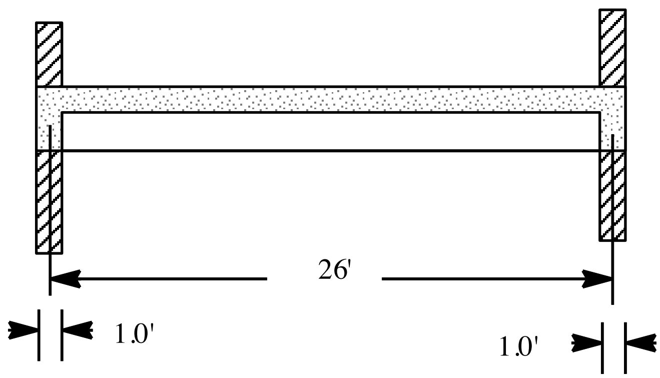
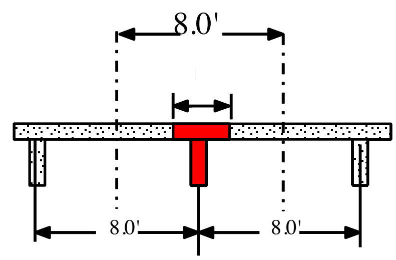
Example of Bar Cutoff

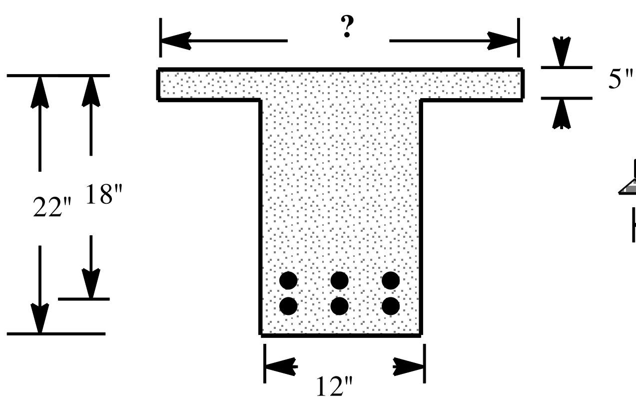
A floor system consists of single span T-beams 8 ft on centers, supported by 12 in masonry walls spaced at 25 ft between inside faces. The general arrangement is shown in below. A 5-inch monolithic slab to be used in heavy storage warehouse. Determine the reinforcement configuration and the cutoff points. Check the provisions of ACI 318 for bar cutoff.

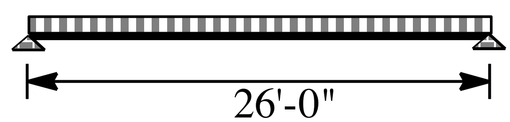
f'_c = 4,000 psi (normal weight)

f_y = 60,000 psi









See ACI 8.9 for definition of span length

Dead Load

Weight of slab $= (\frac{5}{12}\,ft)(7\,ft)(150 lb/ft^3) = 440 \quad lb/ft$

Weight of beam $= (\frac{12}{12}\,ft)(\frac{22}{12}\,ft)((150 lb/ft^3) = 275 \quad lb/ft$

$$w_D = 440 + 275 = 715 \quad lb/ft$$

$$1.2 w_D = 860 \quad lb/ft$$

Live Load

Referring to Table of 1.1 in your notes, for Storage Warehouse – Heavy, $w_L = 250$ psf

$$w_L = (250 lb/ft^2)(8\,ft) = 2{,}000 lb/ft$$

$$1.6 w_L = 3{,}200 \quad lb/ft$$

Find Flange Width

$$L/4 = \frac{26\times12}{4} = 78 \text{ inches}$$

$\leftrightarrow$ Controls b = 48 inches

$$16h_f + b_w = 80 + 12 = 92 \text{ inches}$$

Centerline spacing = $8\times12 = 96$ inches

Determine Factored Load

$$w_u = 1.2w_D + 1.6w_L = 860 + 3,200 = 4060 \quad lb/ft = 4.06\ kips/ft$$

Determine Factored Moment

$$M_u = \frac{1}{8}w_u l^2$$

$$M_u = \frac{1}{8}(4.06)(26)^2 = 343 ft\text{-}kips$$

Design the T-beam

Use a trial and error procedure. First, assume for the first trial that the stress block depth will be equal to the slab thickness (a = 5 inches):

