Chapter 7.
Flexural Analysis of T-Beams

7.1. Reading Assignments
Text Chapter 3.7; ACI 318, Section 8.10.

7.2. Occurrence and Configuration of T-Beams
- Common construction type.- used in conjunction with either one-way or two-way slabs.
- Sections consists of the flange and web or stem; the slab forms the beam flange, while the part of the beam projecting below the slab forms is what is called web or stem.

(a) one-way slab  
(b) two-way slab

7.3. Concepts of the effective width, Code allowable values
In reality the maximum compression stress in T-section varies with distance from section Web.
Code allows the following maximum effective widths:

7.3.1. Symmetrical Beam

ACI318, Section 8.10.2.

\[
\begin{align*}
1) & \quad b \leq \frac{\text{span}}{4} \\
2) & \quad \frac{b - b_w}{2} \leq 8h_f \\
3) & \quad \frac{b - b_w}{2} \leq \frac{1}{2} \text{ clear distance between beams}
\end{align*}
\]

7.3.2. Flange on one side only (Spandrel Beam)

ACI318, Section 8.10.3.

\[
\begin{align*}
1) & \quad b - b_w \leq \frac{\text{span}}{12} \\
2) & \quad b - b_w \leq 6h_f \\
3) & \quad b - b_w \leq \frac{1}{2} \text{ clear distance to next web}
\end{align*}
\]

7.3.3. Isolated T-Beam

ACI318, Section 8.10.2.

\[
\begin{align*}
1) & \quad b \leq 4b_w \\
2) & \quad \frac{b_w}{2} \leq h_f
\end{align*}
\]
7.4. Analysis of T-Beams - (a > hf)

Consider the total section in two parts:

1) Flange overhangs and corresponding steel;
2) Stem and corresponding steel;

For equilibrium we have:

7.4.1. Case I:

\[ A_{sf} f_y = 0.85 f'_c h_f (b - b_w) \]  \hspace{1cm} (8.1)

or

\[ A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} \]  \hspace{1cm} (8.2)

7.4.2. Case II:

\[ (A_s - A_{sf}) f_y = 0.85 f'_c b_w a \]  \hspace{1cm} (8.3)

Solve for “a”:

\[ a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} \]  \hspace{1cm} (8.4)

and nominal moment capacity will be:

\[ M_n = A_{sf} f_y (d - \frac{h_f}{2}) + (A_s - A_{sf}) f_y (d - \frac{a}{2}) \]  \hspace{1cm} (8.5)
7.5. Balanced Condition for T-Beams

See Commentary page 48 of ACI 318-83 (old code).

\[ \epsilon_u = 0.003 \]

\[ c^b = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d = \frac{87,000}{87,000 + f_y d} \]  

(8.6)
7.6. Example.- Analysis of T-Beams in Bending:

Find the nominal moment capacity of the beam given above:

\[ f_c' = 2,400 \text{ psi} \]
\[ f_y = 50,000 \text{ psi} \]

**Solution:**

Check to see if a T-beam analysis is required:

Assume \( a < h_f \)

\[ a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6.88 \times 50}{0.85 \times 2.4 \times 40} = 4.22 \text{ in} \]

Since 4.22 in > 4.00 in, a T-beam analysis is required.

First find the reinforcement area to balance flanges (\( A_{sf} = ? \))

\[ A_{sf} = 0.85 f_c' \frac{f_y}{f_y} (b - b_w) h_f = 0.85 \times \frac{2.4}{50} \times (40 - 10) \times 4 = 4.90 \text{ in}^2 \]

\[ A_s - A_{sf} = 6.88 - 4.90 = 1.98 \text{ in}^2 \]

Solve for “\( a \)”

\[ 0.85 f_c' b_w a = (A_s - A_{sf}) f_y \]

\[ a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.98 \times 50}{0.85 \times 2.4 \times 10} = 4.86 \text{ in} > 4 \text{ in} \text{ o.k.} \]

Assumption is o.k.
\[
\begin{align*}
    c &= \frac{a}{\beta_1} = \frac{4.86}{0.85} = 5.72 \\
    \frac{c}{d} &= \frac{5.72}{20.5} = 0.279 < 0.375 \quad \text{Tension-controlled}
\end{align*}
\]

