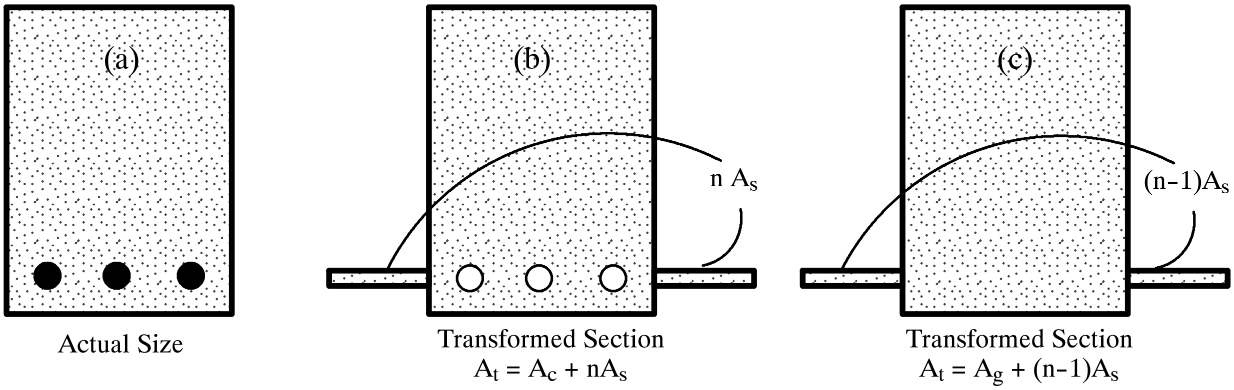
4.8. Method of transformed Sections

a. Limited to consideration of sections in which concrete stress-strain is linear.

b. Applicable to either sections in bending or axial compression.

c. Knowledge or assumption about the depth of cracking of the section is required.

d. General examination of the method

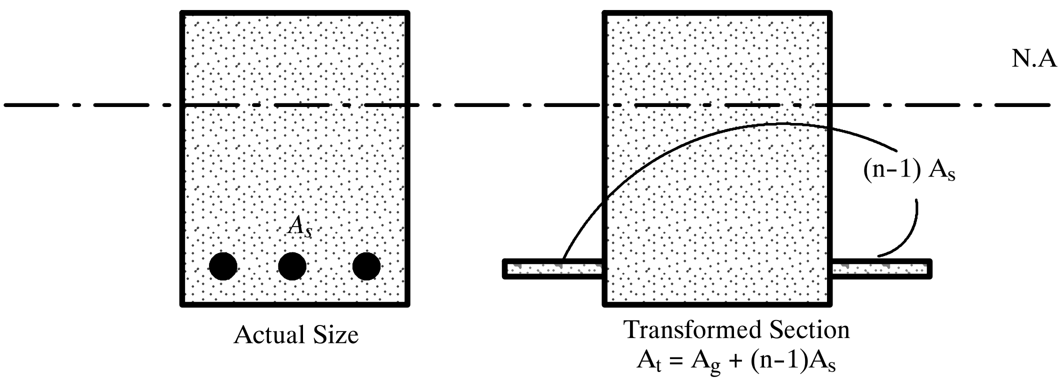


Transformed section in Beams

4.9. Method of Transformed Section for Beams:

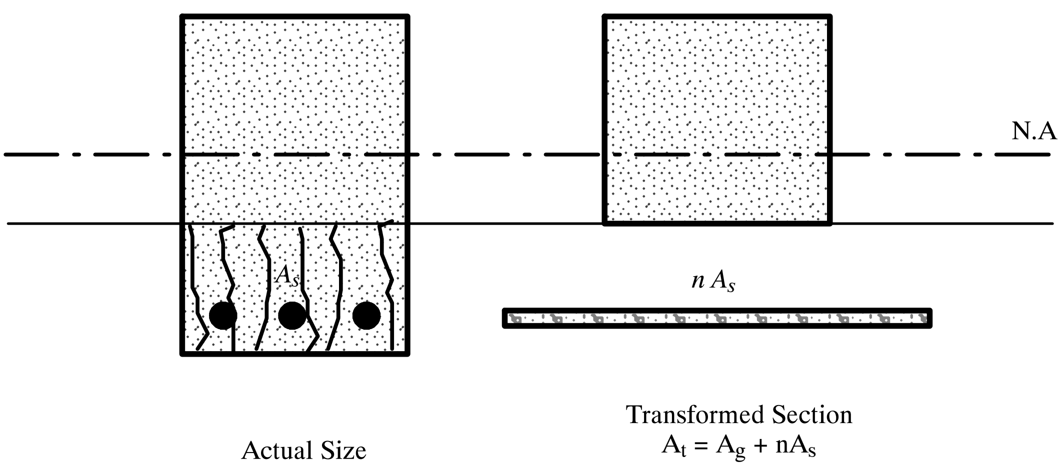
4.9.1. Uncracked Sections (Cracked Moment, M*cr*)