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{343\times12}{0.9\times60(18-5/2)} = \frac{76.2}{18-5/2} = \mathbf{4.92}\ in^2$$

$$a = \frac{A_s f_y}{0.85 f_c^{'} b} = \frac{4.92\times60}{0.85\times4\times78} = 4.92\times0.226 = 1.11 < h_f = 5 \quad inches \rightarrow ok.$$

The stress block depth is less than the slab thickness; therefore, the beam will act as a rectangular beam and the rectangular beam equations are valid.

Adjust trial

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{76.2}{18-1.11/2} = = \mathbf{4.37}\ in^2$$

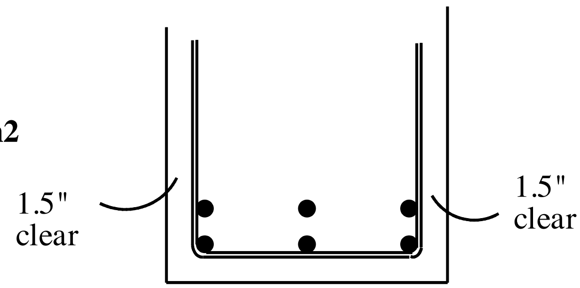
$$a = \frac{A_s f_y}{0.85 f_c^{'} b} = 4.73\times0.226 = 0.99$$

Next trial

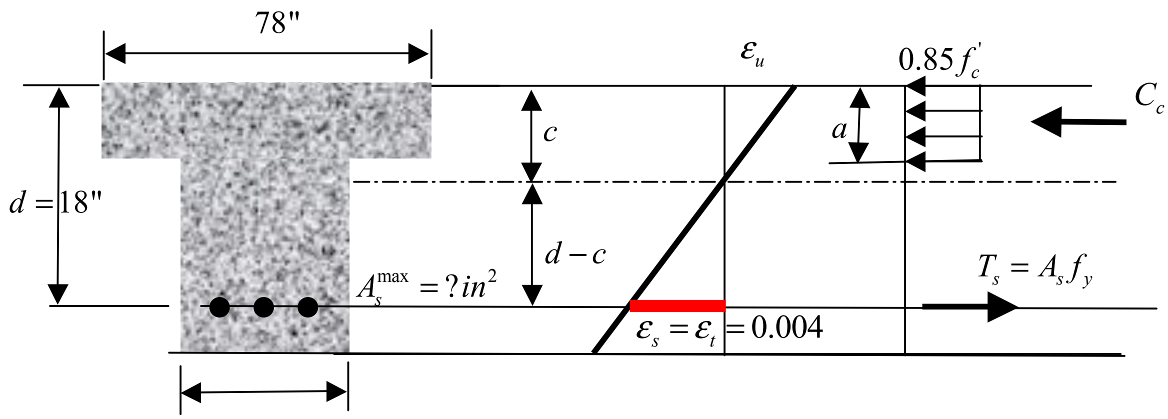
$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{76.2}{18-0.99/2} = = \mathbf{4.35}\ in^2$$

Close enough to previous iteration of **4.37** in^2. Stop here.

