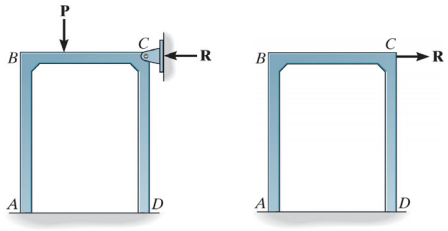
Chapter 11

Displacement Method of Analysis: **Moment Distribution for frames**



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Displacement method of analysis: **moment distribution for frames**

Moment Distribution for Frames: No Sidesway

- Application of the moment-distribution method for frames having ***no sidesway*** follows the same procedure as that given for beams.
- To minimize the chance for errors, it is suggested that the analysis be arranged in a tabular form, as in the previous examples.
- Also, the distribution of moments can be shortened if the stiffness factor of a span can be modified as indicated in the previous section.

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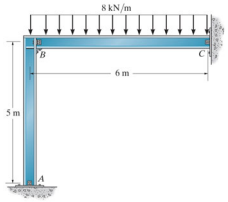
Displacement method of analysis: **moment distribution for frames**

- **Example 11-4**: Determine the internal moments at the joints of the frame shown below. There is a pin at *A* and *C* and a fixed support at *B*. *EI* is constant.



- By inspection, the pin at *C* will prevent the frame from sidesway.
- The stiffness factors of *BC* and *BA* can be calculated using *K = 3EI/L* since the far ends *A* and *C* are pinned.

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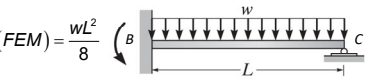
Displacement method of analysis: **moment distribution for frames**

- **Example 11-4**: For span *BC,* the FEM are:

$$(FEM) = \frac{wL^2}{8}$$



$$(FEM)_{BC} = -\frac{wL^2}{8} = -\frac{8\ {}^{kN}/_{m}(6\,m)^2}{8} = -36\,kNm$$

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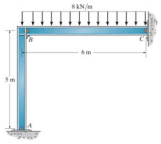
Displacement method of analysis: **moment distribution for frames**

- **Example 11-4**: The DFs for joint *B* are:



- The stiffness for *AB*: $K_{BA} = \frac{3EI}{L} = \frac{3EI}{5m}$
- The stiffness for *BC*: $K_{BC} = \frac{3EI}{L} = \frac{3EI}{6m}$
- The distribution factors are: $DF_{AB} = 1$ $DF_{CB} = 1$

$$DF_{BA} = \frac{K_{BA}}{\sum K} = \frac{3/5}{3/5 + 3/6} = 0.545$$

$$DF_{BC} = \frac{K_{BC}}{\sum K} = \frac{3/6}{3/5 + 3/6} = 0.455$$

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-4**: Putting the data into a table:

Joint	*A*	*B*		*C*
Members	*AB*	*BA*	*BC*	*CB*
DF	1	0.545	0.455	1
FEM				
Dist.				
Σ				

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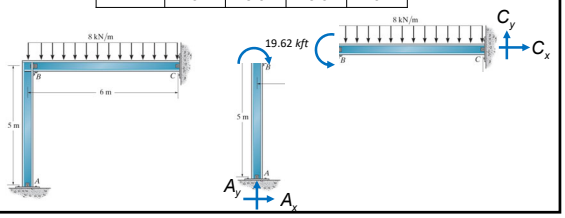
Displacement method of analysis: **moment distribution for frames**

- **Example 11-4**: Putting the data into a table:

Joint	A	B		C
Members	AB	BA	BC	CB
DF	1	0.545	0.455	1
FEM	0		-36	0
Dist.		19.62	16.38	
Σ	**0**	**19.62**	**-19.62**	**0**



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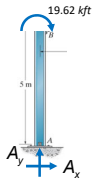
Displacement method of analysis: **moment distribution for frames**

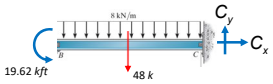
- **Example 11-4**: Solving for reactions in *AB* and *BC*:



$$\circlearrowleft^{+}\sum M_B = 0 \quad = -19.62\,kft + A_x(5m)$$

$$\boxed{A_x = 3.92k}$$

$$\circlearrowleft^{+}\sum M_B = 0 = 19.62\,kft - 48k(3m) + C_y(6m)$$



$$\boxed{C_y = 20.73k}$$

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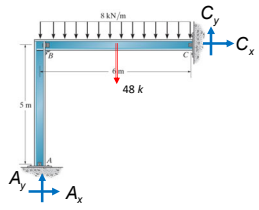
Displacement method of analysis: **moment distribution for frames**

- **Example 11-4**: Solving for the remaining reactions in *ABC*:



$$\circlearrowleft^{+}\sum M_A = 0$$

$$= -48k(3m) + C_y(6m) - C_x(5m)$$

$$\boxed{C_x = -3.92k}$$

$$+\uparrow\sum F_y = 0 = A_y + C_y - 48k$$

$$\boxed{A_y = 27.27k}$$

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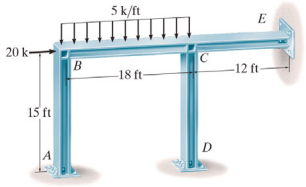
Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: Determine the internal moments at the joints of the frame shown below. There is a pin at *E* and *D* and a fixed support at *A*. *EI* is constant.