Applicable to beams uncracked section when $0 < \mathbf{M} < \mathbf{M_{cr}}$ (where M_{cr} is the crack moment)



4.9.2. Cracked Sections

Applicable to beams cracked section when $\mathbf{M_{cr}} < \mathbf{M} < \mathbf{M_y}$ (where M_y is the yield moment)



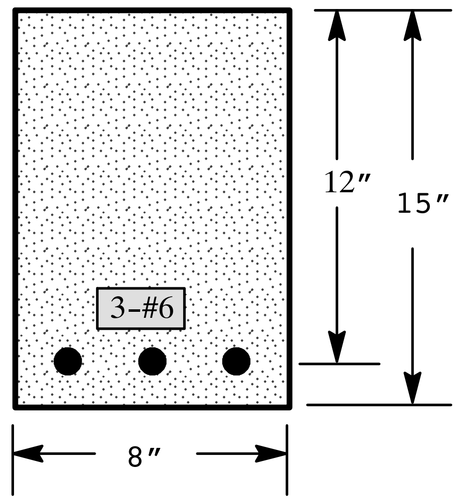
4.10. Example of Transformed Section Applied to Beam

1. Consider the section shown below. Calculate the stress caused by a bending moment of 13.83 ft-kips.

$f'_c = 5$ ksi

$f_y = 60$ ksi

$f_r = 500$ psi



Section Properties:

A_s = *3-#6 bars*

$$A_s = 3 \times 0.44 \ (in^2) = 1.32 \ in^2$$

$$E_c = 57,000\sqrt{5,000} = 4.03 \times 10^6 \ psi$$

$$n = \frac{29,000,000 \ psi}{4.03 \times 10^6 \ psi} = 7.2 \rightarrow use \ n = 7$$

$$\rho = \frac{A_s}{bd} = \frac{1.32 \ in^2}{8 \ (in) \times 12 \ (in)} = 0.014$$

For Uncracked Section (Assume)

Find the location of neutral axis (First Moment of Area = 0).

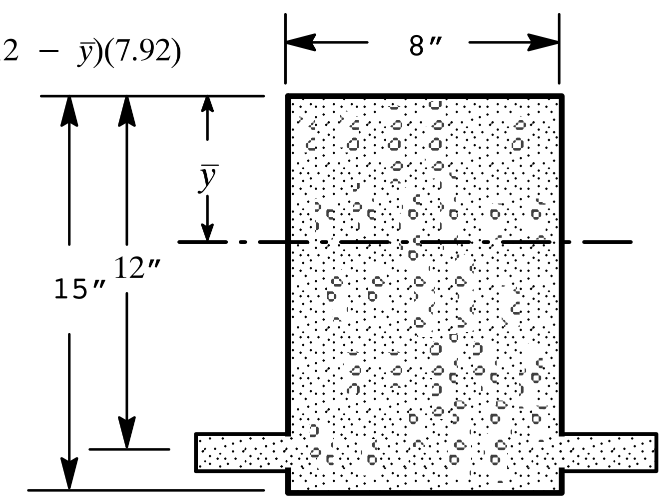
$$(8\bar{y})\left(\frac{\bar{y}}{2}\right) = 8(15 - \bar{y}) \times \left(\frac{15 - \bar{y}}{2}\right) + (12 - \bar{y})(7.92)$$

$$\bar{y} = 7.78 \ in$$

or find the centroidof the cross section by using the top edge as the reference point.

$$\bar{y} = \frac{8 \times 15 \times \frac{15}{2} + 7.92 \times 12}{8 \times 15 + 7.92}$$

$$\bar{y} = 7.78 \ in$$



Area of overhangs; $(n - 1)A_s = 6 \times 1.32 = 7.92 \ in^2$

This value (7.78 in) should be the same as the one we get using Eq. (4.2) found earlier. (see next page for proof).

$$\frac{c}{d} = \frac{2\rho(n - 1) + (h/d)^2}{2\rho(n - 1) + 2(h/d)} \qquad (4.2)$$

