Chapter 8.
Flexural Analysis of T-Beams

8.1. Reading Assignments
Text Chapter 3.7; ACI 318, Section 8.10.

8.2. Occurrence and Configuration of T-Beams

- Common construction type.- used in conjunction with either on-way or two-way slabs.
- Sections consists of the flange and web or stem; the slab forms the beam flange, while the part of the beam projecting below the slab forms is what is called web or stem.

8.3. Concepts of the effective width, Code allowable values

In reality the maximum compression stress in T-section varies with distance from section Web.
Analysis of T-Sections in Bending

a. Two possible locations of equivalent rectangular stress block lead to the following analysis requirements:

1. "a" less than \( h_f \) - Analyze as for rectangular beam.
2. "a" greater than \( h_f \) - Special analysis required.

b. Consider a T-section in which entire stress block lies in flange - we may assume this to be the case for all sections. Statics will tell us if we are correct.

If \( a \leq h_f \), \( C_c = T_s \) → \( 0.85 f' c ab = A_s f_y \)

Therefore

\[
a = \frac{A_s f_y}{0.85 f' c b} = \frac{f_y d}{0.85 f' c}
\]

ask ourselves: if "\( a \)" \( \leq h_f \)?

if answer is "yes", calculate moment capacity for a rectangular beam.

if answer is "no", an extensive analysis is required.
Code allows the following maximum effective widths:

### 8.3.1. Symmetrical Beam

ACI318, Section 8.10.2.

1) \( b \leq \frac{\text{span}}{4} \)

2) \( \frac{b - b_w}{2} \leq 8h_f \)

3) \( \frac{b - b_w}{2} \leq \frac{1}{2} \text{ clear distance between beams} \)

### 8.3.2. Flange on one side only (Spandrel Beam)

ACI318, Section 8.10.3.

1) \( b - b_w \leq \frac{\text{span}}{12} \)

2) \( b - b_w \leq 6h_f \)

3) \( b - b_w \leq \frac{1}{2} \text{ clear distance to next web} \)

### 8.3.3. Isolated T-Beam

ACI318, Section 8.10.4.

1) \( b \leq 4b_w \)

2) \( \frac{b_w}{2} \leq h_f \)
8.4. Analysis of T-Beams - (a > h_f)

Consider the total section in two parts:

1) Flange overhangs and corresponding steel;
2) Stem and corresponding steel;

For equilibrium we have:

8.4.1. Case I:

\[ A_{sf} f_y = 0.85 f'_c h_f (b - b_w) \]  \hspace{1cm} (8.1)

or

\[ A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} \]  \hspace{1cm} (8.2)

8.4.2. Case II:

\[ (A_s - A_{sf}) f_y = 0.85 f'_c b_w a \]  \hspace{1cm} (8.3)

Solve for “a”:

\[ a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} \]  \hspace{1cm} (8.4)

and nominal moment capacity will be:

\[ M_n = A_{sf} f_y (d - \frac{h_f}{2}) + (A_s - A_{sf}) f_y (d - \frac{a}{2}) \]  \hspace{1cm} (8.5)
8.5. Balanced Condition for T-Beams

See Commentary page 48 of ACI 318-83 (old code).

From geometry:

\[
c^b = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d = \frac{87,000}{87,000 + f_y d}
\] (8.6)
8.6. Example.- Analysis of T-Beams in Bending:

Find the nominal moment capacity of the beam given above:

\[ f'_c = 2,400 \text{ psi} \]
\[ f_y = 50,000 \text{ psi} \]

Solution:

Check to see if a T-beam analysis is required:

Assume \( a < h_f \)

\[
a = \frac{A_s f_y}{0.85 f'_c b} = \frac{6.88 \times 50}{0.85 \times 2.4 \times 40} = 4.22 \text{ in}
\]

Since 4.22 in > 4.00 in, a T-beam analysis is required.

First find the reinforcement area to balance flanges \( (A_{sf} = ?) \)

\[
A_{sf} = 0.85 \frac{f'_c}{f_y} (b - b_w) h_f = 0.85 \times \frac{2.4}{50} \times (40 - 10) \times 4 = 4.90 \text{ in}^2
\]

\[
A_s - A_{sf} = 6.88 - 4.90 = 1.98 \text{ in}^2
\]

Solve for “a”

\[
0.85 f'_c b_w a = (A_s - A_{sf}) f_y
\]

\[
a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} = \frac{1.98 \times 50}{0.85 \times 2.4 \times 10} = 4.86 \text{ in} > 4 \text{ in} \ o.k.
\]

