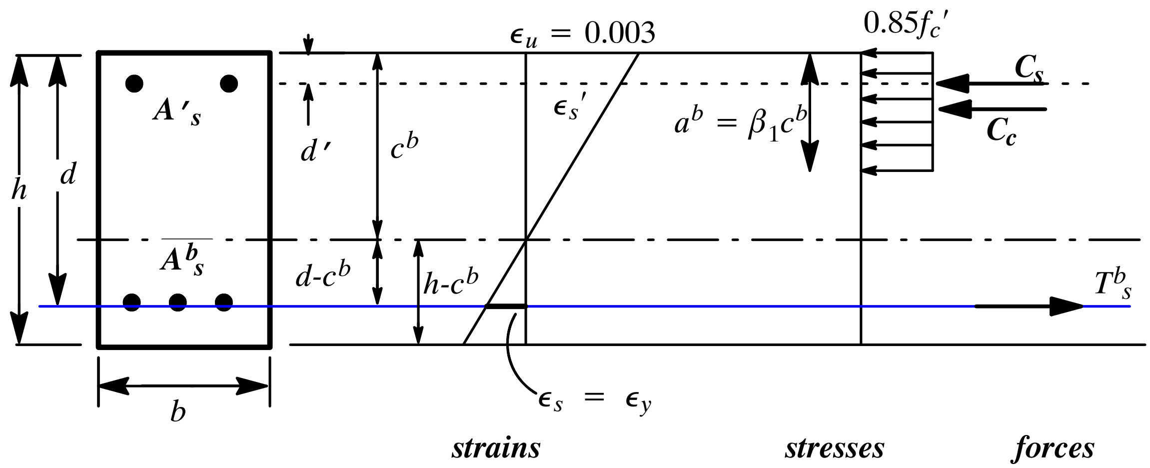
Chapter 6.
Compression Reinforcement - Flexural Members

If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete cannot develop the compression force required to resist the give bending moment. In this case, reinforcing is added in the compression zone, resulting in a so-called **doubly reinforced** beam, i.e., one with compression as well as tension reinforcement. Compression reinforced is also used to improve serviceability, improve long term deflections, and to provide support for stirrups throughout the beam.

6.1. Reading Assignment:

Text Section 5.7; ACI 318, Sections: 10.3.4, 10.3.3, and 7.11.1

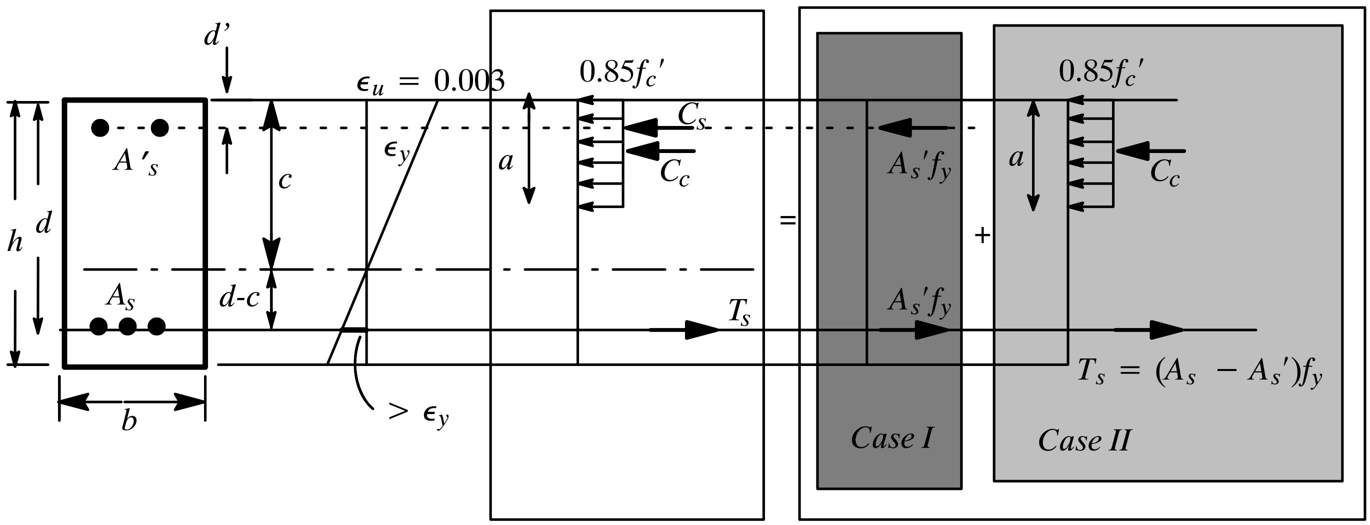
6.2. Strength Calculations



From geometry we can find the strain in compression steel at failure as:

$$\epsilon_s' = 0.003\frac{c - d'}{c} \tag{6.1}$$

6.3. Nominal Resisting Moment When Compression Steel Yields



Doubly Reinforced Rectangular Beam

Total resisting moment can be considered as sum of:

1. Moment from corresponding areas of tension and compression steel
2. The moment of some portion of the tension steel acting with concrete.

$$M_n = (A_s - A_s')f_y\left(d - \frac{\beta_1 c}{2}\right) + A_s' f_y (d - d') \tag{6.2}$$

and from equilibrium:

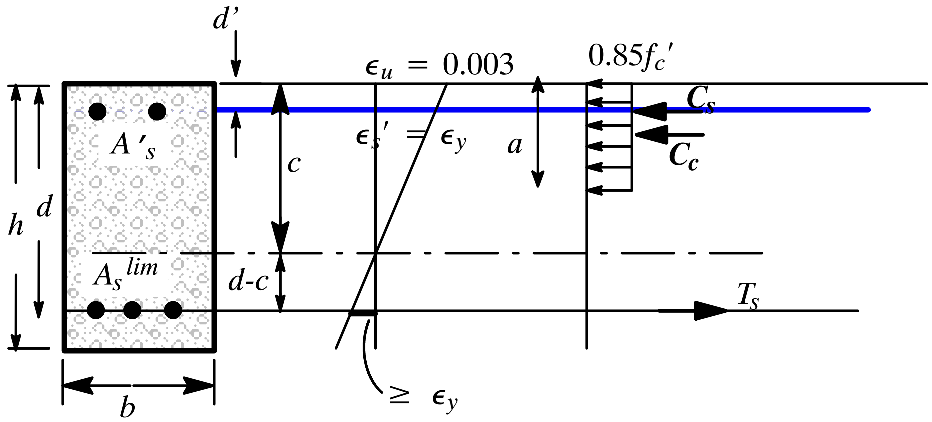
$$0.85f_c'\,ab = (A_s - A_s')f_y \tag{6.3}$$

Solve for "a":

$$a = \frac{A_s - A_s'}{0.85f_c'\,b}f_y \tag{6.4}$$

6.4. Compression Steel below Yield Stress (strain compatibility check).

Whether or not the compression steel will have yielded at failure can be determined as follows:



From geometry: $$\frac{\epsilon_u}{\epsilon'_s} = \frac{c}{c - d'} \qquad (6.5)$$

if compression steel yield $\epsilon'_s = \epsilon_y$ then:

$$\frac{\epsilon_u}{\epsilon_y} = \frac{c}{c - d'} \quad \rightarrow \quad c = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} d' \qquad (6.6)$$

Equilibrium for case II:

$$(A_s^{\lim} - A'_s) f_y = 0.85 \times (\beta_1 c)\, b\, f'_c \qquad (6.7)$$

Substitute for *"c"* from Eq. (6.6) and (6.7) and divide both sides by "*bd*" gives:

$$\frac{(A_s^{\lim} - A'_s) f_y}{bd} = 0.85 \times \beta_1 \times b \times f'_c \left[\left(\frac{\epsilon_u}{\epsilon_u - \epsilon_y}\right) d'\right] \frac{1}{bd} \qquad (6.8)$$

or $$\frac{A_s^{\lim}}{bd} = \frac{A'_s}{bd} + 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \left(\frac{\epsilon_u}{\epsilon_u - \epsilon_y}\right) \frac{d'}{d} \qquad (6.9)$$

$$\rho_{\lim} = \rho'_s + 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \times \left(\frac{87,000}{87,000 - f_y}\right) \frac{d'}{d} \qquad (6.10)$$

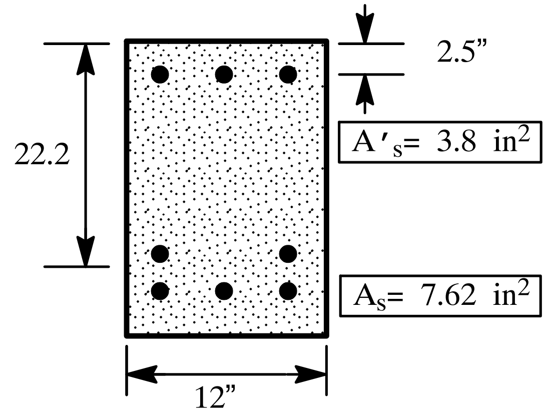
if $\rho_{actual} > \rho_{\lim}$ then compression steel will yield	this is common for shallow beams using high strength steel
if $\frac{A_s - A'_s}{bd} \geq 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \times \left(\frac{87,000}{87,000 - f_y}\right) \frac{d'}{d}$	then compression steel will yield

