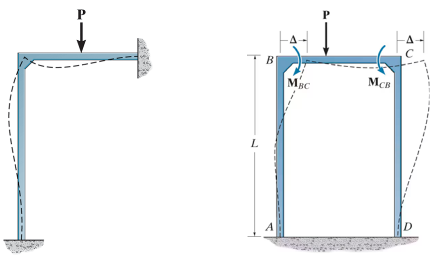
Chapter 10

Displacement Method of Analysis:
Slope-Deflection Equations for *frames*



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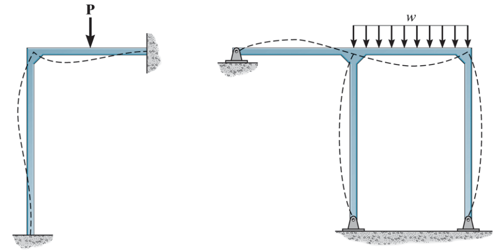
Displacement method of analysis: **slope-defection method**

Analysis of Frames: No Sidesway

➢ A frame will not sidesway, or be displaced to the left or right, provided it is properly restrained.



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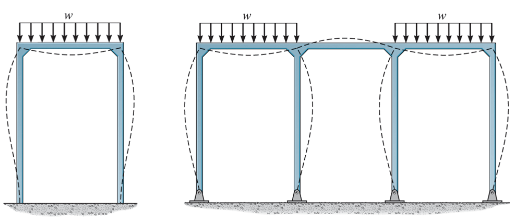
Displacement method of analysis: **slope-defection method**

Analysis of Frames: No Sidesway

➢ Also, no sidesway will occur in an unrestrained frame provided it is symmetric with respect to both ***loading*** and ***geometry***.



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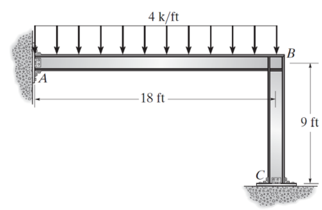
Displacement method of analysis: **slope-defection method**

➢ **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.



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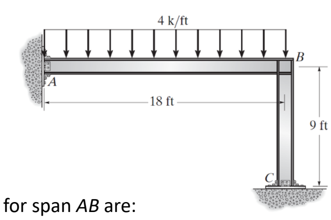
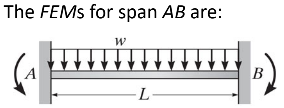
Displacement method of analysis: **slope-defection method**

➢ **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.



➢ The *FEM*s for span *AB* are:



$$(FEM)_{AB} = (FEM)_{BA} = \frac{wL^2}{12}$$

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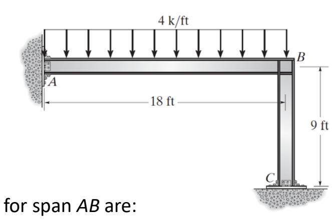
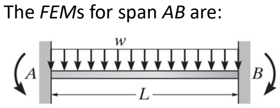
Displacement method of analysis: **slope-defection method**

➢ **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.



➢ The *FEM*s for span *AB* are:



$$(FEM)_{AB} = \frac{(4\,^{k}\!/_{ft})(18\,ft)^2}{12} = 108\,kft$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- Here $(FEM)_{AB}$ is negative since it acts counterclockwise on the beam at *A*.

$$(FEM)_{AB} = -108\,kft \qquad (FEM)_{BA} = 108\,kft$$

- Since there is no load on span *BC*

$$(FEM)_{BC} = (FEM)_{CB} = 0$$

- There are four unknown moments and an unknown slope at *B*.
- Since the supports do not settle, $\psi_{AB} = \psi_{BC} = 0$.

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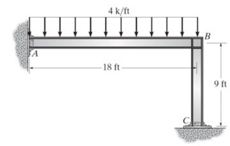
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- For span *AB*, consider *A* to be near and *B* to be far



$$M_N = 2\frac{EI}{L}(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2\frac{EI}{L}(2\cancelto{0}{\theta_A} + \theta_B - 3\cancelto{0}{\psi}) + (FEM)_{AB}$$

$$M_{AB} = 2\frac{EI}{18ft}[2(0) + \theta_B - 3(0)] - 108\,kft$$

$$\boxed{M_{AB} = \frac{EI}{9ft}\theta_B - 108\,kft} \quad (1)$$

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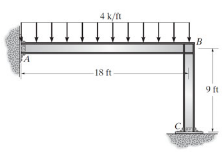
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- For span *AB*, consider *B* to be near and *A* to be far



$$M_N = 2\frac{EI}{L}(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{BA} = 2\frac{EI}{L}(2\theta_B + \cancelto{0}{\theta_A} - 3\cancelto{0}{\psi}) + (FEM)_{BA}$$

$$M_{BA} = 2\frac{EI}{18ft}[2\theta_B] + 108\,kft$$

$$\boxed{M_{BA} = \frac{2EI}{9ft}\theta_B + 108\,kft} \quad (2)$$

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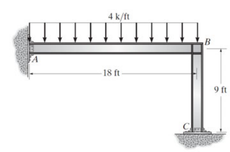
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- For span *BC*, consider *B* to be near and *C* to be far



$$M_N = 2\frac{EI}{L}(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{BC} = 2\frac{EI}{L}(2\theta_B + \cancelto{0}{\theta_C} - 3\cancelto{0}{\psi}) + \cancelto{0}{(FEM)_{BC}}$$