Use 6- #8 bars As = 4.71 in2



Check ACI for Maximum Steel:



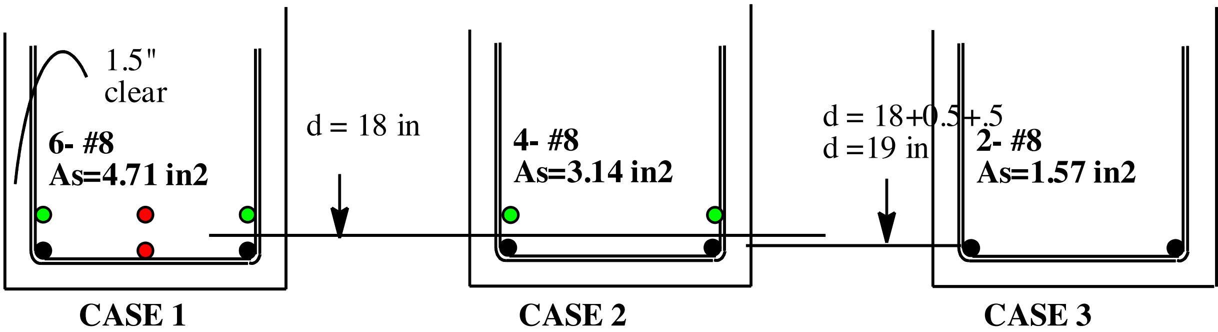
Using similar triangles:

$$\frac{\varepsilon_u}{0.004} = \frac{c}{d-c} \quad \rightarrow \quad \frac{0.007}{0.004} = \frac{c}{18-c} \quad \rightarrow \quad c = 7.71 \quad inches$$

$$a = \beta_1 c = 0.85 \times 7.71 = 6.65 \quad inches$$

$$A_s^{max} f_y = 0.85 f_c^{'} \left[78 \times 5 + 12 \times 1.56\right] \rightarrow A_s^{max} = 23.16 \quad in^2$$

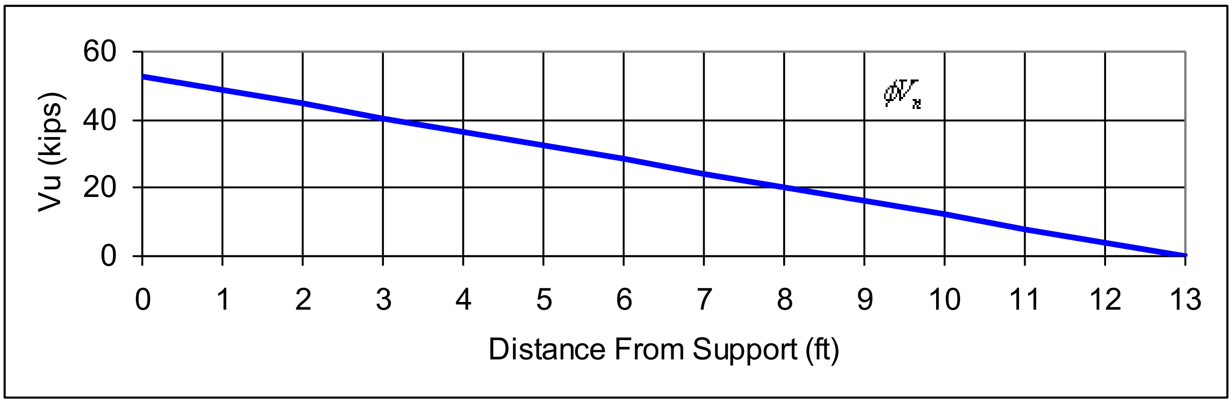
Since $A_s = 4.71 \quad in^2 \leq 23.16 \quad in^2$, we satisfy the ACI code and we will have tension failure.

