5.11. Under-reinforced Beams (Read Sect. 3.4b of your text)

We want the reinforced concrete beams to fail in tension because is not a sudden failure. Therefore, following Figure 5.3, you have to make sure that you stay in the tension-controlled side of the curve. In actual practice the upper limit on *c/d* should be somewhat below 0.375 for the following reasons:

1. material properties are never precisely known;
2. strain-hardening of the reinforcing steel, not accounted for in design may lead to a brittle concrete compression failure;
3. the actual steel area provided, considering standard rebar sizes, will always be equal or larger than required tending toward over-reinforcement.

Therefore, for all members designed according to ACI 318 Code, $f_s = f_y$ at failure, and the nominal strength is given by:

$$\left.\begin{aligned} a &= \frac{A_s f_y}{0.85 f_c{}' \, b} \\ M_n &= A_s f_y \left(d - \frac{a}{2}\right) \end{aligned}\right\} \quad M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c{}'}\right) \tag{5.31}$$

by imposing a strength reduction factor $\phi = 0.9$ for bending, we get

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) \tag{5.32}$$

or

$$M_u = \phi M_n = \phi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c{}'}\right) \tag{5.33}$$

Let us define the strength *coefficient of resistance, R,* as

$$R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c{}'}\right) \tag{5.34}$$

$$M_u \quad = \quad \phi M_n \quad = \quad \phi b\, d^2\, R \tag{5.35}$$

The relationship between ϱ and R for various values of f'_c and f_y is shown in the Figure 5.1.

To use the Figure 5.3, we have to rewrite the Equation (5.31) in terms of c/d

$$a \quad = \quad \frac{A_s\ f_y}{0.85 f_c{}'\ \ b} = \frac{A_s\ f_y}{0.85 f_c{}'\ \ b} \times \frac{d}{d} = \frac{f_y}{0.85 f_c{}'} \times \frac{A_s}{bd} \times d \tag{5.36}$$

$$a \quad = \quad \beta_1 c = \frac{f_y}{0.85 f_c{}'} \times \rho \times d \tag{5.37}$$

$$\frac{c}{d} = \frac{f_y}{f_c{}'} \times \frac{\rho}{0.85\beta_1} \tag{5.38}$$

We can find ϱ to be

$$\rho \quad = \quad 0.85\beta_1 \frac{c}{d} \times \frac{f_c{}'}{f_y} \tag{5.39}$$

5.12. Minimum Steel Area

Another mode of failure is when there is not enough reinforcement in a beam. If flexural strength of the cracked section is less than the moment produced cracking of the previously uncracked section, the beam will fail immediately upon formation of the first cracking. Aci code imposes a minimum tensile steel area of

$$ACI\ \ Eq.\ \ (10 - 3)\ gives \quad \rightarrow \quad A_{\text{min}} \quad = \quad \frac{3\sqrt{f_c{}'}}{f_y} b_w d \geq \frac{200}{f_y} b_w d \tag{5.40}$$

5.13. Design of Rectangular Sections in Bending with Tension Reinforcement Only (Singly Reinforced Beams).

When we design rectangular sections in bending with tension reinforcement only, we need to determine b, d, and A_s from the required value of $M_n = M_u/\Phi$, and the given material properties of f'_c and f_y. There are two approaches in determining b, d, and A_s. Equation (5.31) provides the condition of equilibrium. Since there are three unknowns, but only one equation, there are several possible solutions.

- **Case 1.** Select the optimum steel ratio, ϱ, determine concrete dimensions.
- **Case 2.** Select concrete dimensions, b and d, then determine the required reinforcement.

5.13.1. Case 1. Select the optimum steel ratio, ϱ, determine concrete dimensions.

1. Set the required strength M_u, equal to the design strength ϕM_n from equation (5.31) or (5.35)

$$M_u = \phi M_n = \phi b\, d^2 R \tag{5.41}$$

2. Using a c/d usually 0.3, we can determine the value of R from Equation (5.34):

   Find ρ from Equation (5.39)

$$\rho = 0.85\beta_1 \frac{c}{d} \times \frac{f_c'}{f_y} \tag{5.42}$$

$$R = \rho f_y \left(1 - 0.59\frac{\rho\ f_y}{f_c'}\right) \tag{5.43}$$

3. Knowing R we can determine the required bd^2

$$bd^2 = \frac{M_u}{\phi R} = \frac{M_n}{R} \tag{5.44}$$

4. Compute A_s from

$$A_s = \rho bd \tag{5.45}$$

5. Select reinforcement and check strength of the section to make sure that

$$\phi M_n \geq M_u \tag{5.46}$$

5.13.2. Case 2. Select concrete dimensions, *b* and *d*, then determine the required reinforcement.

This is similar to case 1 except steps taken will be a little different.

