Homework 6 Solution

Problem 8.6

The groundwater table starts out at the bottom of the sand layer. If it rises to the middle of the sand layer, two things happen: (a) the total stress goes up because the dry sand in the lower half of the layer has been replaced by saturated sand, which weighs more, and (b) the porewater pressure goes up because the clay layer is under an additional 2 m of water.

The dry unit weight of the sand can be calculated from Equation 3.18 as

$$\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{(2.68)(9.81)}{1+0.6} = 16.43\,\frac{\text{kN}}{\text{m}^3}$$

and the saturated unit weight of the sand can be calculated from Equation 3.20 as

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.68+0.6)(9.81)}{1+0.6} = 20.11\,\frac{\text{kN}}{\text{m}^3}$$

So the total stress goes up by

$$\Delta\sigma = 2\left(\gamma_{sat} - \gamma_d\right) = 2(20.11 - 16.43) = 7.36\,\frac{\text{kN}}{\text{m}^2}$$

and the porewater pressure goes up by

$$\Delta u = 2\gamma_w = 2(9.81) = 19.62\,\frac{\text{kN}}{\text{m}^2}$$

Therefore, the effective stress goes up by

$$\Delta\sigma' = \Delta\sigma - \Delta u = 7.36 - 19.62 = -12.26\,\frac{\text{kN}}{\text{m}^2}$$

So the total stress rises and the porewater pressure rises, but the effective stress drops by 12.26 kPa because 2 m of dry sand (with a unit weight of 16.43 kN/m^3) has been replaced by 2 m of buoyant sand (with a unit weight of $\gamma' = \gamma_{sat} - \gamma_w$ = 20.11 – 9.81 = 10.30 kN/m^3).

Problem 8.12

For each of the three loads, the stress increase 3 m below the ground surface at Point D can be found using the Bouissinesq equation for the stress increase due to a point load:

$$\Delta\sigma = \frac{P}{z^2}\left(\frac{3}{2\pi}\right)\left[\left(\frac{r}{z}\right)^2 + 1\right]^{-5/2} = \frac{P}{6\pi}\left[\left(\frac{r}{3}\right)^2 + 1\right]^{-5/2}$$

For the load at Point A, P = 9 kN and r = $\sqrt{3^2 + 1.5^2}$ = 3.35 m:

$$\Delta\sigma = \left(\frac{9}{6\pi}\right)\left[\left(\frac{3.35}{3}\right)^2 + 1\right]^{-5/2} = 0.063\,\frac{\text{kN}}{\text{m}^2}$$

For the load at Point B, P = 18 kN and r = $\sqrt{3^2 + 1.5^2}$ = 3.35 m:

$$\Delta\sigma = \left(\frac{18\text{ kN}}{6\pi}\right)\left[\left(\frac{3.35}{3}\right)^2 + 1\right]^{-5/2} = 0.126\,\frac{\text{kN}}{\text{m}^2}$$

For the load at Point C, P = 27 kN and r = 1.5 m:

$$\Delta\sigma = \left(\frac{27\text{ kN}}{6\pi}\right)\left[\left(\frac{1.5}{3}\right)^2 + 1\right]^{-5/2} = 0.820\,\frac{\text{kN}}{\text{m}^2}$$

Summing the three increases:

$$\Delta\sigma = 0.063 + 0.126 + 0.820 = 1.009\,\frac{\text{kN}}{\text{m}^2}$$

Problem 8.14

For each of the line loads, the stress increase 2 m below the ground surface at Point A can be found using the Bouissinesq equation for the stress increase due to a line load:

$$\Delta\sigma = \frac{2q}{\pi z}\left[\left(\frac{x}{z}\right)^2 + 1\right]^{-2} = \frac{q}{\pi}\left[\left(\frac{x}{2}\right)^2 + 1\right]^{-2}$$

For the line load furthest from Point A, q_1 = 100 kN/m and x = x_1 + x_2 = 5 m:

$$\Delta\sigma = \frac{100}{\pi}\left[\left(\frac{5}{2}\right)^2 + 1\right]^{-2} = 0.606\,\frac{\text{kN}}{\text{m}^2}$$

For the line load closest to Point A, q_1 = 200 kN/m and x = x_2 = 2 m:

$$\Delta\sigma = \frac{200}{\pi}\left[\left(\frac{2}{2}\right)^2 + 1\right]^{-2} = 15.915\,\frac{\text{kN}}{\text{m}^2}$$

Summing the two increases:

$$\Delta\sigma = 0.606 + 15.915 = 16.52\,\frac{\text{kN}}{\text{m}^2}$$

Problem 8.15

Using the Bouissinesq equation for the stress increase beneath the center of a circularly load area:

$$\Delta\sigma = q\left\{1 - \left[\left(\frac{R}{z}\right)^2 + 1\right]^{-3/2}\right\} = 250\frac{\text{kN}}{\text{m}^2}\left\{1 - \left[\left(\frac{3}{5}\right)^2 + 1\right]^{-3/2}\right\} = 92.37\,\frac{\text{kN}}{\text{m}^2}$$

