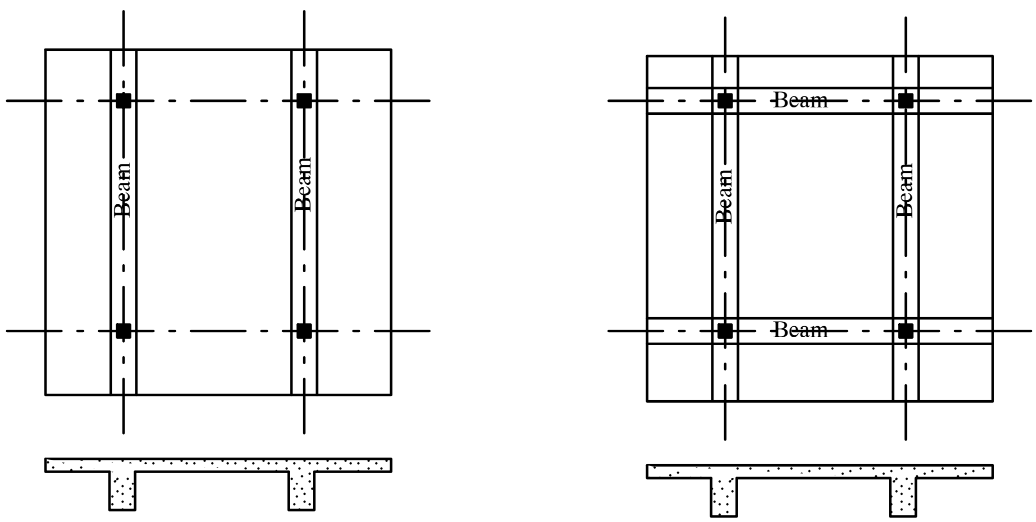
Chapter 8.
Flexural Analysis of T-Beams

8.1. Reading Assignments

Text Chapter 3.7; ACI 318, Section 8.10.

8.2. Occurrence and Configuration of T-Beams

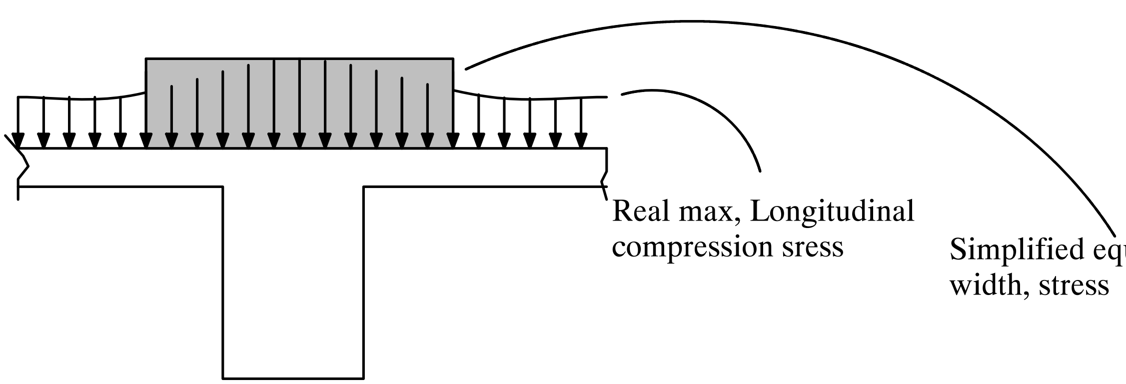
- Common construction type.- used in conjunction with either on-way or two-way slabs.
- Sections consists of the flange and web or stem; the slab forms the beam flange, while the part of the beam projecting below the slab forms is what is called web or stem.



(a) one-way slab (b) two-way slab

8.3. Concepts of the effective width, Code allowable values

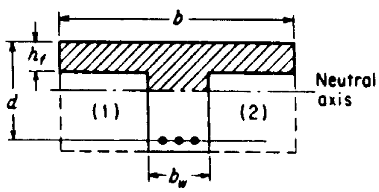
In reality the maximum compression stress in T-section varies with distance from section Web.



Analysis of T-Sections in Bending

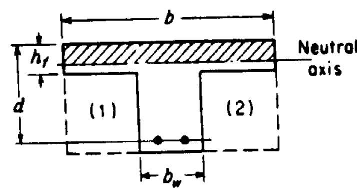
a. Two possible locations of equivalent rectangular stress block lead to the following analysis requirements:

1. "a" less than h_f	-	Analyze as for rectangular beam.
2. "a" greater than h_f	-	Special analysis required.



