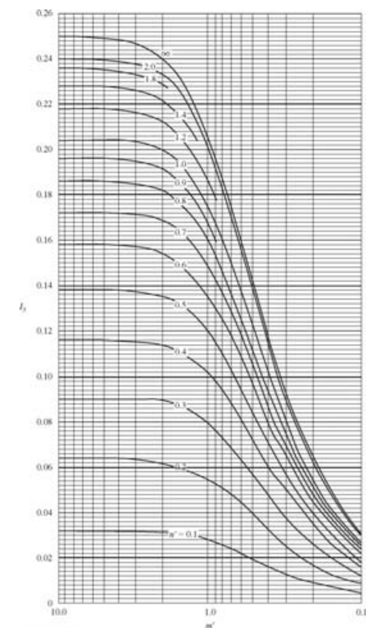


Figure 8.13 Variation of I_3 with m' and n'

Example

A uniform load of 80 kPa is applied at the ground surface over a rectangular area with a length of 5 m and a width of 2 m. What is the stress increase beneath the corner of the loaded area at a depth of 10 m?

$$m' = \frac{B}{z} = \frac{2}{10} = 0.2$$

$$n' = \frac{L}{z} = \frac{5}{10} = 0.5$$

$$\Delta\sigma = qI_3 = (80\ kPa)(0.04) = 3.2\ kPa$$

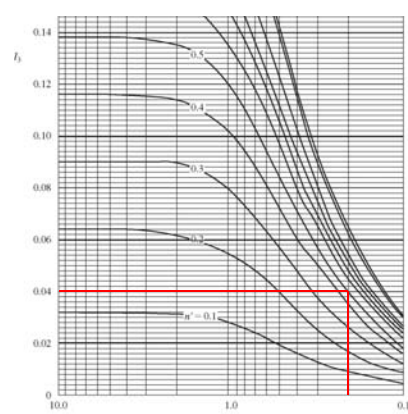


Figure 8.13 Variation of I_3 with m' and n'

Example

A uniform load of 80 kPa is applied at the ground surface over a rectangular area with a length of 5 m and a width of 2 m. What is the stress increase beneath the corner of the loaded area at a depth of 10 m?

$$m' = \frac{B}{z} = \frac{5}{10} = 0.5$$

$$n' = \frac{L}{z} = \frac{2}{10} = 0.2$$

$$\Delta\sigma = qI_3 = (80\ kPa)(0.04) = 3.2\ kPa$$

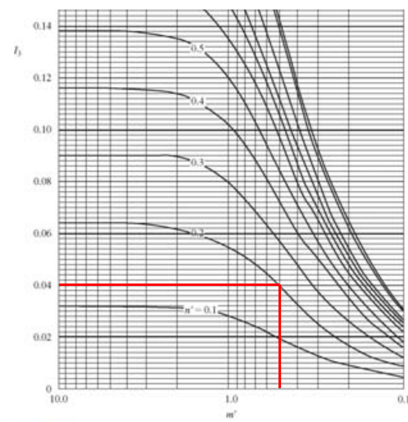
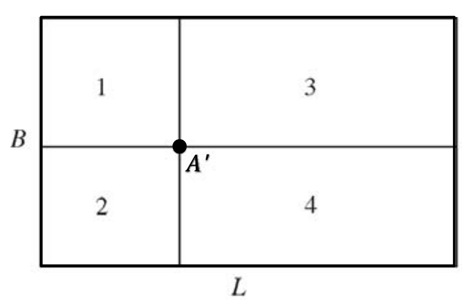
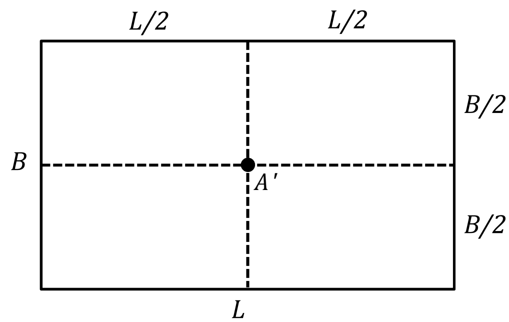


Figure 8.13 Variation of I_3 with m' and n'

