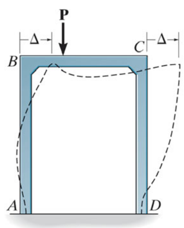
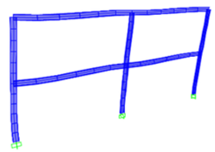
Chapter 11

Displacement Method of Analysis:
Moment Distribution for frames with sidesway





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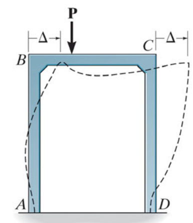
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway

- It has been shown that frames that are nonsymmetrical or subjected to nonsymmetrical loadings tend to sidesway.
- An example of one such case is shown below:



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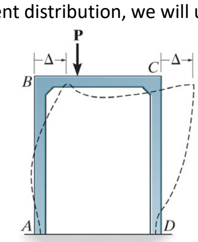
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway

- Here the applied loading **P** will create unequal moments at joints *B* and *C* such that the frame will sidesway Δ to the right.
- To determine this deflection and the internal moments at the joints using moment distribution, we will use the principle of superposition.



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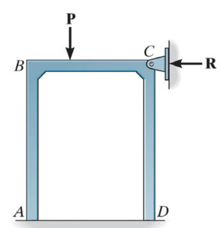
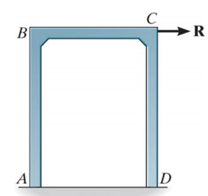
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway

- The frame is first considered held from sidesway by applying an artificial joint support at *C*.
- Using moment distribution and statics the restraining force **R** is determined.





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Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway

- One method for doing this is to assume a numerical value for one of the internal moments, say $\mathbf{M'_{BA}}$.
- Using moment distribution and statics, the deflection Δ' and external force **R'** corresponding to this assumed value of $\mathbf{M'_{BA}}$ are calculated.
- Since the force **R'** develops moments in the frame that are *proportional* to those developed by **R**, then the moment at *B* developed by **R** will be $\mathbf{M_{BA} = M'_{BA}\,(R/R')}$.
- Finally, addition of the joint moments for both cases, will yield the actual moments in the frame.

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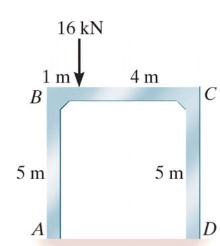
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Determine the internal moments at the joints of the frame shown below. All supports are fixed. *EI* is constant.



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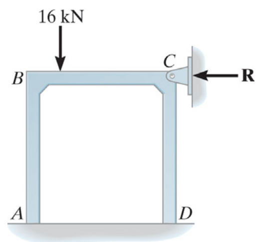
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: First, we consider the frame held from sidesway as shown below:



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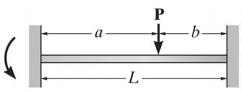
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: For the *BC* span, the FEMs are:

$$(FEM) = \frac{Pb^2a}{L^2}$$



$$(FEM) = \frac{Pa^2b}{L^2}$$

$$(FEM)_{BC} = -\frac{Pb^2a}{L^2} = -\frac{16kN(4m)^2(1m)}{(5m)^2} = -10.24\,kNm$$

$$(FEM)_{CB} = \frac{Pa^2b}{L^2} = -\frac{16kN(1m)^2(4m)}{(5m)^2} = 2.56\,kNm$$

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: The stiffnesses are:

- The stiffness for *BA*: $K = \frac{4EI}{L}$ $\quad K_{BA} = \frac{4EI}{5m}$

- The stiffness for *BC*: $K = \frac{4EI}{L}$ $\quad K_{BC} = \frac{4EI}{5m}$

- The stiffness for *CD*: $K = \frac{4EI}{L}$ $\quad K_{CD} = \frac{4EI}{5m}$

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: The distribution factors are:

$DF_{AB} = 0 \qquad DF_{DC} = 0$

$$DF_{BA} = \frac{K_{BA}}{\sum K} = \frac{4/5}{4/5 + 4/5} = 0.5 \qquad DF_{BC} = 1 - DF_{AB} = 0.5$$

$$DF_{CB} = \frac{K_{CB}}{\sum K} = \frac{4/5}{4/5 + 4/5} = 0.5 \qquad DF_{CD} = 1 - DF_{CB} = 0.5$$

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Putting the data into a table:

Joint	*A*	*B*		*C*		*D*
Members	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	0	0.5	0.5	0.5	0.5	0
FEM			-10.24	2.56		
Dist.						
CO						
Dist.						
CO						
Dist.						
CO						
Dist.						
Σ						