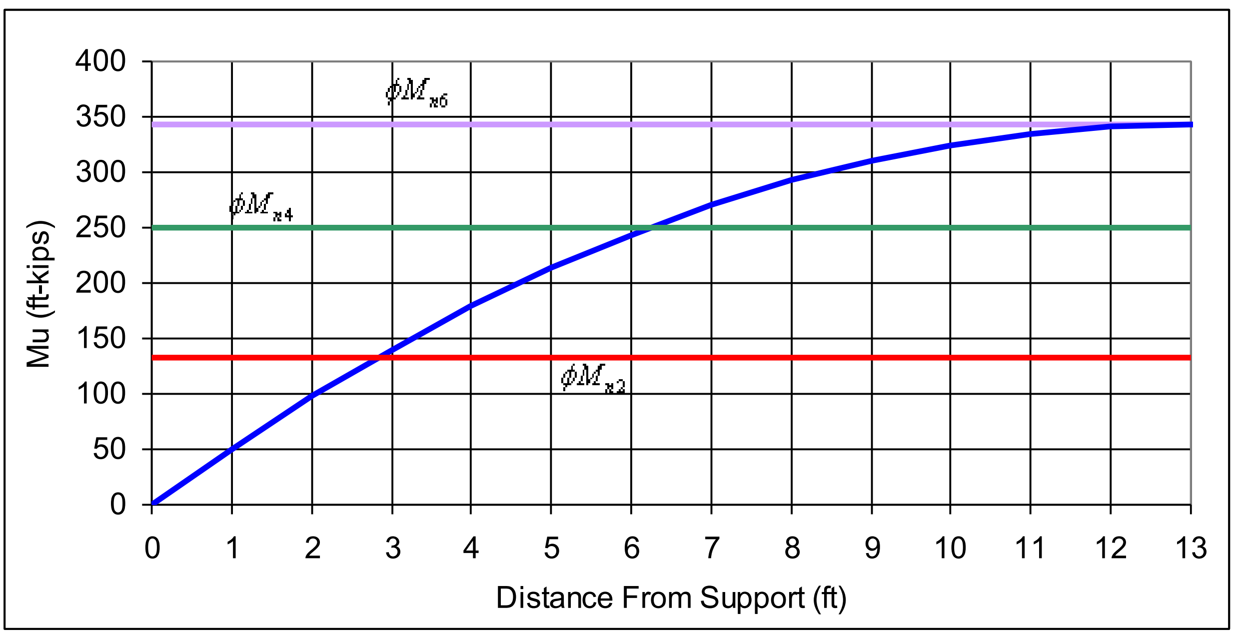


6 bars
As = 4.71
d = 18 in

4 bars
As = 3.14
d = 18 in

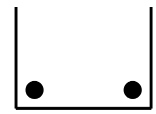
2 bars
As = 1.57
d = 19 in





Note: Code allows discontinuing 2/3 of longitudinal bars for simple spans. Therefore, let's cut 4 bars.

Capacity of section after 4 bars are discontinued:

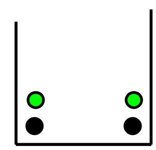


$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.57 \times 60}{0.85 \times 4 \times 78} = 0.355 \quad inches$$

$$M_u(2\ bars) = \phi M_n = \phi A_s f_y (d - \frac{a}{2})$$

$$M_u(2\ bars) = 0.9 \times 1.57 \times 60(19 - \frac{0.355}{2}) \times \frac{1}{12} = 133 \quad ft - kips$$

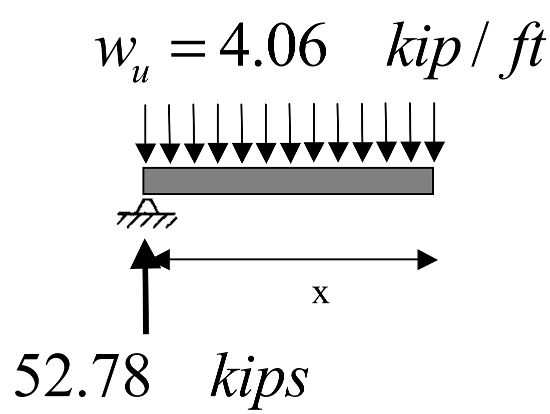
Capacity of section after 2 bars are discontinued:



$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.14 \times 60}{0.85 \times 4 \times 78} = 0.71 \quad inches$$

$$M_u(4\ bars) = \phi M_n = \phi A_s f_y (d - \frac{a}{2})$$

$$M_u(4\ bars) = 0.9 \times 3.14 \times 60(18 - \frac{0.71}{2}) \times \frac{1}{12} = 250 \quad ft - kips$$



Find the location where the moment is equal to $M_u(2\ bars)$:

$$M = 52.7x - \frac{1}{2}(4.06)x^2$$

$$M = 52.7x - 2.03x^2$$

$$M_u(2bars) = 52.7x - 2.03x^2 = 133$$

$$2.03x^2 - 52.78x + 133 = 0 \rightarrow x = \frac{52.78 \pm \sqrt{52.78^2 - 4 \times 133 \times 2.03}}{2 \times 2.03} = 2.8\ ft$$

Find the location where the moment is equal to $M_u(4\ bars)$:

$$M_u(4\ bars) = 52.7x - 2.03x^2 = 250$$

$$2.03x^2 - 52.78x + 250 = 0 \rightarrow x = \frac{52.78 \pm \sqrt{52.78^2 - 4 \times 250 \times 2.03}}{2 \times 2.03} = 6.3\ ft$$

Input information

As=	4.71	in^2
Av =	0.22	in^2
f'c =	4000	psi
fy =	60000	psi
bar length =	26	ft
s.s. (1), Cant. (2)	1	
W_L =	2.000	k/ft
W_d =	0.715	k/ft
b_w =	12	in
d =	18	in
W_u =	4.058	k/ft
Reaction at Support =	52.75	
rho =	0.0218	
Vs_max =	22.47	kips
4*sqrt(f'c)*bw*d =	54.64	kips

beta_1 = 0.85

$3.5\phi\sqrt{f'_c}b_w d =$ 35.86 kips

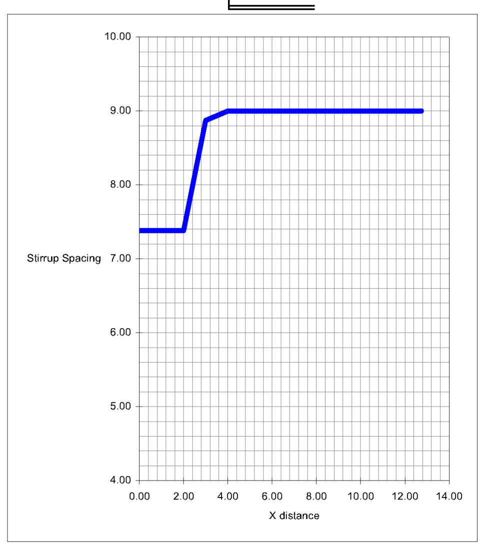
Vs<4sqrt(f'c)(bw)d

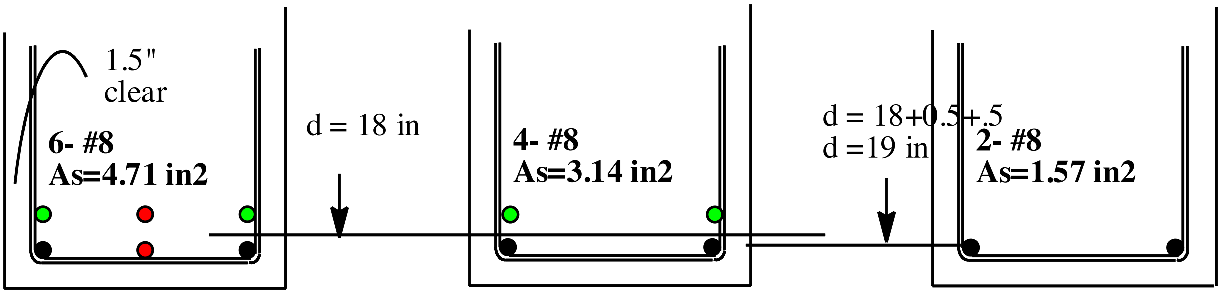
Smax		
d/2	9	inches
24	24	inches
Av*fy/(50b)=	22.00	inches

