Chapter 10.
BOND AND ANCHORAGE

10.1. Reading Assignment

Chapter 5 of text

ACI 318 Chapter 12.

10.2. Introduction

Reinforcement for concrete to develop the strength of a section in tension depends on the compatibility of the two materials to act together in resisting the external load. The reinforcing element, such as a reinforcing bar, has to undergo the same strain or deformation as the surrounding concrete in order to prevent the discontinuity or separation of the two materials under load. The modulus of elasticity, the ductility, and the yield or rupture strength of the reinforcement must also be considerably higher than those of the concrete to raise the capacity of the reinforced concrete section to a meaningful level. Consequently, materials such as brass, aluminum, rubber, or bamboo are not suitable for developing the bond or adhesion necessary between the reinforcement and the concrete. Steel and fiber glass do possess the principal factors necessary: yield strength, ductility, and bond value.

Bond strength results from a combination of several parameters, such as the mutual adhesion between the concrete and steel interfaces and the pressure of the hardened concrete against the steel bar or wire due to the drying shrinkage of the concrete. Additionally, friction interlock between the bar surface deformations or projections and the concrete caused by the micro movements of the tensioned bar results in increased resistance to slippage. The total effect of this is known as *bond.* In summary, bond strength is controlled by the following major factors:

1. Adhesion between the concrete and the reinforcing elements
2. Gripping effect resulting from the drying shrinkage of the surrounding concrete and the shear interlock between the bar deformations and the surrounding concrete
3. Frictional resistance to sliding and interlock as the reinforcing element is subjected to tensile stress
4. Effect of concrete quality and strength in tension and compression

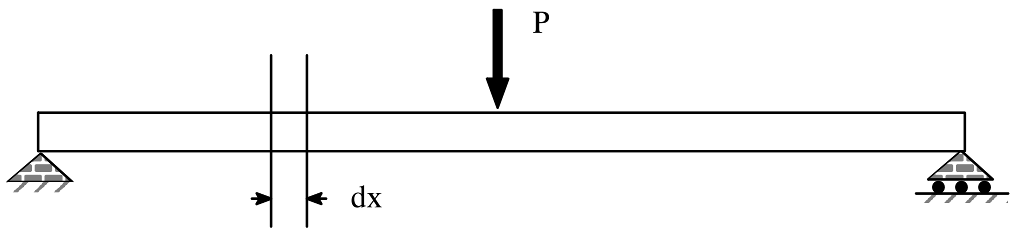
5. Mechanical anchorage effect of the ends of bars through development length, splicing, hooks, and crossbars

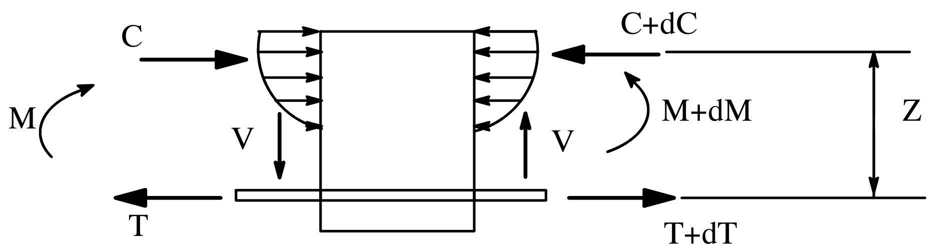
6. Diameter, shape, and spacing of reinforcement as they affect crack development

The individual contributions of these factors are difficult to separate or quantify. Shear interlock, shrinking confining effect, and the quality of the concrete can be considered as major factors.

10.3. Bond Stress Development

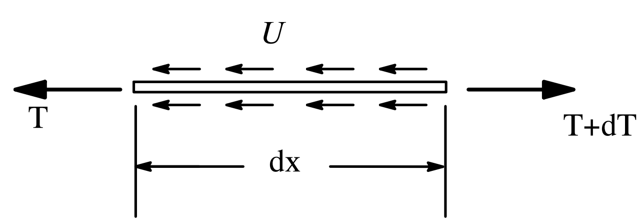
Bond stress is primarily the result of the shear interlock between the reinforcing element and the enveloping concrete caused by the various factors previously enumerated. It can be described as a local shearing stress per unit area of the bar surface. This direct stress is transferred from the concrete to the bar interface so as to change the tensile stress in the reinforcing bar along its length.