Find the nominal moment capacity of the beam:

\[
M_n = A_{sf}f_y (d - \frac{h_f}{2}) + f_y(A_s - A_{sf}) (d - \frac{a}{2})
\]

\[
M_n = 4.9(in^2) \times 50(ksi) \times (20.5 - \frac{4}{2}) + 50(ksi) \times 1.98(in^2) \times (20.5 - \frac{4.86}{2})
\]

\[
M_n = 4530 + 1790 = 6,320 \text{ in-k}
\]

Note:

This could have been done by statics with

\[
T_s = A_{sf}f_y
\]

\[
C_c = (b - b_w)(h_f) \times 0.85f_c' + ab_w(0.85)f_c'
\]
7.7. Example.- Design of T-Beams in Bending- Determination of Steel Area for a given Moment:

A floor system consists of a 3 in. concrete slab supported by continuous T beams of 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are \( b_w = 11 \) in. and \( d = 20 \) in. What tensile steel area is required at midspan to resist a moment of 6,400 in-kips if \( f_y = 60,000 \) psi and \( f'_c = 3,000 \) psi.

\[
\begin{align*}
A_s & = \frac{M_u}{\phi f_y (d - a/2)} = \frac{6400}{0.9 \times 60 \times (20 - 3/2)} = 6.40 \text{ in}^2 \\
\end{align*}
\]

Solution

First determining the effective flange width from Section (7.3.1.) or ACI 8.10.2

1) \( b \leq \frac{\text{span}}{4} = \frac{24 \times 12}{4} = 72 \text{ in} \)

2) \( b \leq 16h_f + b_w = (16 \times 3) + 11 = 59 \text{ in} \)

3) \( b \leq \text{clear spacing between beams} + b_w = \text{center to center spacing between beams} = 47 \text{ in} \)

The centerline T beam spacing controls in this case, and \( b = 47 \) inches.

Assumption: Assuming that stress-block depth equals to the flange thickness of 3 inches (beam behaves like a rectangular shape).
Solve for “a”:

\[ a = \frac{A_{sf} f_y}{0.85 f_c' b} = \frac{6.40 \times 60}{0.85 \times 3 \times 47} = 3.2 \text{ in} > h_f = 3.0 \text{ Assumption incorrect} \]

Therefore, the beam will act as a T-beam and must be designed as a T-beam. From Case I given above and Section (7.4.1.) we have

\[ A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = \frac{0.85 \times (3\text{ksi}) \times (3\text{in}) \times (47 - 11)}{60(\text{ksi})} = 4.58 \text{ in}^2 \quad (8.8) \]

\[ \phi M_{n1} = \phi A_{sf} f_y (d - \frac{h_f}{2}) = 0.9 \times 4.58 \times (60\text{ksi}) \times (20 - 3/2) = 4570 \text{ in-kips} \quad (8.9) \]

\[ \phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4570 = 1830 \text{ in-kips} \quad (8.10) \]

Find “a” value by iteration. Assume initial a = 3.5 inches

\[ A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.5/2)} = 1.86 \text{ in}^2 \quad (8.11) \]

Find an improve “a” value

\[ a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.97 \text{ in} \quad (8.12) \]

Iterate with the new a = 3.97 in.

\[ A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.97/2)} = 1.88 \text{ in}^2 \quad (8.13) \]

Find an improve “a” value

\[ a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.88 \times 60}{0.85 \times 3 \times 11} = 4.02 \text{ in} \quad (8.14) \]

\[ A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 4.02/2)} = 1.88 \text{ in}^2 \quad (8.15) \]
Since there is no change between equations (8.13) and (8.15) we have arrived at the answer. Therefore,

\[ A_s = A_{sf} + (A_s - A_{sf}) = 4.58 + 1.88 = 6.46 \text{ in}^2 \]  

(8.16)

Check with ACI requirements for maximum amount of steel (Tension-Controlled)

\[ c = \frac{a}{\beta_1} = \frac{4.02}{0.85} = 4.73 \]  

(8.17)

\[ \frac{c}{d} = \frac{4.73}{20} = 0.237 < 0.375 \quad \text{Tension-controlled} \]

Therefore, the T-beam satisfies the ACI provisions for tension failure. Next steps will be to select the reinforcement and check all the spacing requirements and detail the beam.