$$s_{\max} = \frac{A_v f_y}{0.75\sqrt{f'_c}\,b_w} \le \frac{A_v f_y}{50 b_w}$$

----> **Smax=** 9 inches

Dist. from support (ft)	Mu (ft-kips)	Vu (kips)	phi*Vc (kips)	Vu-phi*Vc (kips)	req'd spacing	Vu*d/Mu	phi*Vc (kips)	Vu-phi*Vc (kips)	req'd spacing
	w_u*(L*x-x^2)/2	w_u*L/2-w_u*x	**Eq 11-3**				**Eq 11-5**		
0.00	0.01	44.64	20.49	24.15	7.38	1.00	28.30	16.34	9.00
1.00	50.73	44.64	20.49	24.15	7.38	1.00	28.30	16.34	9.00
1.50	74.57	44.64	20.49	24.15	7.38	0.90	27.40	17.24	9.00
2.00	97.39	44.64	20.49	24.15	7.38	0.69	25.54	19.10	9.00
3.00	140.00	40.58	20.49	20.09	8.87	0.43	23.31	17.27	9.00
4.00	178.55	36.52	20.49	16.03	9.00	0.31	22.18	14.35	9.00
5.00	213.05	32.46	20.49	11.97	9.00	0.23	21.49	10.98	9.00
6.00	243.48	28.41	20.49	7.91	9.00	0.18	21.01	7.39	9.00
7.00	269.86	24.35	20.49	3.86	9.00	0.14	20.66	3.69	9.00
8.00	292.18	20.29	20.49	0.00	9.00	0.10	20.39	-0.10	-2084.14
9.00	310.44	16.23	20.49	0.00	9.00	0.08	20.16	-3.93	-51.42
10.00	324.64	12.17	20.49	0.00	9.00	0.06	19.96		
11.00	334.79	8.12	20.49	0.00	9.00	0.04	19.79		
12.00	340.87	4.06	20.49	0.00	9.00	0.02	19.62		
12.75	342.77	1.01	20.49	0.00	9.00	0.00	19.51		





CASE 1	CASE 2	CASE 3
6 bars	*4 bars*	*2 bars*
As = 4.71	*As = 3.14*	*As = 1.57*
d = 18 in	*d = 18 in*	*d = 19 in*
clear spacing = 2.63 in	*clear spacing =6.25 in*	*clear spacing = 6.25 in*
center to center spacing = 3.63 in	*center to center spacing = 7.25 in*	*center to center spacing = 6.25 in*

Note: Clear bar spacing is equal to:

$$\frac{1}{\text{no. of bars in one row - 1}} = \left[12 - 2\left(\frac{3}{8}\right) - \text{no. of bars} \times \left(\frac{8}{8}\right) - 2(1.5)\right]$$

Determine the development length

$$\psi_t = 1.0 \qquad \psi_e = 1.0$$

$$\psi_s = 1.0 \qquad \lambda = 1.0$$

$$A_{tr} = 0.22 \quad in^2$$

$$n = 3$$

$$s = 9 \quad in$$

$$k_{tr} = \frac{A_{tr} f_{yt}}{1500 sn} = \frac{0.22 \times 60,000}{1500 \times 9 \times 3} = 0.33$$

$$c = \begin{cases} \frac{1}{2}(3.63) = 1.8 \quad in \quad \leftarrow control \\ 1.5 + 3/8 + 0.5 = 2.375 \quad in \end{cases} \qquad \frac{c + k_{tr}}{d_b} = \frac{1.8 + 0.33}{1.00} = 2.13 < 2.5 \quad ok$$