6.5. Example of analysis of a reinforced concrete section having compression reinforcement.

Determine the nominal moment, M_n, and the ultimate moment capacity, M_u, of the reinforced concrete section shown below.

$$f_c' = 5,000 \; psi$$

$$f_y = 60,000 \; psi$$



Solution

M_n can be calculated if we assume some conditions for compression steel.

Assume that compression steel yields:

$$C_c = 0.85 f'_c \beta_1 cb = 0.85 \times (5 \; ksi) \times (0.80) \times c \times (12) = 40.8c$$

$$C_s = A_s' f_y = 3.8 \times (60 ksi) = 228 \; kips$$

$$T_s = (7.62 \; in^2) \times (60 \; ksi) = 457 \; kips$$

Equilibrium:

$$C_s + C_c = T_s$$

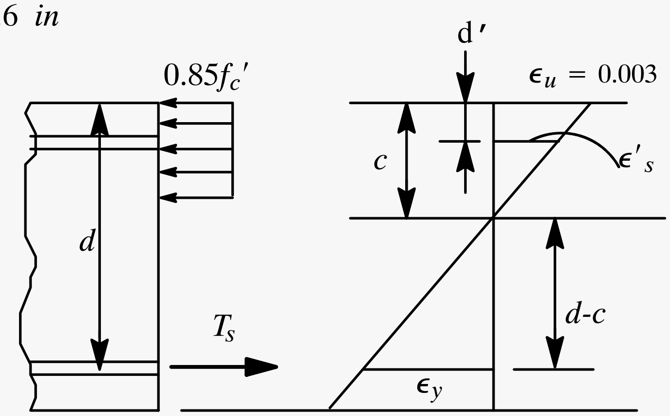
solve for c:

$$c = \frac{457 - 228}{40.8} = 5.6 \; in$$

check assumption

$$\epsilon'_s = 0.003 \frac{c - d'}{c}$$

$$= 0.003 \frac{5.6 - 2.5}{5.6} = 0.0017$$



$$\epsilon'_s = 0.0017 < \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$ **wrong assumption**

This means the compression steel does not yield. Therefore, our initial assumption was wrong. We need to make a new assumption.

Assume $f'_s < f_y$

$$C_s = A'_s f'_s = A'_s \epsilon'_s E_s$$

$$= (3.8\ in^2) \times (0.003 \frac{c - 2.5}{c}) \times (29,000\ ksi) = 330 \frac{c - 2.5}{c}$$

Now for equilibrium: $C_s + C_c = T_s$

$$40.8c + 330 \times \frac{c - 2.5}{c} = 457\ kips \quad \rightarrow solve \text{ for } c \quad \rightarrow \quad c = 6.31\ in$$