$$T = \frac{M}{Z} \quad ; \quad T + dT = \frac{M + dM}{Z} \quad \rightarrow \quad dT = \frac{dM}{Z}$$

For equilibrium of the bar section:



$$T + Udx = T + dT$$

$$Udx = dT$$

$$U = \frac{dT}{dx} = \frac{dM}{Z} \times \frac{1}{dx} \quad kips/inch$$

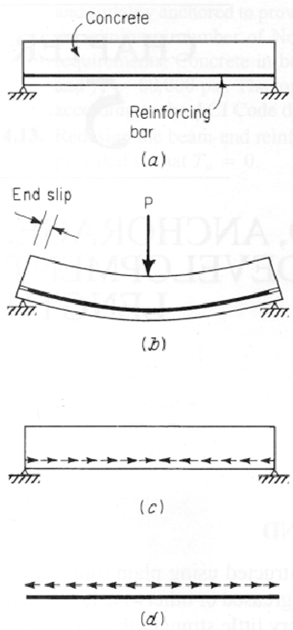
$$\frac{dM}{dx} = V \quad \rightarrow \quad U = \frac{V}{Z}$$

$$u = \frac{U}{\sum_0} = \frac{V}{\sum_0 Z}$$

u = flexural bond stress; kips/in^2

where $\sum_0$ is the perimeter or sum of perimeters of the bars at the section considered.

10.4. Local Bond Effects near Cracks



Bars are greased before pouring concrete.

FIGURE 5.1
Bond stresses due to flexure: (*a*) beam before loading; (*b*) unrestrained slip between concrete and steel; (*c*) bond forces acting on concrete; (*d*) bond forces acting on steel.

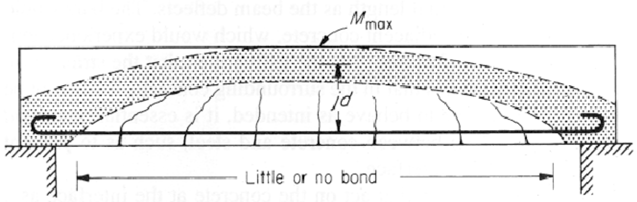
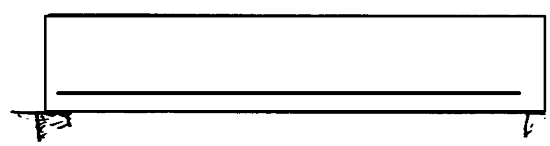


FIGURE 5.2
Tied-arch action in a beam with little or no bond.



<u>Main Reinforcing Bars:</u>

(assume no bond)

- Beam acts as a tied arch, will not collapse

- Tension in bars is uniform and equal

$$T = \frac{M_{\max}}{Z}$$

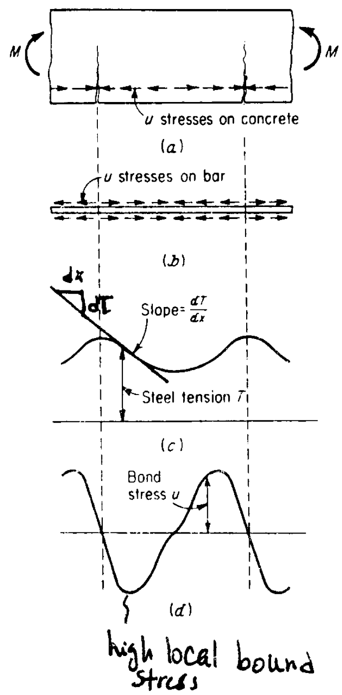
- Linear total deformation results in large beam deflection, large cracks

<u>Deformed Bars</u>

- Stress in stell is maximum only over a short section – less elsewhere

- much smaller total deflection

- Cracks are distributed, narrow



Beam segment subjected to pure bending

$$\textit{Bar force T,} \qquad U = \frac{dT}{dx}$$

$$\text{Bond Stress } u = \frac{U}{\sum_0} = \frac{V}{\sum_0 Z}$$

Figure Variation of steel force and bond stress in reinforced concrete member subject to pure bending: (*a*) cracked concrete segment, (*b*) bond stresses acting on reinforcing bar, (*c*) variation of tensile force in steel, (*d*) variation of bond stresses along steel.