1. Set the required strength M_u, equal to the design strength ϕM_n from equation (5.31) or (5.35)

$$M_u = \phi M_n = \phi b\, d^2\, R \tag{5.47}$$

2. Knowing *b* and *d,* we can determine the strength coefficient of resistance, *R*

$$R = \frac{M_u}{\phi b d^2} = \frac{M_n}{b d^2} \tag{5.48}$$

3. Knowing *R* we can determine the reinforcement ratio, ϱ, from Figure 5.5 or equation (5.43) as

$$\rho = \frac{1 - \sqrt{1 - 2.36R/f'_c}}{1.18 f_y / f'_c} \tag{5.49}$$

4. Compute A_s from

$$A_s = \rho b d \tag{5.50}$$

5. Select refinement and check strength of the section to make sure that

$$\phi M_n \geq M_u \tag{5.51}$$

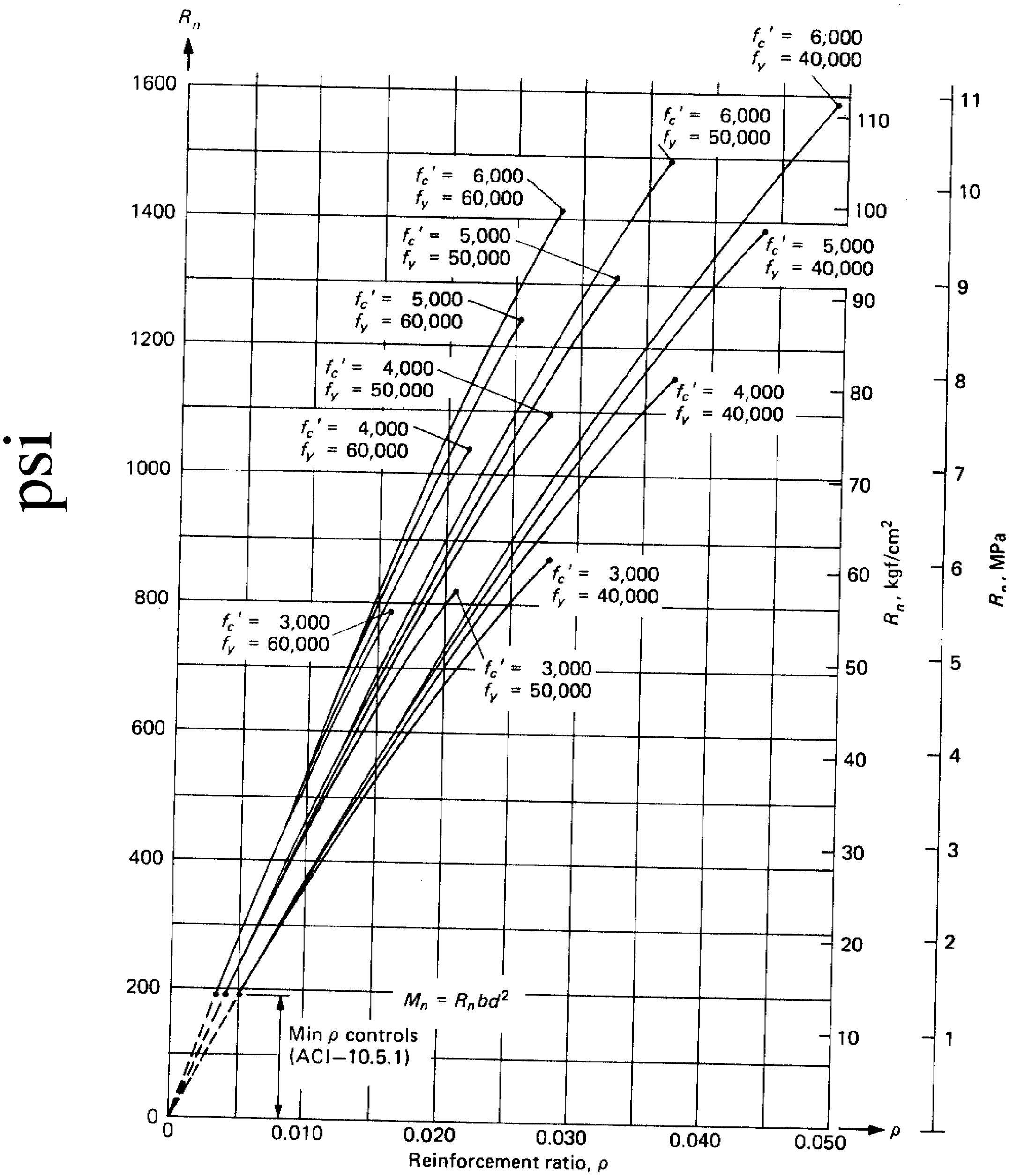


Figure 5.5. Strength Curves for Singly Reinforced Rectangular Sections.