check assumption

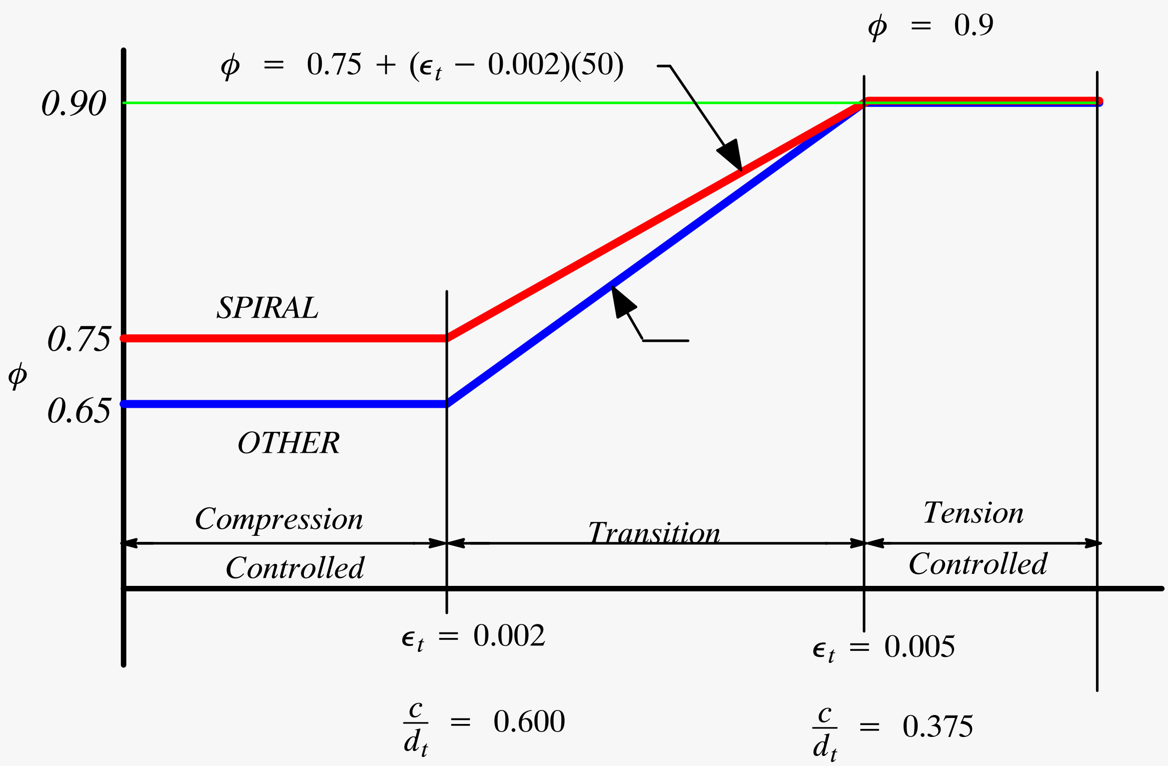
$$f'_s = 0.003 \times \frac{6.31 - 2.5}{6.31} \times 29,000 = 52.5\ ksi < f_y = 60\ ksi$$

assumption o.k.

check ACI Code requirements for tension failure

$$\frac{c}{d} = \frac{6.31}{22.2} = 0.284 < 0.375$$

We are in the tension-controlled section and satisfy the ACI code requirements.



Calculate forces:

$$\left.\begin{aligned} C_c &= 40.8 \times (6.31\ \ in) = 258\ \ kips \\ C_s &= 3.8 \times (52.5 ksi) = 200\ \ kips \end{aligned}\right\} \quad 258+200=458$$

$$T_s = (7.62\ \ in^2) \times (\ 60 ksi) = 457\ \ kips$$

Equilibrium is satisfied

Take moment about tension reinforcement to determine the nominal moment capacity of the section:

$$M_n = C_c\left(d - \frac{\beta_1 c}{2}\right) + C_s(d - d')$$

Nominal moment capacity is:

$$M_n = (258\ \ kips) \times (22.2 - \frac{0.80 \times 6.31}{2}) + 200(22.2 - 2.5)$$

$$= 5080 + 3940 = 9020\ \ in - kips$$

Ultimate moment capacity is:

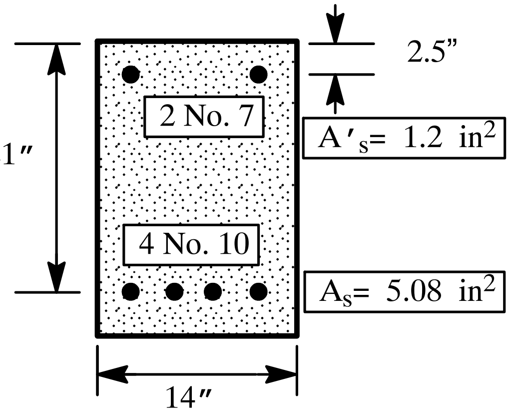
$$M_u = \phi M_n = 0.9 \times 9020 = 8118\ \ in - k$$

6.6. Example of analysis of a doubly reinforced concrete beam for flexure

Determine whether the compression steel yield at failure.

$$f_c' = 5{,}000 \ psi$$

$$f_y = 60{,}000 \ psi$$



Solution

$$\rho = \frac{A_s}{bd} = \frac{5.08}{14 \times 21} = 0.0173$$

$$\rho' = \frac{A'_s}{bd} = \frac{1.2}{14 \times 21} = 0.0041$$

$$\rho - \rho' = 0.0173 - 0.0041 = 0.0132$$

Check whether the compression steel has yielded, use Eq. (6.10):

$$0.0132 \overset{?}{\geq} 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \times \left(\frac{87{,}000}{87{,}000 - f_y}\right)\frac{d'}{d}$$

$$0.0132 \overset{?}{\geq} 0.85 \times 0.80 \times \frac{5}{60} \times \left(\frac{87{,}000}{87{,}000 - 60000}\right)\frac{2.5}{21}$$

$$0.0132 \overset{?}{\geq} 0.0217$$

Therefore, the compression steel does not yield.