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Putting the data into a table:

Joint	*A*	*B*		*C*		*D*
Members	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	0	0.5	0.5	0.5	0.5	0
FEM			-10.24	2.56		
Dist.		5.12	5.12	-1.28	-1.28	
CO	2.56		-0.64	2.56		-0.64
Dist.		0.32	0.32	-1.28	-1.28	
CO	0.16		-0.64	0.16		-0.64
Dist.		0.32	0.32	-0.08	-0.08	
CO	0.16		-0.04	0.16		-0.04
Dist.		0.02	0.02	-0.08	-0.08	
Σ	**2.88**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.32**

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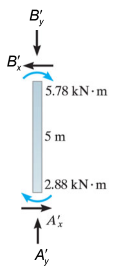
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Using these results, the equations of equilibrium are applied to the free-body diagrams of the columns to determine ***A$_x$'*** and ***D$_x$'***.



$$\circlearrowleft^{+}\sum M_B = 0$$

$$= -5.78\,kNm - 2.88\,kNm + A_x(5m)$$

$$\boxed{A_x = 1.73\,kN}$$

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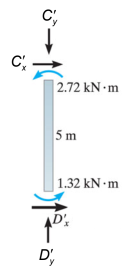
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Using these results, the equations of equilibrium are applied to the free-body diagrams of the columns to determine ***A$_x$'*** and ***D$_x$'***.



$$\circlearrowleft^{+}\sum M_C = 0$$

$$= 2.72\,kNm + 1.32\,kNm + D_x(5m)$$

$$\boxed{D_x = -0.81\,kN}$$

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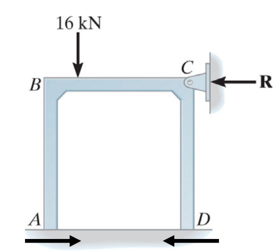
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: From the free-body diagram of the entire frame (not shown) the joint restraint **R**



$$\xrightarrow{+}\sum F_x = 0 = A'_x - D'_x - R$$

$$\boxed{R = 0.92\,kN}$$

$A'_x = 1.73\,kN$ $D'_x = 0.81\,kN$

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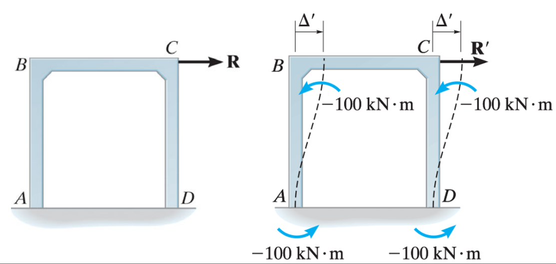
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: An equal but opposite value of **R** = 0.92 *kN* must now be applied to the frame at *C* and the internal moments at the joints must be calculated.
- The easiest way to do this is to *assume* these fixed-end moments have a certain value. The moments are then found by proportion.



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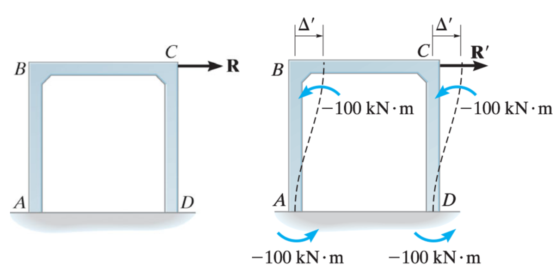
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: To begin, **R'** causes the frame to deflect Δ' as shown.
- Here the joints at *B* and *C* are ***temporarily restrained from rotating.***



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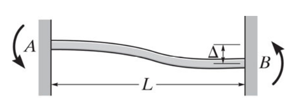
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: As a result, the fixed-end moments at the ends of the columns are determined from the formula for deflection found on the inside back cover:

$$(FEM) = \frac{6EI\Delta}{L^2}$$



$$(FEM) = \frac{6EI\Delta}{L^2}$$

- Since *both B* and *C* happen to be displaced Δ' and *AB* and *DC* have the *same E*, *I*, and *L*, the FEM in *AB* will be the *same* as that in *DC*.

$$(FEM)_{AB} = (FEM)_{BA} = (FEM)_{CD} = (FEM)_{DC} = -100\,kNm$$

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: A ***negative sign*** is necessary since the moment must act ***counterclockwise*** on the column for deflection Δ' to the right.
- The value of **R'** associated with this -100 *kN·m* moment can now be determined.