Load bond stress effects add to the above overal effects:

$$u_{total} = \underbrace{\frac{V}{\sum_0 Z}}_{overal} + \underbrace{\frac{dT/dx}{\Sigma_0}}_{local}$$

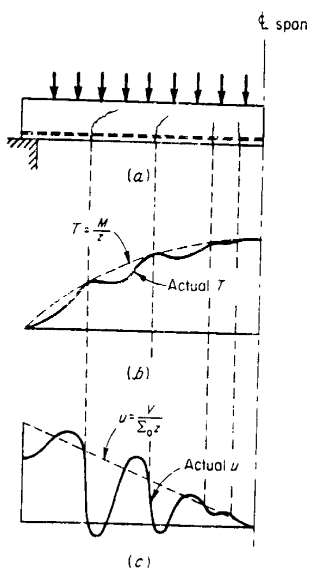
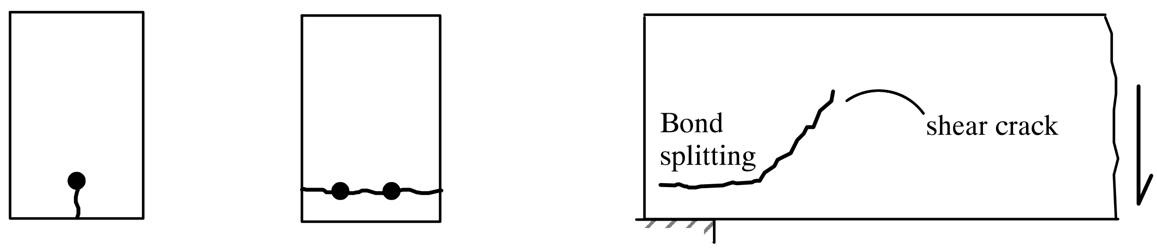


Figure Effect of flexural cracks on bond stresses in beam: (*a*) beam with flexural cracks, (*b*) variation of tensile force *T* in steel along span, (*c*) variation of bond stress *u* along span.

10.5. Bond Failure

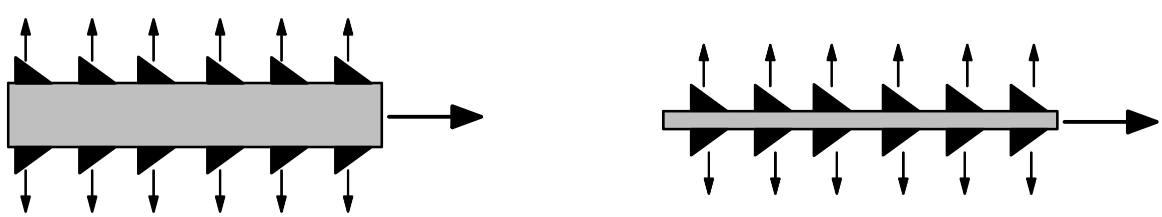
Bond failure is likely to occur near ends of beams, where high flexural bond stresses can combine with high local bond stresses.

Bond failure may take two forms, both of which result from wedging action as the bar is pulled relative to the concrete and often acts in concrete with shear crack and often acts in concrete with shear crack.



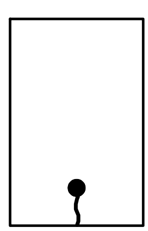
vertical crack Horizontal crack

Tests at N.B.S. (National Bureau of Standards) and University of Texas indicate that bond failure will occur when bond force U reaches a critical value. It is interesting to note that at failure, the force U is independent of bar size. Consistent with concept of "wedge action", when splitting force depends on driving force, not wedge width.



wedge action is when the ribs of deformed bars, bears against the concrete.

