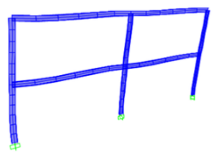
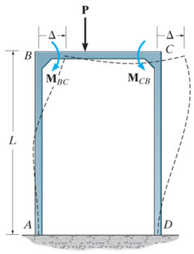
Chapter 10

Displacement Method of Analysis: **Slope-Deflection Equations for *frames***



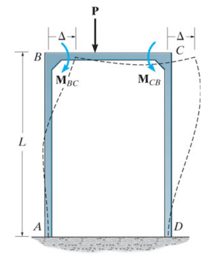


1

Displacement method of analysis: **slope-defection method**

Analysis of Frames: Sidesway

- A frame will sidesway, or be displaced to the side, when it or the loading acting on it is nonsymmetric.



- Here the loading **P** causes *unequal* moments $\mathbf{M}_{BC}$ and $\mathbf{M}_{CB}$ at the joints *B* and *C*, respectively.
- $\mathbf{M}_{BC}$ tends to displace joint *B* to the right, whereas $\mathbf{M}_{CB}$ tends to displace joint *C* to the left.

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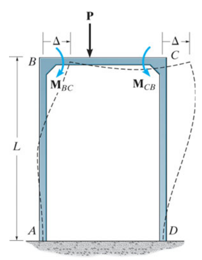
Displacement method of analysis: **slope-defection method**

Analysis of Frames: Sidesway

- A frame will sidesway, or be displaced to the side, when it or the loading acting on it is nonsymmetric.



- Since $\mathbf{M}_{BC}$ will be larger than $\mathbf{M}_{CB}$, the net result is a sidesway of both joints *B* and *C* to the right.
- Due to this deflection, we must therefore consider the column rotation ψ (since $\psi = \Delta/L$) as unknown in the slope-deflection equations.

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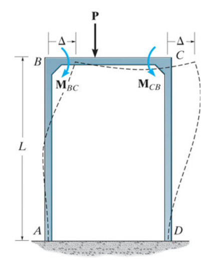
Displacement method of analysis: **slope-defection method**

Analysis of Frames: Sidesway

- A frame will sidesway, or be displaced to the side, when it or the loading acting on it is nonsymmetric.



- As a result, an extra equilibrium equation must be included for the solution.

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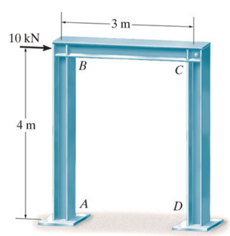
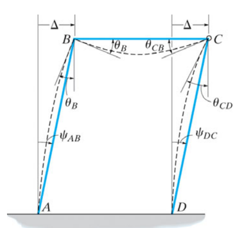
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.





- We can apply the ***fixed-end slope-displacement equation*** for span *AB*

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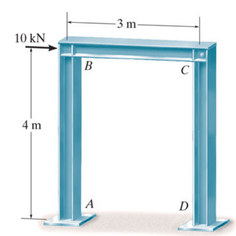
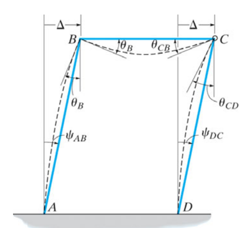
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.





- We can apply the ***pinned-end slope-displacement equation*** for spans *BC* and *CD*

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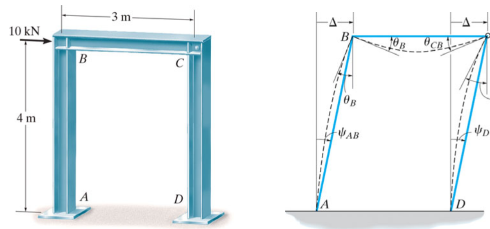
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.



- There is an unknown Δ and θ_B at *B*. The angular displacement θ_{CB} and θ_{CD} at joint *C* are not include since we using the ***pinned-end slope-displacement*** equations.

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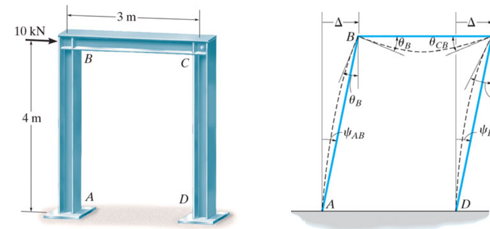
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.