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: The moment distribution of the FEMs is

Joint	*A*	*B*		*C*		*D*
Members	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF						
FEM						
Dist.						
CO						
Dist.						
CO						
Dist.						
CO						
Dist.						
CO						
Dist.						
Σ						

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: The moment distribution of the FEMs is

Joint	*A*	*B*		*C*		*D*
Members	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	0	0.5	0.5	0.5	0.5	0
FEM	-100	-100			-100	-100
Dist.		50	50	50	50	
CO	25		25	25		25
Dist.		-12.5	-12.5	-12.5	-12.5	
CO	-6.25		-6.25	-6.25		-6.25
Dist.		3.125	3.125	3.125	3.125	
CO	1.563		1.563	1.563		1.563
Dist.		-0.781	-0.781	-0.781	-0.781	
CO	-0.391		-0.391	-0.391		-0.391
Dist.		0.195	0.195	0.195	0.195	
Σ	**-80**	**-60**	**60**	**60**	**-60**	**-80**

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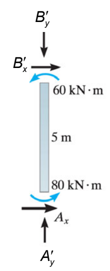
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Using these results, the equations of equilibrium are applied to the free-body diagrams of the columns to determine ***A$_x$'*** and ***D$_x$'***.



$$\circlearrowleft^+ \sum M_B = 0 = 60\,kNm + 80\,kNm + A_x(5\,m)$$

$$A_x = -28\,kN$$

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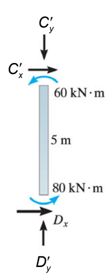
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Using these results, the equations of equilibrium are applied to the free-body diagrams of the columns to determine ***A$_x$'*** and ***D$_x$'***.



$$\circlearrowleft^+ \sum M_C = 0 = 60\,kNm + 80\,kNm + D_x(5\,m)$$

$$D_x = -28\,kN$$

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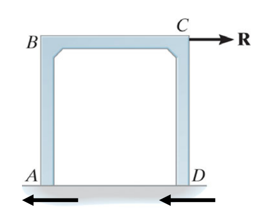
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: From the free-body diagram of the entire frame (not shown) the joint restraint **R**



$A_x = 28\,kN$ $D_x = 28\,kN$

$$^+\!\uparrow \sum F_x = 0 = -A_x - D_x + R$$

$$R = 56\,kN$$

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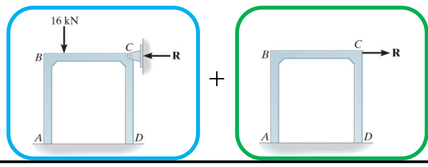
Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Hence, **R'** = 56 kN creates the moments tabulated with the assumed FEM = -100 kN·m.
- Corresponding moments caused by **R** = 0.92 kN can now be determined by proportion.
- The resultant moment in the frame is equal to the ***sum*** of those calculated for the frame in the **no sidesway** plus the **proportionate amount** of those for the frame.



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Displacement method of analysis: **moment distribution for frames**

- **Example 11-6**: Therefore:

$$M_{AB} = \boxed{2.88\,kNm} + \boxed{\left(\frac{0.92\,kN}{56\,kN}\right)(-80\,kNm)} = 1.57\,kNm$$

$$M_{BA} = 5.78\,kNm + \left(\frac{0.92\,kN}{56\,kN}\right)(-60\,kNm) = 4.79\,kNm$$

$$M_{BC} = -5.78\,kNm + \left(\frac{0.92\,kN}{56\,kN}\right)(60\,kNm) = -4.79\,kNm$$

$$M_{CB} = 2.72\,kNm + \left(\frac{0.92\,kN}{56\,kN}\right)(60\,kNm) = 3.71\,kNm$$

$$M_{CD} = -2.72\,kNm + \left(\frac{0.92\,kN}{56\,kN}\right)(-60\,kNm) = -3.71\,kNm$$

$$M_{DA} = -1.32\,kNm + \left(\frac{0.92\,kN}{56\,kN}\right)(-80\,kNm) = -2.63\,kNm$$

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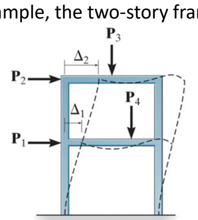
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- Quite often, multistory frameworks may have several *independent* joint displacements, and consequently, the moment-distribution analysis using the above techniques will involve more calculation.
- Consider, for example, the two-story frame shown below:



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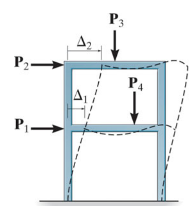
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- This structure can have two independent joint displacements, since the sidesway Δ_1 of the first story is independent of any displacement Δ_2 of the second story.