Rectangular Loaded Area



Rectangular Loaded Area



$$\Delta\sigma = qI_c$$

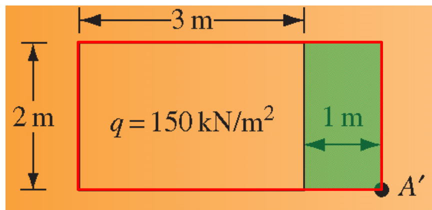
Table 8.6 Variation of I_c with m_1 and n_1 [Eq. (8.33)]

	$m_1 = L/B$									
$z/(B/2) = n_1$	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

Example

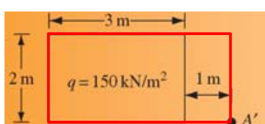
Example 8.5

The plan of a uniformly loaded rectangular area is shown in Figure 8.15a. Determine the vertical stress increase, $\Delta\sigma$, below point A' at a depth $z = 4$ m.



(a)

Example



(a)

$$m' = \frac{B}{z} = \frac{4}{4} = 1.0$$

$$n' = \frac{L}{z} = \frac{2}{4} = 0.5$$

$$\Delta\sigma = qI_3 = (150\ kPa)(0.12) = 18\ kPa$$

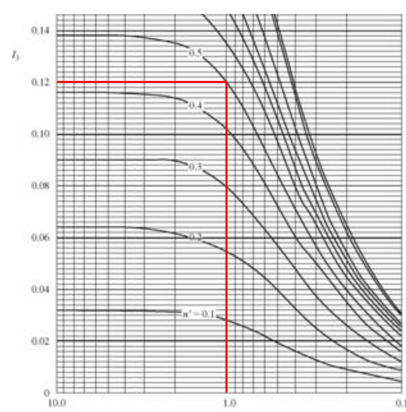


Figure 8.13 Variation of I_3 with m' and n'

Example

$$I_3 = \frac{1}{4\pi}\left[\frac{2m'n'\sqrt{m'^2+n'^2+1}}{m'^2+n'^2+m'^2n'^2+1}\left(\frac{m'^2+n'^2+2}{m'^2+n'^2+1}\right)+\tan^{-1}\left(\frac{2m'n'\sqrt{m'^2+n'^2+1}}{m'^2+n'^2-m'^2n'^2+1}\right)\right]$$

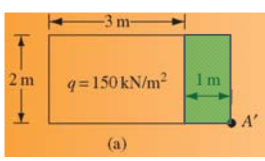
$$m'n' = (1)(0.5) = 0.5$$

$$m'^2 = 1^2 = 1 \qquad n'^2 = 0.5^2 = 0.25$$

$$m'^2n'^2 = (1)(0.25) = 0.25$$

$$I_3 = \frac{1}{4\pi}\left[\frac{2(1)(0.5)\sqrt{1+0.25+1}}{1+0.25+0.25+1}\left(\frac{1+0.25+2}{1+0.25+1}\right)\right]+\tan^{-1}\left(\frac{2(1)(0.5)\sqrt{1+0.25+1}}{1+0.25-0.25+1}\right) = 0.120175$$

Example



$$m' = \frac{B}{z} = \frac{1}{4} = 0.25$$

$$n' = \frac{L}{z} = \frac{2}{4} = 0.5$$

$$\Delta\sigma = qI_3 = (-150\ kPa)(0.048) = -7.2\ kPa$$

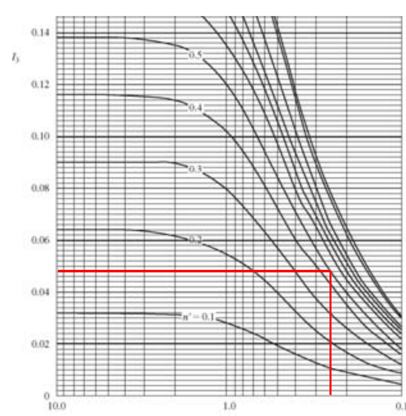


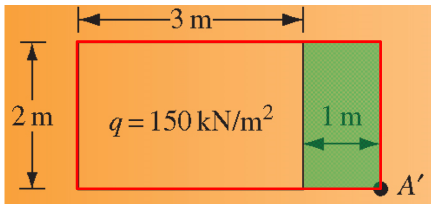
Figure 8.13 Variation of I_3 with m' and n'

Example

Example 8.5

The plan of a uniformly loaded rectangular area is shown in Figure 8.15a. Determine the vertical stress increase, $\Delta\sigma$, below point A' at a depth $z = 4$ m.

$$\Delta\sigma = 18.0 - 7.2 = 10.8\ kPa$$



(a)