- Due to the Δ, the cords pf *AB* and *DC* rotate *clockwise,* $\psi = \psi_{AB} = \psi_{DC} = \Delta/4$ m

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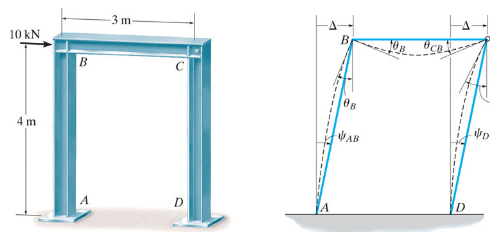
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.



- The rotations θ_A and θ_D are zero (fixed ends), and there are no *FEM* for the members.

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Displacement method of analysis: **slope-defection method**

- **Example 10-4**: For span *AB*, consider *A* to be near and *B* to be far.

$$M_N = 2\frac{EI}{L}(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2\frac{EI}{L}(2\overset{0}{\theta_A} + \theta_B - 3\psi) + (\overset{0}{FEM})_{AB}$$

$$\boxed{M_{AB} = \frac{EI}{2m}[\theta_B - 3\psi]} \quad (1)$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-4**: For span *AB*, consider *B* to be near and *A* to be far.

$$M_N = 2\frac{EI}{L}(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{BA} = 2\frac{EI}{L}(2\theta_B + \overset{0}{\theta_A} - 3\psi) + (\overset{0}{FEM})_{BA}$$

$$\boxed{M_{BA} = \frac{EI}{2m}[2\theta_B - 3\psi]} \quad (2)$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-4**: For span *BC*, consider *B* to be near and *C* to be far.

$$M_N = 3\frac{EI}{L}(\theta_N - \psi) + (FEM)_N$$

$$M_{BC} = 3\frac{EI}{3m}(\theta_B - \overset{0}{\psi}) + (\overset{0}{FEM})_{BC}$$

$$\boxed{M_{BC} = \frac{EI}{1m}(\theta_B)} \quad (3)$$

- For span *DC*, consider *D* to be near and *C* to be far.

$$M_N = 3\frac{EI}{L}(\theta_N - \psi) + (FEM)_N$$

$$M_{DC} = 3\frac{EI}{4m}(\overset{0}{\theta_D} - \psi) + (\overset{0}{FEM})_{DC}$$

$$\boxed{M_{DC} = \frac{3EI}{4m}(-\psi)} \quad (4)$$

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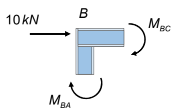
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: These four equations contain six unknowns: θ_B, M_{AB}, M_{BA}, M_{BC}, M_{DC}, and Δ.
- The necessary additional equations comes from the condition of moment equilibrium at support *B* and forces for the ***entire frame***.
- The free-body diagram of a segment of the beam at *B* is:



$$\circlearrowleft^+ \sum M_B = 0 = -M_{BA} - M_{BC}$$

$$\boxed{M_{BA} + M_{BC} = 0} \quad (5)$$

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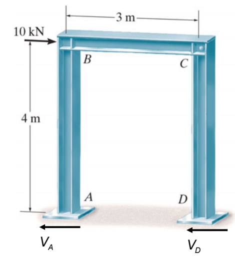
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.
- Summing horizontal forces for the entire frame gives:



$$\xrightarrow{+} \sum F_x = 0 = -V_A - V_D + 10\,kN$$

$$V_A + V_D = 10\,kN$$

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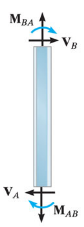
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.
- From the free-body diagram of each column we can get:

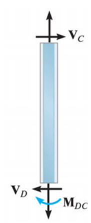


$$\circlearrowleft^+ \sum M_B = 0 = -M_{AB} - M_{BA} - V_A(4\,m)$$

$$V_A = \frac{-M_{AB} - M_{BA}}{4\,m}$$

$$\circlearrowleft^+ \sum M_C = 0 = -M_{DC} - V_D(4\,m)$$

$$V_D = -\frac{M_{DC}}{4\,m}$$



$$V_A + V_D = 10\,kN \quad \Rightarrow \quad \boxed{\frac{-M_{AB} - M_{BA}}{4\,m} - \frac{M_{DC}}{4\,m} = 10\,kN} \quad (6)$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.
- Substituting Equations (2) and (3), into Equation (5):

$$M_{BA} + M_{BC} = 0 \Rightarrow \frac{EI}{2\,m}[2\theta_B - 3\psi] + \frac{EI}{1\,m}(\theta_B) = 0 \qquad \boxed{\theta_B = \frac{3}{4}\psi} \quad (5a)$$