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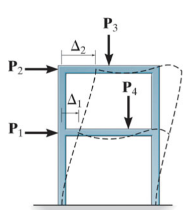
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- Unfortunately, these displacements are not known initially, so the analysis must proceed based on superposition, in the same manner as discussed previously.



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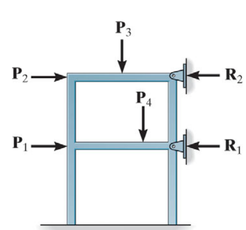
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- In this case, two restraining forces $\mathbf{R_1}$ and $\mathbf{R_2}$ are applied, and the fixed-end moments are determined and distributed.



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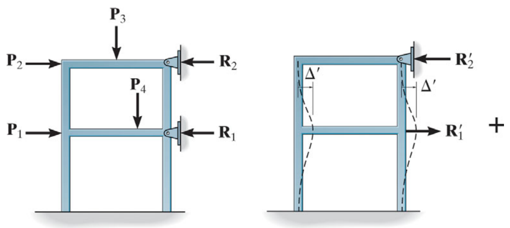
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- Next, the restraint at the ceiling of the first story is removed and the ceiling is given a displacement Δ'.
- This displacement causes fixed-end moments (FEMs) in the frame, which can be assigned specific numerical values.



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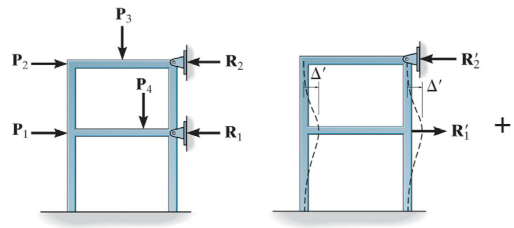
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- By distributing these moments and using the equilibrium equations, the associated numerical values of $\mathbf{R'_1}$ and $\mathbf{R'_2}$ can be determined.



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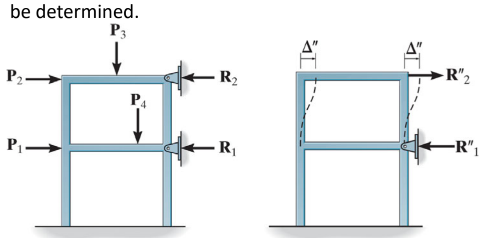
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- In a similar manner, the ceiling of the second story is then given a displacement Δ''.
- By distributing these moments and using the equilibrium equations, the associated numerical values of $\mathbf{R''_1}$ and $\mathbf{R''_2}$ can be determined.



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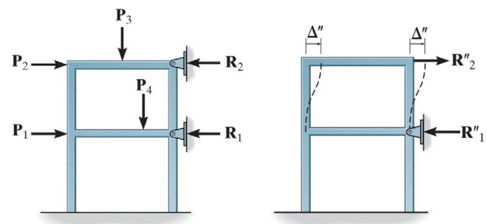
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- Since the last two steps associated with depend on *assumed* values of the FEMs, correction factors *C'* and *C''* must be applied to the distributed moments.



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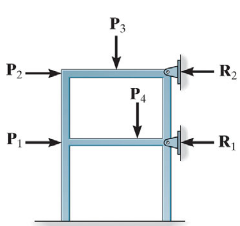
Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- With reference to the restraining forces shown below, we require equal but opposite application of and to the frame, such that

$$R_2 = -C'R_2' + C''R_2'' \qquad R_1 = -C'R_1' + C''R_1''$$



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Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- With reference to the restraining forces shown below, we require equal but opposite application of and to the frame, such that

$$R_2 = -C'R_2' + C''R_2'' \qquad R_1 = -C'R_1' + C''R_1''$$

- Simultaneous solution of these equations yields the values of *C'* and *C''*.
- These correction factors are then multiplied by the internal joint moments found from the moment distribution.

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Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: Sidesway Multistory Frames

- Other types of frames having independent joint displacements can be analyzed using this same procedure; however, it must be admitted that the foregoing method does require ***quite a bit of numerical calculation***.
- Although some techniques have been developed to shorten the calculations, it is best to solve these problems on a computer, preferably using a matrix analysis.
- The techniques for doing this will be discussed in Chapter 14.

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Displacement method of analysis: **moment distribution for frames**

Any questions?

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