5.14. Concrete Protection for Reinforcement

Section 7.7.1 of Code

5.15. Concrete Proportions

To assist the designer further in making choices for beam sizes, bar sizes, and bar placement, the following guideline are suggested. These may be regard as role of thumb, and are not ACI Code requirements. Undoubtedly, situations will arise in which the experienced design will, for good and proper reasons, make a selection not conforming to the guidelines.

1. Use whole inches for overall dimensions; except slabs may be in 1/2 in increments.
2. Beam stem widths are most often in multiples of 2 or 3 in., such as 9, 10, 12,14, 15, 16, and 18.
3. Minimum specified clear cover is measured from outside the stirrup or tie to the face of the concrete. (Thus beam effective depth d has rarely, if ever, a dimension to the whole inch.)
4. An economical rectangular beam proportion is one in which the overall depth-to-width ratio is between about 1.5 to 2.0 unless architectural requirements or construction cost dictates otherwise.
5. For T-shaped beams, typically the flange thickness represents about 20% of overall depth (we will talk about treatment of T-shaped sections in later sections).

5.16. Reinforcing Bar Selection

1. Maintain bar symmetry about the centroidal axis which lies at right angles to the bending axis (i.e., symmetry about the vertical axis in usual situations).
2. Use at least two bars wherever flexural reinforcement is required. Bars #3 to #11 are more common and larger bars of #14 and #18 are mainly used in columns.
3. Use bars # 11 and smaller for usual sized beams.
4. Use no more than two bar sizes and no more than two standard sizes apart for steel in one face at a given location in the span (i.e., #7 and #9 bars may be acceptable, but #9 and #4 bars would not).
5. Place bars in one layer if practicable. Try to select bar size so that no less than two and no more than five or six bars are put in one layer.
6. Follow requirements of ACI-7.6.l and 7.6.2 for clear distance between bars and between layers, and arrangement between layers.
7. When different sizes of bars are used in several layers at a location, place the largest bars in the layer nearest the face of beam.

5.17. Bar Spacing

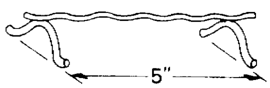
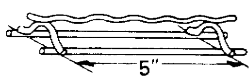
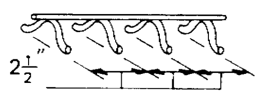
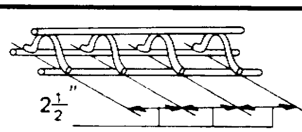
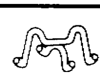
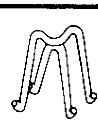
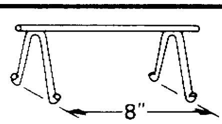
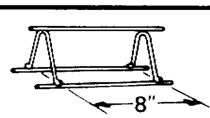
ACI 7.6

Note:

Table 5 through 7, given at the beginning of your notes, gives the maximum number of bars that can be placed in a single layer in beams. Assuming 1.5 in concrete cover and #4 stirrups.

Bars are supported from the bottom of forms, and layers of bars are separated by various types of bar supports, known as "bolsters" and "chairs," some of which are shown in Figure above. Bars supports may be made o concrete, metal, or other approved materials - they are usually factory-made wire bar supports. They remain in place after the concrete is cast and must have special rust protection on the portions nearest the face of the concrete.

Standard Types and Sizes of Bar Supports (Adapted from Ref. 2.26, p. 194)

SYMBOL	BAR SUPPORT ILLUSTRATION	TYPE OF SUPPORT	STANDARD SIZES
SB		Slab Bolster	$\frac{3}{4}$, 1, $1\frac{1}{2}$, and 2 in. heights in 5-ft and 10-ft lengths
SBU		Slab Bolster Upper	Same as SB
BB		Beam Bolster	1, $1\frac{1}{2}$, 2, over 2 to 5 in. heights in increments of $\frac{1}{4}$ in. in lengths of 5 ft
BBU		Beam Bolster Upper	Same as BB
BC		Individual Bar Chair	$\frac{3}{4}$, 1, $1\frac{1}{2}$, and $1\frac{3}{4}$ in. heights
HC		Individual High Chair	2 to 15 in. heights in increments of $\frac{1}{4}$ in.
CHC		Continuous High Chair	Same as HC in 5-ft and 10-ft lengths
CHCU		Continuous High Chair Upper	Same as CHC