(b)

2



(a)

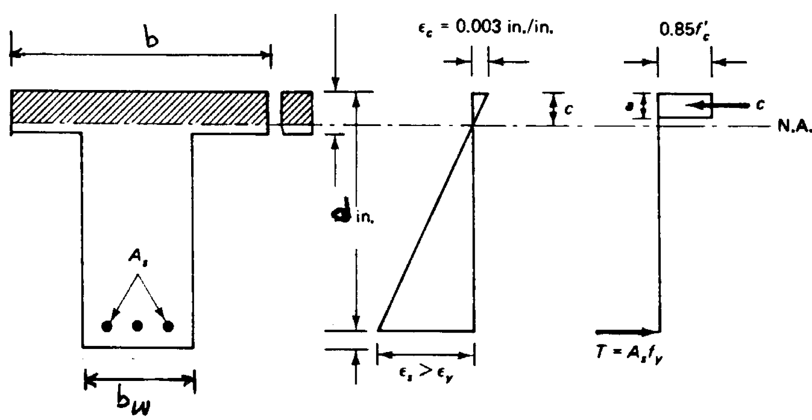
1

b. Consider a T-section in which entire stress block lies in flange - we may assume this to be the case for all sections. Statics will tell us if we are correct.

if $a \leq h_f$, $C_c = T_s$ $\rightarrow$ $0.85\, f'_c ab = A_s f_y$

therefore

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho f_y d}{0.85 f'_c}$$



Ask ourselves: if "a" $\leq h_f$?

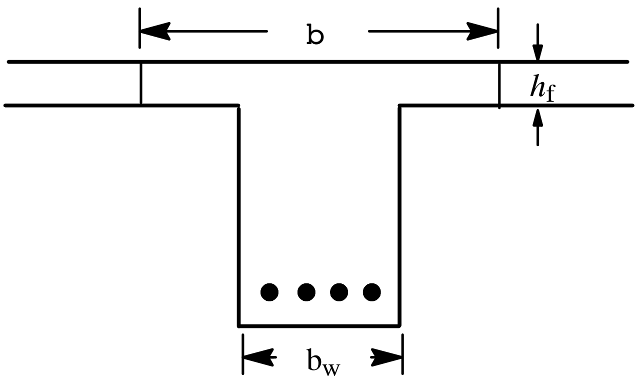
if answer is "yes", calculate moment capacity for a rectangular beam.

if answer is "no", an extensive analysis is required.

Code allows the following maximum effective widths:

8.3.1. Symmetrical Beam

ACI318, Section 8.10.2.



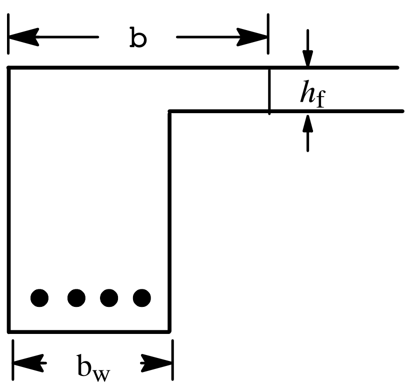
1) $b \le \frac{span}{4}$

2) $\frac{b - b_w}{2} \le 8h_f$

3) $\frac{b - b_w}{2} \le \frac{1}{2}$ *clear distance between beams*

8.3.2. Flange on one side only (Spandrel Beam)

ACI318, Section 8.10.3.



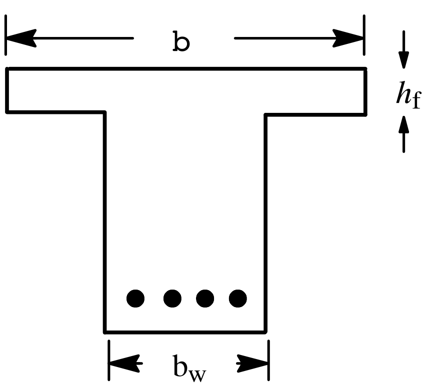
1) $b - b_w \le \frac{span}{12}$

2) $b - b_w \le 6h_f$

3) $b - b_w \le \frac{1}{2}$ *clear distance to next web*

8.3.3. Isolated T-Beam

ACI318, Section 8.10.4.



