Seepage

Chapter 7

Laplace’s Equation
Laplace’s Equation

\[ \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \]

No storage
No drainage
\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \]

Darcy’s Law
\[ v_z = k \frac{\partial h}{\partial z} \]

Anisotropic Soil
\[ k \frac{\partial^2 h}{\partial x^2} + k \frac{\partial^2 h}{\partial z^2} = 0 \]

Isotropic Soil
\[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \]
Solution of Laplace’s Equation

Since water can only flow due to a change in head, and equipotential lines are lines of constant head, the flow lines must be perpendicular to the equipotential lines everywhere.

Flow Nets

Upstream and downstream surfaces are equipotential lines
Flow Nets

All impervious boundaries are flow lines

Flow lines and equipotential lines are orthogonal
Flow Nets

Ideally, the “cells” in the flow net should be “square”

Flow Net Example

Flow lines are ⊥ to upstream ground surface
Flow lines are ⊥ to face of toe filter faces
Equipotential lines are ⊥ to impervious boundaries
Flow lines are ⊥ to equipotential lines everywhere
“Cells in the flow net are “square” everywhere
Common Flow Net Mistakes

- Flow Lines Not Perpendicular
- Extraneous Equipotential Lines
- Not Square
- Disappearing Flow Lines
- Not a right angle
- Equipotential Lines Not Perpendicular
Correctly Drawn Flow Net

\[ \Delta q_1 = k_i A_1 = k \left( \frac{h_1 - h_2}{l_1} \right) (l_1 \times 1) = k (h_1 - h_2) \]
Seepage Calculations

\[ \Delta q_1 = \Delta q_2 = \Delta q_3 = \cdots = \Delta q \]

Seepage Calculations

\[ \Delta q = k(h_1 - h_2) = k(h_2 - h_3) = k(h_3 - h_4) = \cdots \]
Seepage Calculations

\[ \Delta q = k \frac{H}{N_d} \]

\( \Delta q \) — Total head loss

\( k \) — # of head drops

Seepage Calculations

\[ q = N_f \Delta q = kH \frac{N_f}{N_d} \]

\( q \) — 3 flow channels

\( N_f \) — 8 head drops
Flow Nets

Impervious layer

$k_x = k_y = k$
$N_f = 4$
$N_d = 6$

Flow Nets

$k_x = k_y = k$
$N_f = 5$
$N_d = 9$
Example

1. What is the head at points a, b, c, d and e?
2. What is the seepage rate beneath the dam?

Partial Flow Channels

\[ \Delta q = k \left( \frac{h_1 - h_2}{l_1} \right) b_1 = k \left( \frac{h_2 - h_3}{l_2} \right) b_2 = k \left( \frac{h_3 - h_4}{l_3} \right) b_3 = \cdots \]

\[ \frac{b_1}{l_1} = \frac{b_2}{l_2} = \frac{b_3}{l_3} = \cdots = n \]
Partial Flow Channels

\[ \Delta q = k(h_1 - h_2)n = k(h_2 - h_3)n = k(h_3 - h_4)n = \cdots \]

\[ \frac{b_1}{l_1} = \frac{b_2}{l_2} = \frac{b_3}{l_3} = \cdots = n \]

Partial Flow Channels

\[ \Delta q = kH \left( \frac{n}{N_d} \right) \]

\[ \frac{b_1}{l_1} = \frac{b_2}{l_2} = \frac{b_3}{l_3} = \cdots = n \]
Partial Flow Channels

\[ q = k \frac{HN_f}{N_d} \]

- 2.38 flow channels
- 6 head drops

Flow channel 1
- \( b \approx 1 \)
- \( T = 1 \)

Flow channel 2
- \( b \approx 1 \)
- \( T = 1 \)

Flow channel 3
- \( b \approx 0.38 \)
- Impervious layer

2.38 flow channels