Substitute

$$\frac{c}{d} = \frac{2(0.014)(7 - 1) + (15/12)^2}{2(0.014)(7 - 1) + 2(15/12)} = 0.65$$

Solve for c

$$\rightarrow \frac{c}{d} = 0.65 \ \ therefore \ \ c = 0.65d = 7.78 \ in$$

Note:

For a homogenous section, we can relate bending moment to stresses at distance "y" from the neutral axis as the following

$$f = M\frac{Y}{I}$$

where

f	=	stress
M	=	bending moment
y	=	distance from neutral axis to the point where stresses are to be calculated
I	=	moment of inertia of the cross section

Calculate $I_{n.a.}$

$$I_{N.A.} = \frac{1}{3} \times 8 \times 7.78^3 + \frac{1}{3} \times 8 \times (15 - 7.78)^3 + 7.92 \times (12 - 7.78)^2$$

$$I_{N.A.} = 2,400 \ (in^4)$$

Calculate stresses

Now, find the stress in top fiber (compression stress at top fiber):

$$f_{top} = \frac{M\bar{y}}{I_{N.A.}} = \frac{(13.83 \ ft - kips) \times (12 \ in/ft) \times (7.78 \ in)}{2,400 \ in^4} = 0.54 \ k/in^2$$

$$f_{bot} = \frac{M(15 - \bar{y})}{I_{N.A.}} = \frac{(13.83 \ ft - kips) \times (12 \ in/ft) \times (15 - 7.78 \ in)}{2,400 \ in^4} = 0.50 \ k/in^2$$

$$f_{bot} = 0.50 \ k/in^2 \le f_r = 0.5 \ ksi$$

Therefore, the assumption of uncracked section was correct, since tension stresses are smaller than f_r given in the problem.

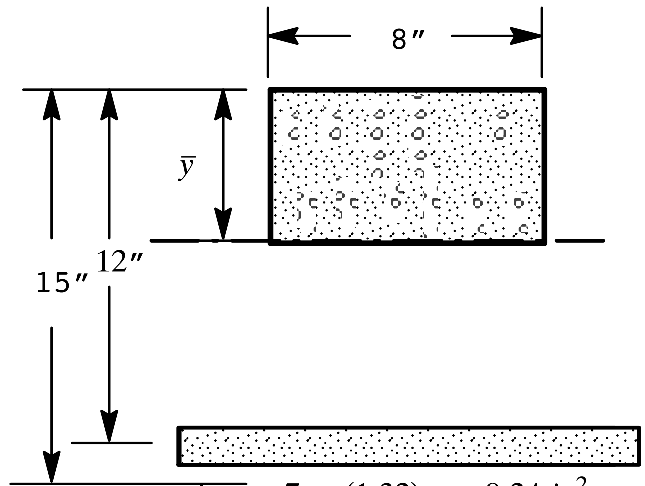
4.11. Example 2. Calculate Yield Moment for the Beam

Assume concrete accepts no tension. (yield moment is when steel is starting to yield).

Locate N.A.

$$(8\bar{y})(\frac{\bar{y}}{2}) = (12 - \bar{y})(9.24\ in^2)$$

$$\bar{y} = 4.24\ in$$



$$nA_s = 7 \times (1.32) = 9.24\ in^2$$

Calculate $I_{n.a.}$

$$I_{N.A.} = \frac{1}{3} \times 8 \times 4.24^3 + 9.24 \times 7.76^{\ 2}$$

$$I_{N.A.} = 760\ in^4$$

At level of steel, if f_y = 60,000 psi, then the stress in the transformed section will be

$$\frac{f_y}{n} = \frac{60,000}{7} = 8,570\ \ psi$$

and

$$M = \frac{fI}{y} = \frac{(8,570\ psi)(760\ in^{4)}}{7.76\ in} = 839,000\ in - lb = 839\ in - kips$$

See next page for check with previous methods that we have learned.

Check

Check the moment found in the previous page with Eq. :

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \tag{4.3}$$

$$k = \sqrt{(0.014 \times 7)^2 + 2(0.014)(7)} - (0.014)(7)$$

$$k = 0.355$$

therefore

$$kd = 0.355(12) = 4.22\ in$$

this is very close to what we calculated for $\bar{y}$ = 4.24 in the last page. The slight difference is due to significant digit calculations.

Therefore

$$M_y = A_s f_y d\left(1 - \frac{k}{3}\right) = (1.32\ in^2)(60\ ksi)(12\ in)\left(1 - \frac{0.355}{3}\right)$$

$$M_y = 838\ in\text{-}kips$$