$$M_{BC} = 2\frac{EI}{9ft}[2\theta_B]$$

$$\boxed{M_{BC} = \frac{4EI}{9ft}\theta_B} \quad (3)$$

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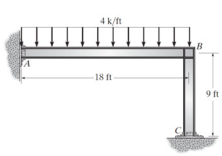
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- For span *BC*, consider *C* to be near and *B* to be far



$$M_N = 2\frac{EI}{L}(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{CB} = 2\frac{EI}{L}(2\cancelto{0}{\theta_C} + \theta_B - 3\cancelto{0}{\psi}) + \cancelto{0}{(FEM)_{CB}}$$

$$M_{CB} = 2\frac{EI}{9ft}[\theta_B]$$

$$\boxed{M_{CB} = \frac{2EI}{9ft}\theta_B} \quad (4)$$

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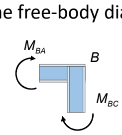
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- These four equations contain five unknowns: θ_B , M_{AB}, M_{BA}, M_{BC}, and M_{CB}.
- The necessary fifth equation comes from the condition of moment equilibrium at support *B*.
- The free-body diagram of a segment of the beam at *B* is:



$$\circlearrowleft^{+} \sum M_B = 0 = -M_{BA} - M_{BC}$$

$$\boxed{M_{BA} + M_{BC} = 0} \quad (5)$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- To solve, substitute Equations (2) and (3), into Equation (5):

$$M_{BA} + M_{BC} = 0 \quad \Rightarrow \underbrace{\frac{2EI}{9ft}\theta_B + 108\,kft}_{M_{BA}} + \underbrace{\frac{4EI}{9ft}\theta_B}_{M_{BC}} = 0$$

$$\Rightarrow \frac{2}{3}\theta_B + \frac{108\,kft^2}{EI} = 0 \qquad \boxed{\theta_B = -\frac{162\,kft^2}{EI}}$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.
- Substituting θ_B into Equations (1) - (4) gives:

$$M_{AB} = \frac{EI}{18ft}\theta_B - 108\,kft = \frac{EI}{18ft}\left[-\frac{162kft^2}{EI}\right] - 108\,kft \qquad \boxed{M_{AB} = -126\,kft}$$

$$M_{BA} = \frac{4EI}{18ft}\theta_B + 108\,kft = \frac{4EI}{18ft}\left[-\frac{162kft^2}{EI}\right] + 108\,kft \qquad \boxed{M_{BA} = 72\,kft}$$

$$M_{BC} = \frac{4EI}{9ft}\theta_B = \frac{4EI}{9ft}\left[-\frac{162kft^2}{EI}\right] \qquad \boxed{M_{BC} = -72\,kft}$$

$$M_{CB} = \frac{2EI}{9ft}\theta_B = \frac{2EI}{9ft}\left[-\frac{162kft^2}{EI}\right] \qquad \boxed{M_{CB} = -36\,kft}$$

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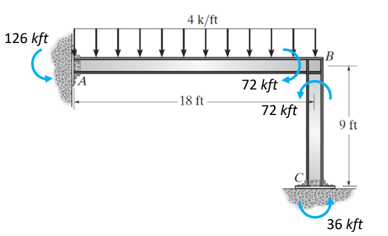
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the moments at *A*, *B*, and *C*. All joints are fixed, and *EI* is constant.



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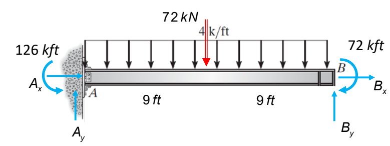
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the reactions at *A* and *C*. Consider section *AB*



$$\circlearrowleft^{+} \sum M_B = 0 = -72\,kft + 126\,kft + 72\,k(9ft) - A_y(18ft) \qquad \boxed{A_y = 39\,k}$$

$$+\uparrow \sum F_y = 0 = A_y + B_y - 72\,k \qquad \boxed{B_y = 33\,k}$$

$$\overset{+}{\rightarrow} \sum F_x = 0 = A_x + B_x \qquad \boxed{A_x = -B_x}$$

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Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the reactions at *A* and *C*. Consider section *BC*

$$\circlearrowleft^{+} \sum M_C = 0 = 72\,kft + 36\,kft + B_x(9ft)$$

$$\boxed{B_x = -12\,k}$$

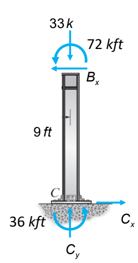
$$\boxed{A_x = 12\,k}$$

$$\overset{+}{\rightarrow} \sum F_x = 0 = -B_x + C_x$$

$$\boxed{C_x = -12\,k}$$

$$+\uparrow \sum F_y = 0 = C_y - 33\,k$$

$$\boxed{C_y = 33\,k}$$



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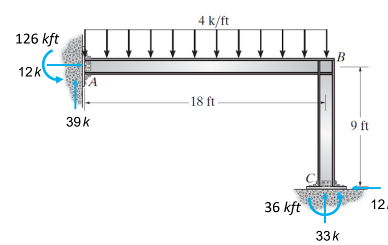
Displacement method of analysis: **slope-defection method**

- **Example 10-3**: Determine the reactions at *A* and *C*.



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Displacement method of analysis: **slope-defection method**

Let's work some problems

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Displacement method of analysis: **slope-defection method**

Any questions?

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