Tests have shown that for single bars causing vertical splits or for bars spaced further than 6 inches apart

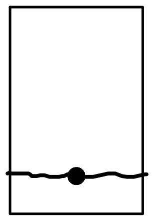


vertical crack

$$U_n = 35\sqrt{f_c'}$$

Ultimate average bond force per inch of length of bar

For bars spaced less than 6 inches apart, (causing horizontal splitting)



$$U_n = 0.80 \times 35\sqrt{f_c'} = 28\sqrt{f_c'}$$

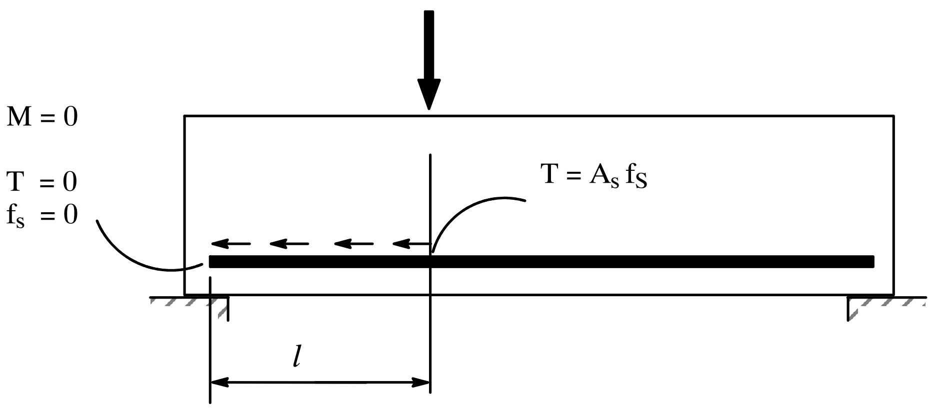
Ultimate average bond force per inch of length of bar

In terms of stresses rather than forces

$$u_n = \frac{U_n}{\sum_0} = \frac{35\sqrt{f_c'}}{\pi d_b} = \frac{11\sqrt{f_c'}}{d_b}$$

10.6. Development Length

Consider a beam similar to that used to obtain the results above:



$$T_s = A_b f_s = Ul$$ (Average bond force per inch) * length

or

$$U = \frac{T_s}{l} = \frac{A_b f_s}{l}$$ Average bond force per unit length

We may also solve for l to obtain the critical **development length**.

$$l_d = \frac{A_b f_s}{U_n}$$ U_n is the ultimate bond force per unit length

Two criteria control development length calculation:

1) Bond must be counted on to develop bar yield force ($f_s = f_y$)

2) Average ultimate bond force is limited to 35sqrt(f'$_c$) or 28sqrt(f'$_c$)

for spacing of greater than 6 inches

$$l_d = \frac{A_b f_y}{35\sqrt{f_c'}} = \frac{0.029 A_b f_y}{\sqrt{f_c'}}$$

for spacing of less than 6 inches:

$$l_d = \frac{A_b f_y}{28\sqrt{f_c'}} = \frac{0.0357 A_b f_y}{\sqrt{f_c'}}$$

If these lengths are provided, bond failure will not occur, obviously, small bars have less bond problem than large bars. Smaller bars require less development length because

$$A_b = \frac{1}{4}\pi d_b^2$$

therefore, the development length, l_d, is proportional to squared of bar diameter. the smaller the bar diameter the smaller will be the required development length.

According to ACI, the development length for design is obtained by a basic development length as given above and then it is modified by a series of modification factors.

10.7. Example of Embedment Length of Deformed bars

Calculate the required embedment length of the deformed bars in the following two cases: (12 inches of concrete below top reinforcement). Assume that #3 stirrups are used for shear and stirrup spacing based on shear calculations is 6.0 in. throughout the beam, S=6.0 in., d=15 in., $A_s^{required} = 1.6$ in.^{2}

A) 3#7 bars top reinforcement in single layer in a beam with No. 3 stirrups
f'_c = 4,000 psi (normal weight)
f_{yt} = 60,000 psi and f_y = 60,000 psi
clear spacing between bars are $2d_b$, clear side cover is 1.5 inches on each side.

B) Same as part (A), except that the clear spacing between bars is equal to *one* inch. The bars are epoxy coated.