- Substituting Equations (1), (2) and (4), into Equation (6):

$$\frac{-M_{AB} - M_{BA}}{4\,m} - \frac{M_{DC}}{4\,m} = 10\,kN \qquad (-M_{AB} - M_{BA}) - M_{DC} = 40\,kN\,m$$

$$-\frac{EI}{2m}[\theta_B - 3\psi] - \frac{EI}{2m}[2\theta_B - 3\psi] - \frac{3EI}{4m}(-\psi) = 40\,kN\,m$$

$$\boxed{EI\left[-\frac{3}{2}\theta_B + \frac{15}{4}\psi\right] = 40\,kN\,m^2} \quad (6a)$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.
- Substituting Equation (5a) into Equation (6a):

$$\boxed{\theta_B = \frac{3}{4}\psi} \Rightarrow \boxed{EI\left[-\frac{3}{2}\theta_B + \frac{15}{4}\psi\right] = 40\,kN\,m^2}$$

$$-\frac{3}{2}\left(\frac{3}{4}\psi\right) + \frac{15}{4}\psi = \frac{40\,kN\,m^2}{EI}$$

$$\frac{21}{8}\psi = \frac{40\,kN\,m^2}{EI}$$

- Solving for ψ gives: $\boxed{\psi = \frac{320\,kN\,m^2}{21\,EI}}$ $\boxed{\theta_B = \frac{240\,kN\,m^2}{21\,EI}}$

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Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.
- Substituting $\boxed{\psi = \frac{320\,kN\,m^2}{21EI}}$ $\boxed{\theta_B = \frac{240\,kN\,m^2}{21EI}}$ into moment equations give

$$M_{AB} = \frac{EI}{2\,m}[\theta_B - 3\psi] = \frac{EI}{2\,m}\left[\frac{240\,kN\,m^2}{21EI} - 3\left(\frac{320\,kN\,m^2}{21EI}\right)\right]$$

$$\boxed{M_{AB} = -17.14\,kNm}$$

$$M_{BA} = \frac{EI}{2\,m}[2\theta_B - 3\psi] = \frac{EI}{2\,m}\left[2\left(\frac{240\,kN\,m^2}{21EI}\right) - 3\left(\frac{320\,kN\,m^2}{21EI}\right)\right]$$

$$\boxed{M_{BA} = -11.43\,kNm}$$

$$V_A = \frac{-M_{AB} - M_{BA}}{4\,m} = \frac{-(-17.14\,kNm) - (-11.43kNm)}{4\,m} \qquad \boxed{V_A = 7.14\,kN}$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.
- Substituting $\psi = \frac{320\,kN\,m^2}{21EI}$ $\theta_B = \frac{240\,kN\,m^2}{21EI}$ into moment equations give

$$M_{BC} = EI(\theta_B) = EI\left(\frac{240\,kN\,m^2}{21EI}\right) \qquad M_{BC} = 11.43\,kNm$$

$$M_{DC} = \frac{3EI}{4}(-\psi) = \frac{3EI}{4}\left(-\frac{320\,kN\,m^2}{21EI}\right) \qquad M_{DC} = -11.43\,kNm$$

$$V_D = -\frac{M_{DC}}{4m} = -\frac{(-11.43\,kNm)}{4m} \qquad V_D = 2.86\,kN$$

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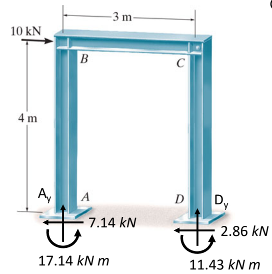
Displacement method of analysis: **slope-defection method**

- **Example 10-4**: Determine the moments at the joint. The supports *A* and *D* are fixed, *B* is fixed, *C* is pinned, and *EI* is constant.



$$\circlearrowleft^+ \sum M_A = 0$$

$$= -10kN(4m) + 11.43\,kNm + 17.14\,kNm + D_y(3m)$$

$$D_y = -3.81kN$$

$$\circlearrowleft^+ \sum M_D = 0$$

$$= -10kN(4m) + 11.43\,kNm + 17.14\,kNm - A_y(3m)$$

$$A_y = 3.81kN$$

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Displacement method of analysis: **slope-defection method**

Let's work some problems

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Displacement method of analysis: **slope-defection method**

Any questions?

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