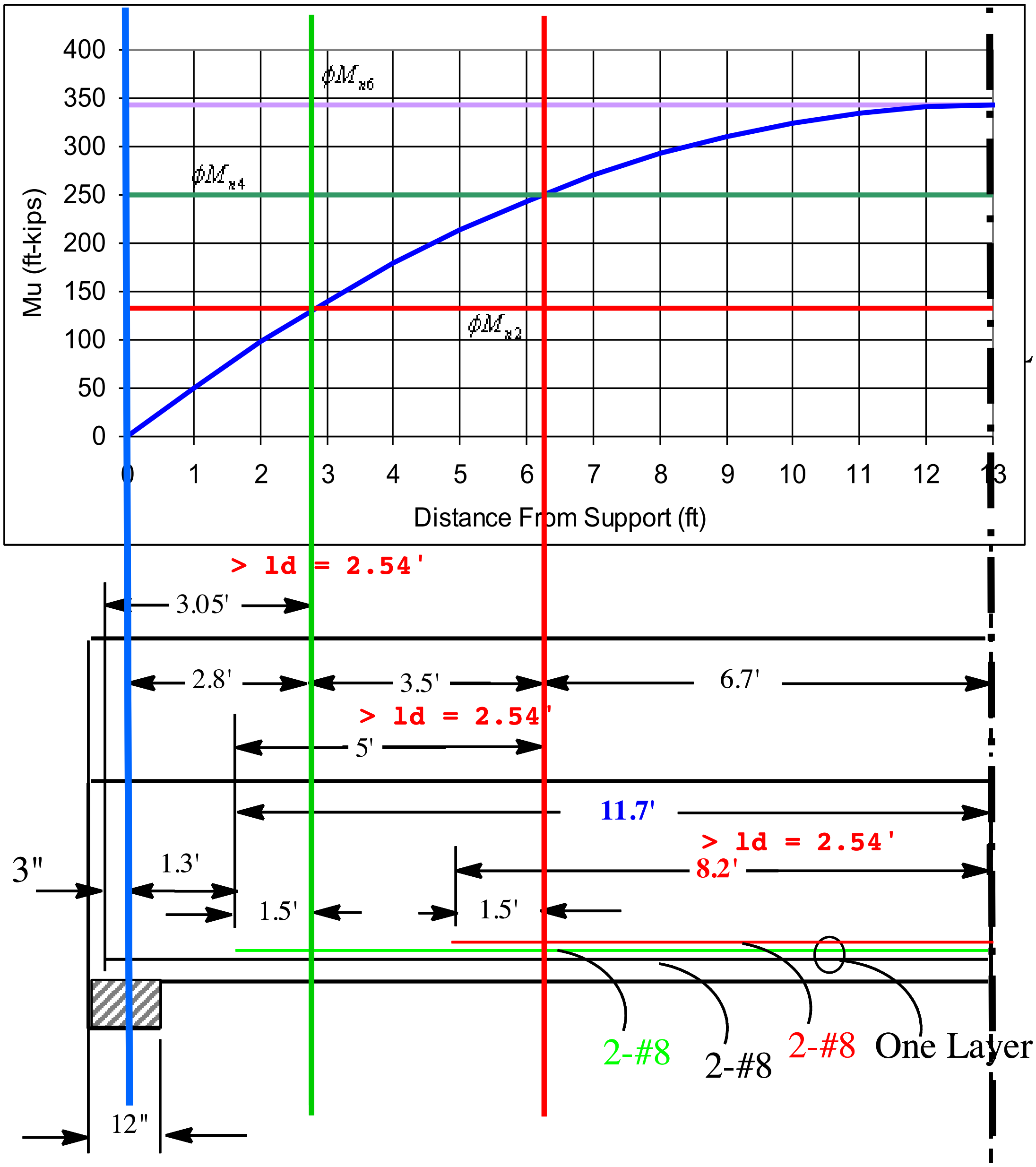
$$l_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f_c^{'}}} \frac{\psi_t \psi_e \psi_s \lambda}{\left(\frac{c + k_{tr}}{d_b} \right)} \right) d_b = \left(\frac{3}{40} \frac{60,000}{\sqrt{4,000}} \frac{1 \times 1 \times 1 \times 1}{2.13} \right) \times 1 = 33 \quad in$$