Solution (A)

ACI Sect. 12.2.3
$$l_d = \left[\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c+K_{tr}}{d_b}\right)}\right]d_b$$

$\Psi_t = 1.3$ Top bars

$\Psi_e = 1.0$ Uncoated reinforcement

$\Psi_s = 1.0$ No. 7 and larger bars

$\lambda = 1.0$ Normal weight concrete

$d_b = 0.875$ *in*

$$c = \text{spacing or cover dimension} = \begin{cases} \dfrac{\text{center to center spacing}}{2} = 3 \times \dfrac{0.875}{2} = 1.31 & \text{in} \leftarrow \text{Controls} \\ 1.5 + \dfrac{0.875}{2} + \dfrac{3}{8} = 2.31 \end{cases}$$

$A_{tr} = 0.22$ in.^{2}
$n = 3$
$s = 6$ *in.*

$$K_{tr} = \frac{A_{tr}40}{s\,n}$$

$$K_{tr} = \frac{0.22 \times 40}{3 \times 6} = 0.49$$

$$l_d = \left[\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c+K_{tr}}{d_b}\right)}\right]d_b$$

$$\frac{c + K_{tr}}{d_b} = \frac{1.31 + 0.49}{0.875} = 2.06 < 2.5 \text{ ok}$$

$$l_d = \left[\frac{3}{40}\frac{60,000}{1 \times \sqrt{4,000}}\frac{1.3 \times 1 \times 1}{(2.06)}\right] \times 0.875 = 39.3 \; in.$$

$$l_d = 50 \times \frac{A_s^{req'd}}{A_s^{provided}}$$ ACI Section 12.2.5

$A_s{}^{required}$ = 1.6 in.^{2}
$A_s{}^{provided}$ = *3-#7*= 1.8 in.^{2}

$$\boxed{l_d = 39.3 \times \frac{1.6}{1.8} = 35} \; in. > \; 12 \; in.$$

Alternative Solution I.

Can use K_{tr} = *0* as a design simplification even if transvers reinforcements are present

$$\frac{c + K_{tr}}{d_b} = \frac{1.31}{0.875} = 1.5 < 2.5 \text{ ok}$$

$$l_d = \left[\frac{3}{40}\frac{60,000}{1 \times \sqrt{4,000}}\frac{1.3 \times 1 \times 1}{\mathbf{1.5}}\right] \times 0.875 = 54 \; in.$$

$$\boxed{l_d = 54 \times \frac{1.6}{1.8} = 48} \; in. > \; 12 \; in.$$

Alternative Solution II.

ACI 12.2.2

$$l_d = \frac{f_y\Psi_t\Psi_e}{20\,\lambda\sqrt{f_c'}}d_b = \frac{60000 \times 1.3 \times 1.0}{20 \times 1 \times \sqrt{4000}} \times 0.875 = 61.6 \; in.$$

$$\boxed{l_d = 61.6 \times \frac{1.6}{1.8} = 55} \; in. > \; 12 \; in.$$

Solution (B)

ACI Sect. 12.2.3
$$l_d = \left[\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c{}'}}\frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c+K_{tr}}{d_b}\right)}\right]d_b$$

