Example of Bar Cutoff

A floor system consists of single span T-beams 8 ft on centers, supported by 12 in masonry walls spaced at 25 ft between inside faces. The general arrangement is shown below. A 5-inch monolithic slab to be used in heavy storage warehouse. Determine the reinforcement configuration and the cutoff points. Check the provisions of ACI 318 for bar cutoff.

\[
f'_{c} = 4,000 \text{ psi (normal weight)}
\]

\[
f_{y} = 60,000 \text{ psi}
\]

Dead Load

Weight of slab \( = \left( \frac{5}{12} \text{ ft})(7 \text{ ft})(150 \text{ lb/ft}^{3}) \right) = 440 \text{ lb/ft} \)

Weight of beam \( = \left( \frac{12}{12} \text{ ft})(\frac{22}{12} \text{ ft})(150 \text{ lb/ft}^{3}) \right) = 275 \text{ lb/ft} \)

\[
w_{D} = 440 + 275 = 715 \text{ lb/ft}
\]

\[
1.2w_{D} = 860 \text{ lb/ft}
\]

Live Load

Referring to Table of 1.1 in your notes, for Storage Warehouse – Heavy, \( w_{L} = 250 \text{ psf} \)

\[
w_{L} = (250 \text{ lb/ft}^{2})(8 \text{ ft}) = 2,000 \text{ lb/ft}
\]

\[
1.6w_{L} = 3,200 \text{ lb/ft}
\]
Find Flange Width

\[ L/4 = \frac{26 \times 12}{4} = 78 \text{ inches} \]

\[ 16h_f + b_w = 80 + 12 = 92 \text{ inches} \]

Centerline spacing = 8 \times 12 = 96 \text{ inches} \[ \leftrightarrow \text{Controls } b = 48 \text{ inches} \]

Determine Factored Load

\[ w_u = 1.2w_D + 1.6w_L = 860 + 3,200 = 4060 \text{ lb/ft} = 4.06 \text{ kips/ft} \]

Determine Factored Moment

\[ M_u = \frac{1}{8} w_u l^2 \]

\[ M_u = \frac{1}{8} (4.06)(26)^2 = 343 \text{ft-kips} \]

Design the T-beam

Use a trial and error procedure. First, assume for the first trial that the stress block depth will be equal to the slab thickness (a = 5 inches):

\[ A_i = \frac{M_u}{\phi f_y (d-a/2)} = \frac{343 \times 12}{0.9 \times 60 (18 - 5/2)} = \frac{76.2}{18 - 5/2} = 4.92 \text{ in}^2 \]

\[ a = \frac{A_i f_y}{0.85 f_c b} = \frac{4.92 \times 60}{0.85 \times 4 \times 78} = 4.92 \times 0.226 = 1.11 h_f = 5 \text{ inches} \rightarrow \text{ok.} \]

The stress block depth is less than the slab thickness; therefore, the beam will act as a rectangular beam and the rectangular beam equations are valid.

Adjust trial

\[ A_i = \frac{M_u}{\phi f_y (d-a/2)} = \frac{76.2}{18 - 1.11/2} = 4.37 \text{ in}^2 \]

\[ a = \frac{A_i f_y}{0.85 f_c b} = 4.73 \times 0.226 = 0.99 \]

Next trial

\[ A_i = \frac{M_u}{\phi f_y (d-a/2)} = \frac{76.2}{18 - 0.99/2} = 4.35 \text{ in}^2 \]

Close enough to previous iteration of 4.37 in². Stop here.
Use 6-#8 bars As = 4.71 in²

Check ACI for Maximum Steel:

Using similar triangles:
\[
\frac{\varepsilon_u}{0.004} = \frac{c}{d - c} \rightarrow \frac{0.007}{0.004} = \frac{c}{18 - c} \rightarrow c = 7.71 \text{ inches}
\]

\[ a = \beta_c = 0.85 \times 7.71 = 6.65 \text{ inches} \]

\[ A_y^{\text{max}} f_y = 0.85 f_c [78 \times 5 + 12 \times 1.56] \rightarrow A_y^{\text{max}} = 23.16 \text{ in}^2 \]

Since \( A_y = 4.71 \text{ in}^2 \leq 23.16 \text{ in}^2 \), we satisfy the ACI code and we will have tension failure.
Note: Clear bar spacing is equal to:

- stirrups
- bars
- cover

1.5" clear
6- #8
As=4.71 in²

Case 1
6 bars
As = 4.71
d = 18 in

Case 2
4- #8
As = 3.14
d = 18 in

Case 3
2- #8
As = 1.57
d = 19 in

Determine the Development Length

6 bars
As = 4.71
d = 18 in

4 bars
As = 3.14

2 bars
As = 1.57

d = 19 in

CASE 1
CASE 2
CASE 3

clear spacing = 2.63 in
center to center spacing = 3.63 in

Determine the Development Length

Vu (kips)
Distance From Support (ft)

Mu (ft-kips)
Distance From Support (ft)
Note: Code allows discontinuing 2/3 of longitudinal bars for simple spans. Therefore, let’s cut 4 bars.

Capacity of section after 4 bars are discontinued:
\[ a = \frac{A_f f_y}{0.85 f_c b} = \frac{1.57 \times 60}{0.85 \times 4 \times 78} = 0.355 \text{ inches} \]
\[ M_u(2 \text{ bars}) = \phi M_n = \phi A_f f_y (d - \frac{a}{2}) \]
\[ M_u(2 \text{ bars}) = 0.9 \times 1.57 \times 60 (19 - \frac{0.355}{2}) \times \frac{1}{12} = 133 \text{ ft - kips} \]

Capacity of section after 2 bars are discontinued:
\[ a = \frac{A_f f_y}{0.85 f_c b} = \frac{3.14 \times 60}{0.85 \times 4 \times 78} = 0.71 \text{ inches} \]
\[ M_u(4 \text{ bars}) = \phi M_n = \phi A_f f_y (d - \frac{a}{2}) \]
\[ M_u(4 \text{ bars}) = 0.9 \times 3.14 \times 60 (18 - \frac{0.71}{2}) \times \frac{1}{12} = 250 \text{ ft - kips} \]

Find the location where the moment is equal to \( M_u(2 \text{ bars}) \):
\[ w_u = 4.06 \text{ kip / ft} \]
\[ M = 52.7x - \frac{1}{2} (4.06)x^2 \]
\[ M = 52.7x - 2.03x^2 \]
\[ M_u(2 \text{ bars}) = 52.7x - 2.03x^2 = 133 \]
\[ 2.03x^2 - 52.78x + 133 = 0 \rightarrow x = \frac{52.78 \pm \sqrt{52.78^2 - 4 \times 133 \times 2.03}}{2 \times 2.03} = 2.8 \text{ ft} \]

Find the location where the moment is equal to \( M_u(4 \text{ bars}) \):
\[ M_u(4 \text{ bars}) = 52.7x - 2.03x^2 = 250 \]
\[ 2.03x^2 - 52.78x + 250 = 0 \rightarrow x = \frac{52.78 \pm \sqrt{52.78^2 - 4 \times 250 \times 2.03}}{2 \times 2.03} = 6.3 \text{ ft} \]
### Input information

- $A_s =$ \( 4.71 \text{ in}^2 \)
- $A_v =$ \( 0.22 \text{ in}^2 \)
- $f_c =$ \( 4000 \text{ psi} \)
- $f_y =$ \( 60000 \text{ psi} \)
- Bar length = 26 ft
- s.s. (1), Cant. (2) 1
- $W_{L} =$ 2.000 k/ft
- $W_{d} =$ 0.715 k/ft
- $b_w =$ 12 in
- $d =$ 18 in
- $W_{u} =$ 4.058 k/ft
- Reaction at Support = 52.75
- $\rho =$ 0.0218
- $V_{s_{\text{max}}} =$ 22.47 kips
- $4\sqrt{f_c}\cdot b\cdot d =$ 54.64 kips

### Smx

- $\delta_{\text{run}} = \frac{A_{f_r}}{0.75\sqrt{f_c}\cdot b\cdot d}$

#### Smax

- $\delta_{\text{run}} = \frac{A_{f_r}}{0.75\sqrt{f_c}\cdot b\cdot d}$

#### Dist. from support (ft) | Mu (kips) | Vu (kips) | phi*Vc (kips) | Vu-phi*Vc (kips) | req'd | Vu*d/Mu (kips) | phi*Vc (kips) | Vu-phi*Vc (kips) | req'd | spacing
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### Diagrams