$$l_d = 33 \quad in = 2.75 \quad ft$$

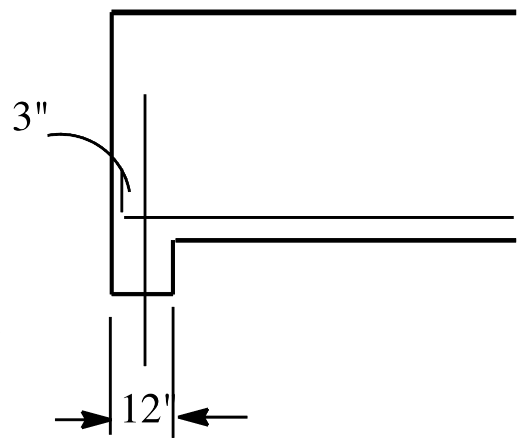
$$l_d = 2.75 \times \frac{A_s^{required}}{A_s^{provided}} = 2.75 \times \frac{4.35}{4.71} = 2.54 \quad ft$$

Extend bars:

$$\begin{cases} 12d_b = 12 \times 1.00 = 12 \quad inches = 1 \quad ft \\ d = 18 \quad inches = 1.5 \quad ft \quad \leftarrow controls \end{cases}$$



Check Zero Moment:



$$l_d \le 1.3\frac{M_n}{V_u} + l_a$$

$$M_n = \frac{M_u}{\phi} = \frac{343}{0.9} = 381\ ft.kips$$

$$l_d \le 1.3\frac{381\times12}{52.78} + 3.00 = 116 \quad inches$$

$$l_d = 2.54\ ft = 2.54\times12 = 31 \quad inches \le 116 \quad inches \rightarrow ok$$

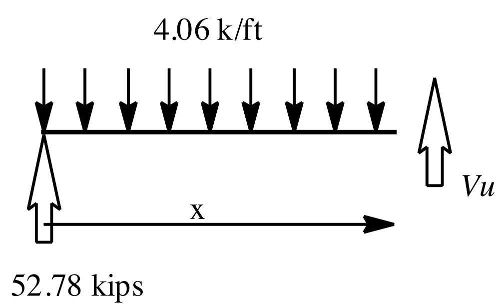
This is to ensure that the continued steel is of sufficiently small diameter and the required anchorage requirement of the ACI code is satisfied.

Check for shear Complication (ACI 12.10.5)

$$V_c = 2\sqrt{f_c^{'}}b_w d = 2\sqrt{4,000}\times12\times18 = 27.3 \quad kips$$

$$V_s = \frac{A_v f_y d}{s} = \frac{(0.22)\times60\times18}{9} = 26.4 \quad kips$$

$$V_u = \phi\left(V_c + V_s\right) = 0.75\left(27.3 + 26.4\right) = 40.3 \quad kips$$



Vu(x = 1.3) = 52.78 - 4.06 x 1.3 = 47.5 kips

Vu(x = 1.3) = 47.5 kips > (2/3) x 40.3 = 26.9

Vu(x = 4.8) = 52.78 - 4.06 x 4.8 = 33.3 kips

Vu(x = 4.8) = 33.3 kips > (2/3) x 40.3 = 26.9

Need additional reiforcements at both cutoff points.

Check for Shear Complications (ACI12.10.5), Continued

$$s = \frac{A_v f_y}{60 b_w} = \frac{(0.22) b_w 60{,}000}{60 \times 12} = 18.33 \quad in$$

$$s = \frac{d}{8\beta_d} = \frac{18}{8 \times \left(\frac{2}{6}\right)} = 6.7 \quad in \leftarrow controls \text{ use 6 inches}$$

Provide additional reinforcement for a length of (3/4)d/

$$\frac{3}{4} d = \frac{3}{4} \times 18 = 13.5 \quad inches$$