$\Psi_t = 1.3$ Top bars

$\Psi_e = 1.5$ Epoxy coated reinforcement

$\Psi_s = 1.0$ No. 7 and larger bars

$\Psi_t \times \Psi_e = 1.3 \times 1.5 = 1.95 < 1.7$ *use* 1.7

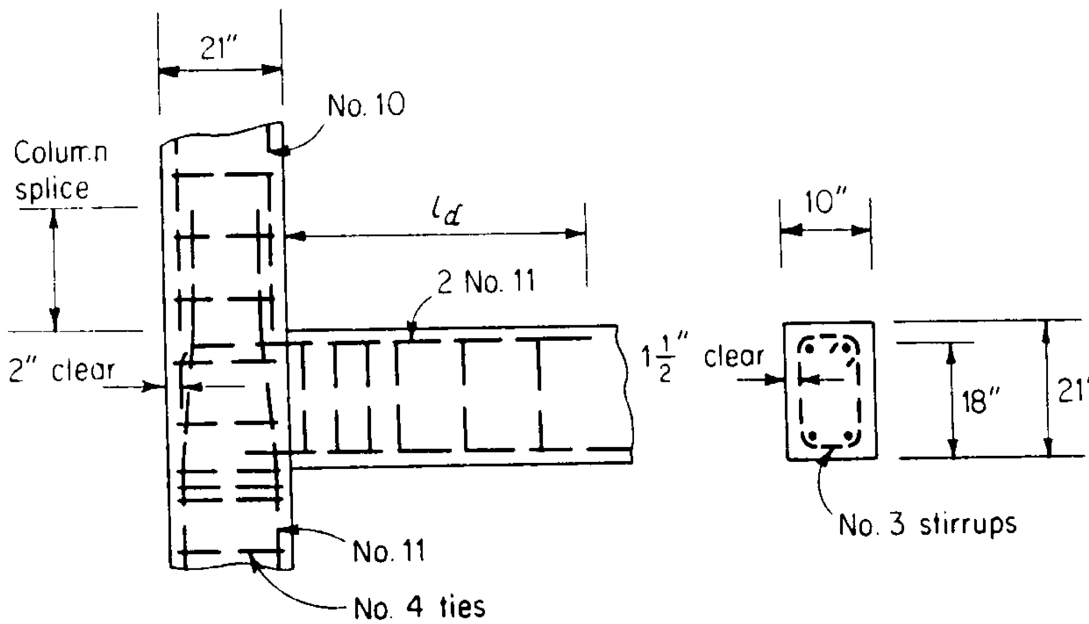
$\lambda = 1.0$ Normal weight concrete

$d_b = 0.875$ *in.*

$$l_d = \left[\frac{3}{40}\frac{60,000}{1 \times \sqrt{4,000}}\frac{1.7 \times 1 \times 1}{(2.06)}\right] \times 0.875 = 51.4\ in.$$

$$l_d = 51.4 \times \frac{1.6}{1.8} = 45.7 \quad in. > \quad 12\ in.$$

10.8. Example. Development length in tension. Figure below shows a beam–column joint in a continuous building frame. Based on frame analysis, the negative steel required at the end of the beam is 2.90 in^2 and two No. 11 bars are used. providing A, = 3.12 in^2. Beam dimensions are b = 10 in d = 18 in and h = 21 in. The design will include No. 3 stirrups spaced four at 3 inches followed by a constant 5 inches spacing in the region of the support. with 1.5 in. clear cover. Normal density concrete is to be used, with f'_c = 4000 psi. and rebars have f_y = 60,000 psi. Find the minimum distance l_d at which the negative bars can be cut off based on development of the required steel area at the face of the column.

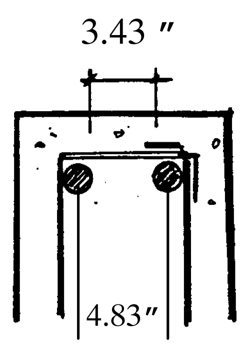


Center to Center Spacing:

$$10 - 2(1.5) - 2(\tfrac{3}{8}) - 1.41 = 4.83 \ \ inches$$

Clear spacing:

$$4.83 - 1.41 = 3.43 \ \ inches$$



ACI Sect. 12.2.3

$$l_d = \left[\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\Psi_t\Psi_e\Psi_s}{\left(\frac{c+K_{tr}}{d_b}\right)}\right]d_b$$

$$c = \text{spacing or cover dimension} = \begin{cases} \dfrac{\text{center to center spacing}}{2} = \dfrac{1}{2}(4.83) = 2.41 \ in & \leftarrow \text{Controls} \\ 1.5 + 3/8 + 1.41/2 = 2.58\ in \end{cases}$$

$$A_{tr} = 0.22$$

$$K_{tr} = \frac{A_{tr}40}{s\,n}$$

$$K_{tr} = \frac{0.22 \times 40}{5 \times 2} = 0.88$$

$$\frac{c + K_{tr}}{d_b} = \frac{2.41 + 0.88}{1.41} = 2.33 < 2.5 \text{ ok}$$

$\Psi_t = 1.3$ Top bars

$\Psi_e = 1.0$ Not Epoxy coated

$\Psi_s = 1.0$ No. 7 and larger bars

$\alpha \times \beta = 1.3 \times 1.0 = 1.3 < 1.7$

$\lambda = 1.0$ Normal weight concrete

$d_b = 1.41 \quad in$

$$l_d = \left[\frac{3}{40}\frac{60,000}{1 \times \sqrt{4,000}}\frac{1.3 \times 1 \times 1}{(2.33)}\right] \times 1.41 = 56\ in$$

$$l_d = 56 \times \frac{2.9}{3.12} = 52 \quad in > \quad 12\ in$$