- By inspection, the pin at *E* will prevent the frame from sidesway.
- The stiffness factors of *CD* and *CE* can be calculated using *K = 3EI/L* since the far ends *D* and *E* are pinned.

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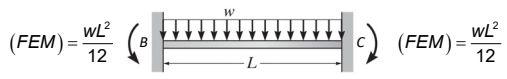
Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: For span *BC* the FEM are:



$$(FEM)_{BC} = -\frac{wL^2}{12} = -\frac{5k/ft(18ft)^2}{12} = -135\,kft$$

$$(FEM)_{CB} = \frac{wL^2}{12} = \frac{5k/ft(18ft)^2}{12} = 135\,kft$$

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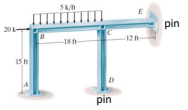
Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: The stiffnesses are:



- The stiffness for *AB*: $K = \frac{4EI}{L}$ $K_{BA} = \frac{4EI}{15ft}$
- The stiffness for *BC*: $K = \frac{4EI}{L}$ $K_{BC} = \frac{4EI}{18ft}$
- The stiffness for *CD*: $K = \frac{3EI}{L}$ $K_{CD} = \frac{3EI}{15ft}$
- The stiffness for *CE*: $K = \frac{3EI}{L}$ $K_{CE} = \frac{3EI}{12ft}$

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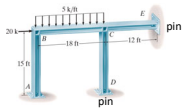
Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: The distribution factors are:



$$DF_{AB} = 0 \qquad DF_{DC} = 1 \qquad DF_{EC} = 1$$

$$DF_{BA} = \frac{K_{BA}}{\sum K} = \frac{4/15}{4/15 + 4/18} = 0.545 \qquad DF_{BC} = 1 - DF_{AB} = 0.455$$

$$DF_{CE} = \frac{K_{CE}}{\sum K} = \frac{3/12}{4/18 + 3/15 + 3/12} = 0.372$$

$$DF_{CD} = \frac{K_{CD}}{\sum K} = \frac{3/15}{4/18 + 3/15 + 3/12} = 0.298$$

$$DF_{CB} = 1 - DF_{CE} - DF_{CD} = 0.330$$

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: Putting the data into a table:

Joint	A	B		C			D	E
Members	AB	BA	BC	CB	CD	CE	DC	EC
DF	0	0.545	0.455	0.330	0.298	0.372	1	1
FEM								
Dist.								
CO								
Dist.								
CO								
Dist.								
CO								
Dist.								
CO								
Dist.								
Σ								

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Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: Putting the data into a table:

Joint	A	B		C			D	E
Members	AB	BA	BC	CB	CD	CE	DC	EC
DF	0	0.545	0.455	0.330	0.298	0.372	1	1
FEM			-135	135			0	0
Dist.		73.6	61.4	-44.6	-40.2	-50.2		
CO	36.8		-22.3	30.7				
Dist.		12.2	10.1	-10.1	-9.1	-11.5		
CO	6.1		-5.1	5.1				
Dist.		2.8	2.3	-1.7	-1.5	-1.9		
CO	1.4		-0.8	1.2				
Dist.		0.4	0.4	-0.4	-0.4	-0.4		
CO	0.2		-0.2	0.2				
Dist.		0.1	0.1	-0.1	0.0	-0.1		
Σ	**44.5**	**89.1**	**-89.1**	**115**	**-51.2**	**-64.1**	**0**	**0**

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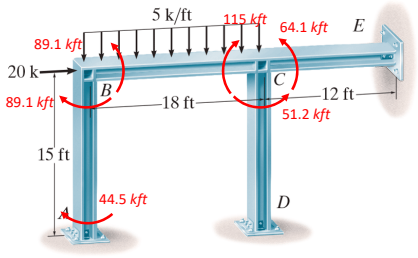
Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: Determine the internal moments at the joints of the frame shown below. There is a pin at *E* and *D* and a fixed support at *A*. *EI* is constant.



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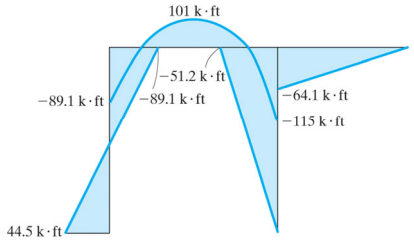
Displacement method of analysis: **moment distribution for frames**

- **Example 11-5**: Determine the internal moments at the joints of the frame shown below. There is a pin at *E* and *D* and a fixed support at *A*. *EI* is constant.
- The resulting moment diagram is:



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Displacement method of analysis: **moment distribution for frames**

Let's work some problems

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Displacement method of analysis: **moment distribution for frames**

Any questions?

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