- Shear diagram
- Bimodal Spacing

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Bar Cutoff
Note: Clear bar spacing is equal to:

\[
\frac{1}{\text{no. of bars in one row} - 1} = \left[ 12 - 2\left(\frac{3}{8}\right) - \frac{\text{no. of bars}}{2} \times \frac{8}{8} \right] - 2(1.5)
\]

**Determine the development length**

\[
\psi_t = 1.0 \quad \psi_e = 1.0
\]

\[
\psi_s = 1.0 \quad \lambda = 1.0
\]

\[
A_f = 0.22 \text{ in}^2
\]

\[
n = 3
\]

\[
s = 9 \text{ in}
\]

\[
k_u = \frac{A_f f_{ct}}{1500 s n} = \frac{0.22 \times 60,000}{1500 \times 9 \times 3} = 0.33
\]

\[
c = \begin{cases} 
1.5 + 3/8 + 0.5 = 2.375 \text{ in} & \text{control} \\
1.8 \text{ in} & \end{cases}
\]

\[
\frac{c + k_u}{d_b} = \frac{1.8 + 0.33}{1.0} = 2.13 < 2.5 \text{ ok}
\]
\[ l_d = \left( \frac{3}{40} \frac{f_y}{\sqrt{f_c}} \frac{\psi_0 \psi_1 \lambda}{\left( \frac{c + k_{tr}}{d_b} \right)} \right) \times 1 = 33 \text{ in} \]

\[ d_b = \left( \frac{3}{40} \frac{60,000}{\sqrt{4,000}} \frac{1 \times 1 \times 1}{2.13} \right) \times 1 = 33 \text{ in} \]

\[ l_d = 33 \text{ in} = 2.75 \text{ ft} \]

\[ l_d = 2.75 \times \frac{A_{\text{required}}}{A_{\text{provided}}} = 2.75 \times \frac{4.35}{4.71} = 2.54 \text{ ft} \]
Extend bars:
\[
\begin{align*}
12d_b &= 12 \times 1.00 = 12 \text{ inches} = 1 \text{ ft} \\
d &= 18 \text{ inches} = 1.5 \text{ ft}
\end{align*}
\]
Check Zero Moment:

\[ l_d \leq 1.3 \frac{M_n}{V_u} + l_a \]

\[ M_n = \frac{M_u}{\phi} = \frac{343}{0.9} = 381 \text{ ft.kips} \]

\[ l_d \leq 1.3 \frac{381 \times 12}{52.78} + 3.00 = 116 \text{ inches} \]

\[ l_d = 2.54 \text{ ft} = 2.54 \times 12 = 31 \text{ inches} \leq 116 \text{ inches} \rightarrow \text{ok} \]

This is to ensure that the continued steel is of sufficiently small diameter and the required anchorage requirement of the ACI code is satisfied.

Check for shear Complications (ACI 12.10.5)

\[ V_c = 2 \sqrt{f_c b_w d} = 2 \sqrt{4,000 \times 12 \times 18} = 27.3 \text{ kips} \]

\[ V_s = \frac{A_s f_s d}{s} = \frac{(0.22) \times 60 \times 18}{9} = 26.4 \text{ kips} \]

\[ V_u = \phi (V_c + V_s) = 0.75 (27.3 + 26.4) = 40.3 \text{ kips} \]

\[ V_u(x = 1.3) = 52.78 - 4.06 \times 1.3 = 47.5 \text{ kips} \]

\[ V_u(x = 1.3) = 47.5 \text{ kips} > (2/3) \times 40.3 = 26.9 \]

\[ V_u(x = 4.8) = 52.78 - 4.06 \times 4.8 = 33.3 \text{ kips} \]

\[ V_u(x = 4.8) = 33.3 \text{ kips} > (2/3) \times 40.3 = 26.9 \]

Need additional reinforcements at both cutoff points.
Check for Shear Complications (ACI12.10.5), Continued

\[ s = \frac{A_v f_v}{60 b_w} = \frac{(0.22)b_w \times 60,000}{60 \times 12} = 18.33 \text{ in} \]

\[ s = \frac{d}{8\beta_d} = \frac{18}{8 \times \left( \frac{2}{6} \right)} = 6.7 \text{ in} \leftarrow \text{controls use 6 inches} \]

Provide additional reinforcement for a length of \((3/4)d/\)

\[ \frac{3}{4} d = \frac{3}{4} \times 18 = 13.5 \text{ inches} \]