1) $b \le 4b_w$

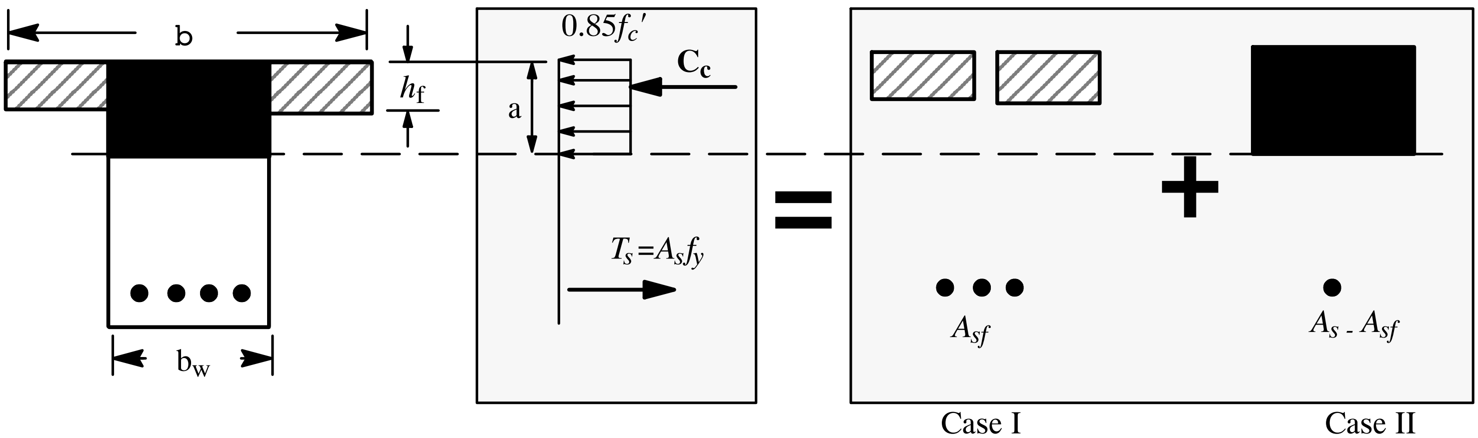
2) $\frac{b_w}{2} \le h_f$

8.4. Analysis of T-Beams - (a > h_f)

Consider the total section in two parts:

1) Flange overhangs and corresponding steel;

2) Stem and corresponding steel;



For equilibrium we have:

8.4.1. Case I:

$$A_{sf} f_y = 0.85 f_c' h_f (b - b_w) \tag{8.1}$$

or

$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} \tag{8.2}$$

8.4.2. Case II:

$$(A_s - A_{sf}) f_y = 0.85 f_c' b_w a \tag{8.3}$$

Solve for "a":

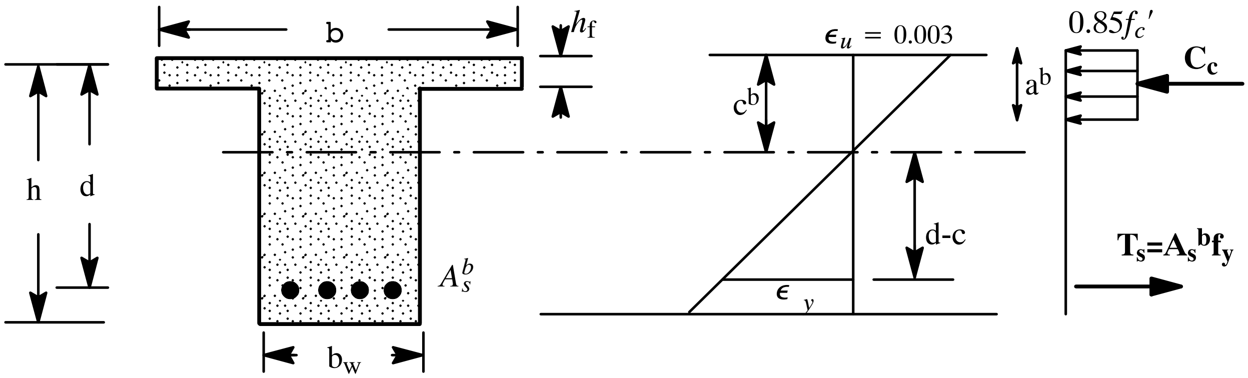
$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} \tag{8.4}$$

