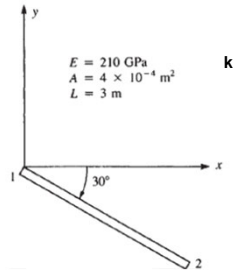
Problem - Stiffness Matrix for a Bar Element

Develop the element stiffness matrices for the bar element shown below.



$$\mathbf{k} = \frac{AE}{L}\begin{array}{c} \\ N_x \\ N_y \\ F_x \\ F_y \end{array}\begin{array}{c} \begin{array}{cccc} N_x & N_y & F_x & F_y \end{array} \\ \begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ -\lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \end{array}$$

$$\lambda_x = \cos\theta_x \qquad \lambda_y = \cos\theta_y$$

1

Problem - Stiffness Matrix for a Bar Element

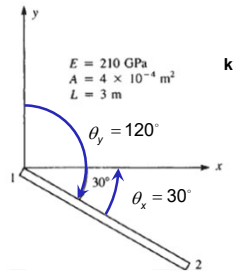
Develop the element stiffness matrices for the bar element shown below.

$$k = \left[\quad\right]\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

2

Problem - Stiffness Matrix for a Bar Element

Develop the element stiffness matrices for the bar element shown below.



$$\mathbf{k} = \frac{AE}{L}\begin{array}{c} \\ N_x \\ N_y \\ F_x \\ F_y \end{array}\begin{array}{c} \begin{array}{cccc} N_x & N_y & F_x & F_y \end{array} \\ \begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ -\lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \end{array}$$

$$\lambda_x = \cos\theta_x \qquad \lambda_y = \cos\theta_y$$

$$\lambda_x = \frac{\sqrt{3}}{2} \qquad \lambda_y = -\frac{1}{2}$$

3

Problem - Stiffness Matrix for a Bar Element

Develop the element stiffness matrices for the bar element shown below.

$$k = \frac{\left(210\times10^6\right)\left(4\times10^{-4}\right)}{3}\left[\begin{array}{c|c|c|c} \frac{3}{4} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} \\ \hline -\frac{\sqrt{3}}{4} & \frac{1}{4} & \frac{\sqrt{3}}{4} & -\frac{1}{4} \\ \hline -\frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{3}{4} & -\frac{\sqrt{3}}{4} \\ \hline \frac{\sqrt{3}}{4} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & \frac{1}{4} \end{array}\right]\frac{kN}{m}$$

4

Problem - Stiffness Matrix for a Bar Element

Develop the element stiffness matrices for the bar element shown below.

$$k = 7{,}000\left[\begin{array}{c|c|c|c} 3 & -\sqrt{3} & -3 & \sqrt{3} \\ \hline -\sqrt{3} & 1 & \sqrt{3} & -1 \\ \hline -3 & \sqrt{3} & 3 & -\sqrt{3} \\ \hline \sqrt{3} & -1 & -\sqrt{3} & 1 \end{array}\right]\frac{kN}{m}$$

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