Problem 8.16a

For the stress increase 5 m below Point A, we can use the equation for the stress increase beneath the corner of a uniformly loaded rectangular area:

$$\Delta\sigma = qI_3$$

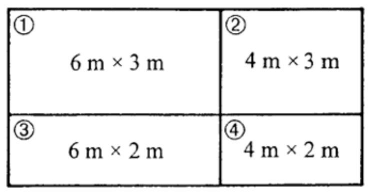
From the geometry of the rectangle, $m' = B/z = 5/5 = 1$ and $n' = L/z = 10/5 = 2$. From Figure 8.13 we can estimate $I_3 = 0.202$ (the equation gives $I_3 = 0.200$) so

$$\Delta\sigma = qI_3 = 400\frac{kN}{m^2}(0.202) = 80.8\frac{kN}{m^2}$$

Since the influence factor only has 3 significant digits, I can only express the answer to 3 significant digits.

Problem 8.16b

For the stress increase 5 m below Point B, we have to break the large rectangle into four small rectangles and sum the contributions from each:



For Rectangle ①, $m' = B/z = 3/5 = 0.6$ and $n' = L/z = 6/5 = 1.2$. From Figure 8.13 it is hard to estimate I_3 because the $n' = 1.2$ curve disappears. If we swap m' and n', though, we can estimate $I_3 = 0.142$ from the curve for $n' = 0.6$[*] (the equation gives $I_3 = 0.143$).

For Rectangle ②, $m' = B/z = 3/5 = 0.6$ and $n' = L/z = 4/5 = 0.8$. From Figure 8.13, $I_3 = 0.125$ (the equation gives $I_3 = 0.125$).

For Rectangle ③, $m' = B/z = 2/5 = 0.4$ and $n' = L/z = 6/5 = 1.2$. From Figure 8.13 it is hard to estimate I_3 because the $n' = 1.2$ curve disappears. If we swap m' and n', though, we can estimate $I_3 = 0.106$ from the curve for $n' = 0.4$ (the equation gives $I_3 = 0.106$).

For Rectangle ④, $m' = B/z = 2/5 = 0.4$ and $n' = L/z = 4/5 = 0.8$. From Figure 8.13, $I_3 = 0.094$ (the equation gives $I_3 = 0.093$).

Summing the contributions from the four rectangular areas:

$$\Delta\sigma = qI_3 = 400\frac{kN}{m^2}(0.142 + 0.125 + 0.106 + 0.094) = 400\frac{kN}{m^2}(0.467) = 187\frac{kN}{m^2}$$

Since the influence factor only has 3 significant digits, I can only express the answer to 3 significant digits.

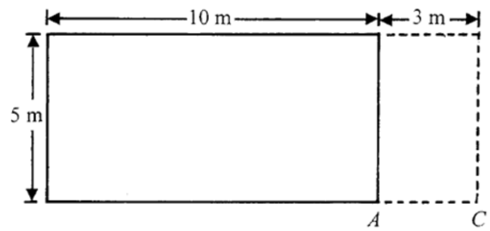
[*] To estimate where $m' = 1.2$ falls between $m' = 1$ and $m' = 2$ on the logarithmic axis, note that

$$\frac{\log 1.2 - \log 1}{\log 2 - \log 1} = 0.26$$

so the point would fall approximately one quarter of the way between the gridlines at $m' = 1$ and $m' = 2$.

Problem 8.16c

For the stress increase 5 m below Point C, we have to extend the foundation rectangle out to Point C and calculate the resulting stress increase, then subtract out the stress increase produced by the extension:



For the extended rectangle, $m' = B/z = 5/5 = 1.0$ and $n' = L/z = 13/5 = 2.6$. In Figure 8.13 there is no curve for $n' = 2.6$, but if we swap m' and n', there is a curve for $n' = 1$ and we can estimate $I_3 = 0.202$ (the equation gives $I_3 = 0.203$).

For the extension, $m' = B/z = 5/5 = 1.0$ and $n' = L/z = 3/5 = 0.6$. In Figure 8.13 we can estimate $I_3 = 0.137$ (the equation gives $I_3 = 0.136$).

Subtracting the contribution of the extension from that of the extended rectangle:

$$\Delta\sigma = qI_3 = 400\frac{kN}{m^2}(0.202 - 0.136) = 400\frac{kN}{m^2}(0.066) = 26.4\frac{kN}{m^2}$$

Again, I've rounded the final answer to 3 significant digits because the influence factors are only accurate to 3 significant digits.

Problem 8.17

For this problem we have to use Table 8.5, which gives the influence factors for a uniformly loaded circular area as a function of depth and radial offset from the axis of symmetry. Given the geometry of the problem, $R = z = r = 4$, so $r/R = z/R = 1$. From the table, $I_2 = 0.332$ so

$$\Delta\sigma = qI_2 = 320\frac{kN}{m^2}(0.332) = 106\frac{kN}{m^2}$$

Again, I've had to round the answer to 3 significant figures to accommodate the accuracy of the influence factor.