and <u>nominal moment capacity will be:</u>

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right) \tag{8.5}$$

8.5. Balanced Condition for T-Beams

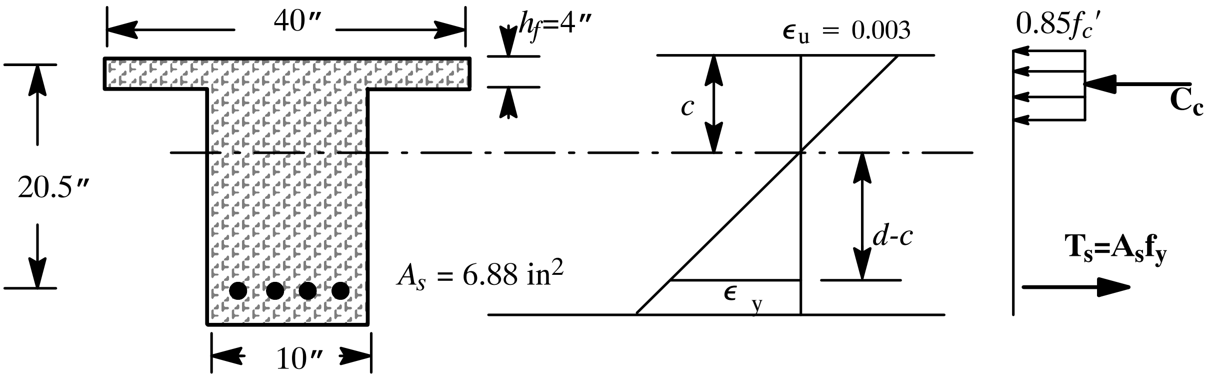
See Commentary page 48 of ACI 318-83 (old code).



From geometry:

$$c^b = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d = \frac{87,000}{87,000 + f_y} d \tag{8.6}$$

8.6. Example.- Analysis of T-Beams in Bending:



Find the nominal moment capacity of the beam given above:

$$f_c' = 2,400 \ psi$$
$$f_y = 50,000 \ psi$$

Solution:

Check to see if a T-beam analysis is required:

Assume $a < h_f$

$$a = \frac{A_s f_y}{0.85 f_c' \, b} = \frac{6.88 \times 50}{0.85 \times 2.4 \times 40} = 4.22 \ in$$

Since 4.22 *in* > 4.00 *in*, a T-beam analysis is required.

First find the reinforcement area to balance flanges (A_{sf} = ?)

$$A_{sf} = 0.85 \frac{f_c'}{f_y}(b - b_w)h_f = 0.85 \times \frac{2.4}{50} \times (40 - 10) \times 4 = 4.90 \ in^2$$

$$A_s - A_{sf} = 6.88 - 4.90 = 1.98 \ in^2$$

Solve for “a”

$$0.85 f_c' b_w a = (A_s - A_{sf}) f_y$$

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.98 \times 50}{0.85 \times 2.4 \times 10} = 4.86 \ in > 4 in \quad o.k.$$

Assumption is o.k.

$$c = \frac{a}{\beta_1} = \frac{4.86}{0.85} = 5.72$$

$$\frac{c}{d} = \frac{5.72}{20.5} = .279 < 0.375 \qquad \textit{Tension-controlled}$$

Find the nominal moment capacity of the beam:

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + f_y(A_s - A_{sf})\left(d - \frac{a}{2}\right)$$

$$M_n = 4.9(in^2) \times 50(ksi) \times (20.5 - \frac{4}{2}) + 50(ksi) \times 1.98(in^{2)} \times (20.5 - \frac{4.86}{2})$$

$$M_n = 4530 + 1790 = 6,320 \ in - k$$

Note:

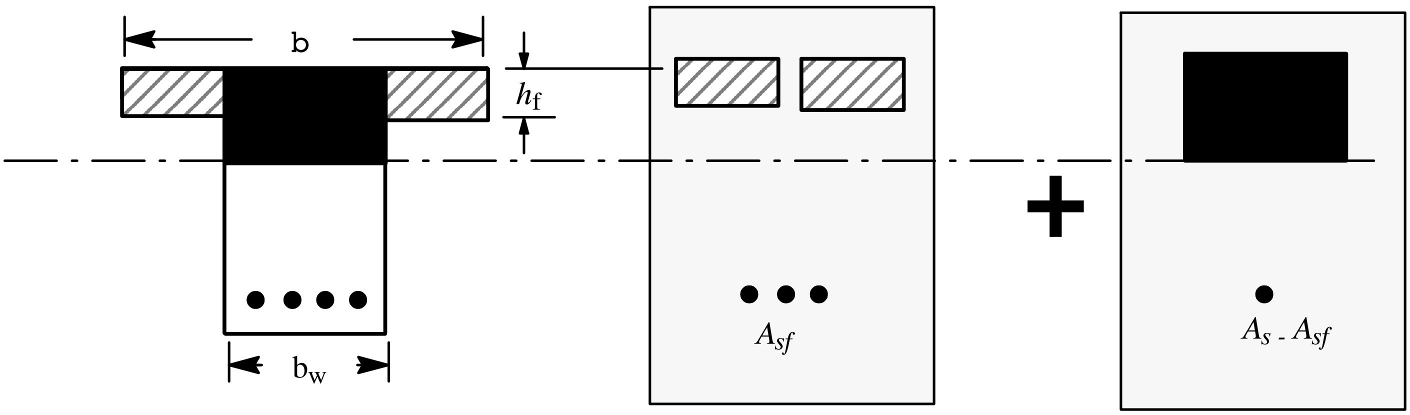
This could have been done by statics with

$$T_s = A_s f_y$$

$$C_c = (b - b_w)(h_f) \times 0.85 f_c' + a b_w (0.85) f_c'$$

8.7. Example.- Design of T-Beams in Bending- Determination of Steel Area for a given Moment:

A floor system consists of a 3 in. concrete slab supported by continuous T beams of 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are b_w = 11 in. and d = 20 in. What tensile steel area is required at midspan to resist a moment of 6,400 in-kips if f_y = 60,000 psi and f'_c = 3,000 psi.



Case I

Case II

Solution

First determining the effective flange width from Section (8.3.1.) or ACI 8.10.2

1) $b \leq \frac{span}{4} = \frac{24 \times 12}{4} = 72 \ in$

2) $b \leq 16h_f + b_w = (16 \times 3) + 11 = 59 \ in$

3) $b \leq$ clear spacing between beams $+ \ b_w =$ center to center spacing between beams = 47 in

The centerline T beam spacing controls in this case, and b = 47 inches.

Assumption: Assuming that stress-block depth equals to the flange thickness of 3 inches (beam behaves like a rectangular shape).

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{6400}{0.9 \times 60 \times (20 - 3/2)} = 6.40 \ in^2 \tag{8.7}$$

Solve for "a":

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6.40 \times 60}{0.85 \times 3 \times 47} = 3.2 \ in > h_f = 3.0 \quad \text{Assumption incorrect}$$

Therefore, the beam will act as a T-beam and must be designed as a T-beam. From Case I given above and Section (8.4.1.) we have

$$A_{sf} = \frac{0.85 f_c' \ h_f (b - b_w)}{f_y} = \frac{0.85 \times (3ksi) \times (3in) \times (47 - 11)}{60(ksi)} = 4.58 \ in^2 \tag{8.8}$$

$$\phi M_{n1} = \phi A_{sf} f_y (d - \frac{h_f}{2}) = 0.9 \times 4.58 \times (60ksi) \times (20 - 3/2) = 4570 \ \text{in-kips} \tag{8.9}$$

$$\phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4570 = 1830 \ \text{in-kips} \tag{8.10}$$

Find "a" value by iteration. Assume initial a = 3.5 inches

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.5/2)} = 1.86 \ in^2 \tag{8.11}$$

Find an improve "a" value

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' \ b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.97 \ in \tag{8.12}$$

Iterate with the new a = 3.97 in.

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.97/2)} = 1.88 \ in^2 \tag{8.13}$$

Find an improve "a" value

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' \ b_w} = \frac{1.88 \times 60}{0.85 \times 3 \times 11} = 4.02 \ in \tag{8.14}$$

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 4.02/2)} = 1.88 \ in^2 \tag{8.15}$$

Since there is no change between equations (8.13) and (8.15) we have arrived at the answer. Therefore,

$$A_s = A_{sf} + (A_s - A_{sf}) = 4.58 + 1.88 = 6.46 \ \ in^2 \tag{8.16}$$

Check with ACI requirements for maximum amount of steel (Tension-Controlled)

$$c = \frac{a}{\beta_1} = \frac{4.02}{0.85} = 4.73 \tag{8.17}$$

$$\frac{c}{d} = \frac{4.73}{20} = .237 < 0.375 \qquad \textit{Tension-controlled}$$

Therefore, the T-beam satisfies the ACI provisions for tension failure. Next steps will be to select the reinforcement and check all the spacing requirements and detail the beam.