Assumption is o.k.
\[ c = \frac{a}{\beta_1} = \frac{4.86}{0.85} = 5.72 \]

\[ \frac{c}{d} = \frac{5.72}{20.5} = 0.279 < 0.375 \quad \text{Tension-controlled} \]

Find the nominal moment capacity of the beam:

\[
M_n = A_{sf} f_y \left( d - \frac{h_f}{2} \right) + f_y (A_s - A_{sf}) \left( d - \frac{a}{2} \right)
\]

\[
M_n = 4.9 (in^2) \times 50 (ksi) \times (20.5 - \frac{4}{2}) + 50 (ksi) \times 1.98 (in^2) \times (20.5 - \frac{4.86}{2})
\]

\[
M_n = 4530 + 1790 = 6,320 \text{ in} - k
\]

Note:

This could have been done by statics with

\[
T_s = A_{sf} f_y
\]

\[
C_c = (b - b_w)(h_f) \times 0.85 f_c' + ab_w(0.85)f_c'
\]
8.7. Example.- Design of T-Beams in Bending- Determination of Steel Area for a given Moment:

A floor system consists of a 3 in. concrete slab supported by continuous T beams of 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are $b_w = 11$ in. and $d = 20$ in. What tensile steel area is required at midspan to resist a moment of 6,400 in-kips if $f_y = 60,000$ psi and $f'c = 3,000$ psi.

\[
\begin{align*}
&b_w \\
&h_f \\
&\text{Case I} \\
&\text{Case II}
\end{align*}
\]

Solution

First determining the effective flange width from Section (8.3.1.) or ACI 8.10.2

1) \( b \leq \frac{\text{span}}{4} = \frac{24 \times 12}{4} = 72 \text{ in} \)

2) \( b \leq 16h_f + b_w = (16 \times 3) + 11 = 59 \text{ in} \)

3) \( b \leq \text{clear spacing between beams} + b_w = \text{center to center spacing between beams} = 47 \text{ in} \)

The centerline T beam spacing controls in this case, and $b = 47$ inches.

Assumption: Assuming that stress-block depth equals to the flange thickness of 3 inches (beam behaves like a rectangular shape).

\[
A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{6400}{0.9 \times 60 \times (20 - 3/2)} = 6.40 \text{ in}^2 \quad (8.7)
\]
Solve for “a”:

\[
a = \frac{A_{sf} f_y}{0.85 f_c' b} = \frac{6.40 \times 60}{0.85 \times 3 \times 47} = 3.2 \text{ in} > h_f = 3.0 \text{ Assumption incorrect}
\]

Therefore, the beam will act as a T-beam and must be designed as a T-beam. From Case I given above and Section (8.4.1.) we have

\[
A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = \frac{0.85 \times (3 \text{ksi}) \times (3 \text{in}) \times (47 - 11)}{60(\text{ksi})} = 4.58 \text{ in}^2 \tag{8.8}
\]

\[
\phi M_{n1} = \phi A_{sf} f_y (d - \frac{h_f}{2}) = 0.9 \times 4.58 \times (60 \text{ksi}) \times (20 - 3/2) = 4570 \text{ in-kips} \tag{8.9}
\]

\[
\phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4570 = 1830 \text{ in-kips} \tag{8.10}
\]

Find “a” value by iteration. Assume initial a = 3.5 inches

\[
A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.5/2)} = 1.86 \text{ in}^2 \tag{8.11}
\]

Find an improve “a” value

\[
a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.97 \text{ in} \tag{8.12}
\]

Iterate with the new a = 3.97 in.

\[
A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.97/2)} = 1.88 \text{ in}^2 \tag{8.13}
\]

Find an improve “a” value

\[
a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.88 \times 60}{0.85 \times 3 \times 11} = 4.02 \text{ in} \tag{8.14}
\]

\[
A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 4.02/2)} = 1.88 \text{ in}^2 \tag{8.15}
\]
Since there is no change between equations (8.13) and (8.15) we have arrived at the answer. Therefore,

\[ A_s = A_{sf} + (A_s - A_{sf}) = 4.58 + 1.88 = 6.46 \text{ in}^2 \]  

(8.16)

Check with ACI requirements for maximum amount of steel (Tension-Controlled)

\[ c = \frac{a}{\beta_1} = \frac{4.02}{0.85} = 4.73 \]  

(8.17)

\[ \frac{c}{d} = \frac{4.73}{20} = .237 < 0.375 \quad \text{Tension-controlled} \]

Therefore, the T-beam satisfies the ACI provisions for tension failure. Next steps will be to select the reinforcement and check all the spacing requirements and detail the beam.