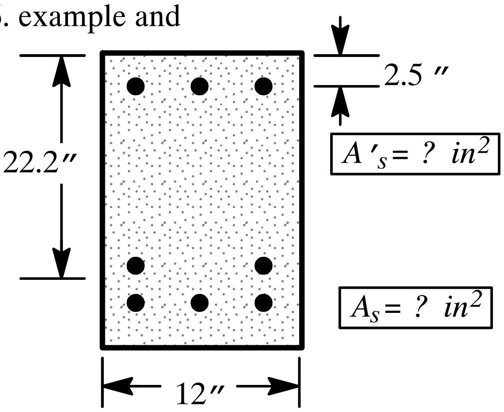
6.7. Example: Design of a member to satisfy a nominal moment capacity.

Assume we have the same size beam as Section 6.6. example and wish to satisfy the same nominal conditions:

$$f_y = 60,000 \ psi$$

$$f_c' = 5,000 \ psi$$

$$Required \quad M_n = 9020 \ in - k$$



Solution

For singly reinforced section:

$$use \quad \frac{c}{d} = 0.375$$

$$\rho = 0.85\beta_1 \frac{c}{d} \times \frac{f_c'}{f_y}$$

$$\rho = (0.85)(0.80)(0.375)\frac{5 \ ksi}{60 \ ksi} = 0.0213$$

Maximum A_{s1} for singly reinforced section then is:

$$A_{s1} = \rho \times b \times d = (0.0213) \times (12) \times (22.2) = 5.66 \ in^2$$

$$M_n = \rho f_y bd^2 \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$M_n = (0.0213 \ in^2)(60 \ ksi)(12 \ in)(22.2 \ in)^2\left(1–0.59(0.0213) \times \frac{60}{5}\right) = 6409 \ in.kips$$

$$M_{u2} = \phi M_n = 0.9 \times 6409 = 5747 \quad in.kips$$

Moment which must be resisted by additional compression and tension reinforcement

$$M_{u1} = M_{u1} - M_{u2}$$

$$M_{u1} = 0.9 \times 9020 - 5747 = 2365 \ in.kips$$

Assuming compression steel yields we will have:

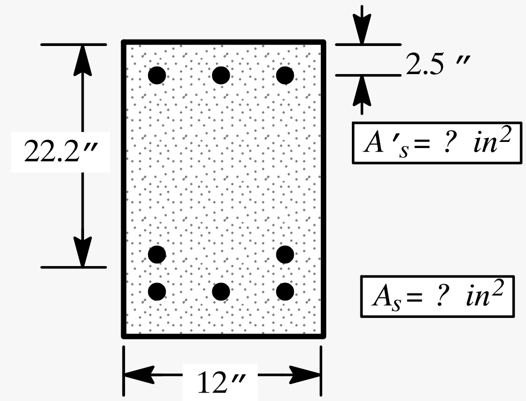
$$M_{u1} = \phi A'_s f_y (d - d') = 0.9 \times A'_s \times (60) \times (22.2 - 2.5) = 1063.8 \times A'_s$$

$$2365 \ in–k = 1063.8 \times A'_s \quad \rightarrow \quad A'_s = \frac{2365}{1063.8} = 2.23 \ in^2$$

Therefore, the design steel area for tension and compression reinforcement will be:

$$A_s = 5.66 + 2.23 = 7.89 \ in^2 \qquad 8\text{-}\#9$$

$$A'_s = 2.23 \ in^2 \qquad 3\text{-}\#8$$



Check whether the compression steel has yielded, use Eq. (6.10):

$$\frac{A_s - A'_s}{bd} \geq 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \times \left(\frac{87,000}{87,000 - f_y}\right)\frac{d'}{d}$$

$$\frac{8 - 2.37}{22.2 \times 12} \geq 0.85 \times 0.80 \times \frac{5}{60} \times \left(\frac{87,000}{87,000 - 60000}\right)\frac{2.5}{22.2}$$

$$0.0211 \geq 0.206$$

Therefore the compression steel yields at failure

Check to make sure that the final design will fall under “tension-controlled”

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b}$$

$$a = \frac{(8.00–2.37)60}{0.85(5)(12)} = 6.62 \ in$$

$$c = \frac{a}{\beta_1} = \frac{6.62}{0.80} = 8.28 \ in$$

$$\frac{c}{d} = \frac{8.28}{22.2} = 0.373 < 0.375$$ Tension controlled

see the following page for the rest of the solution done in a speadsheet.