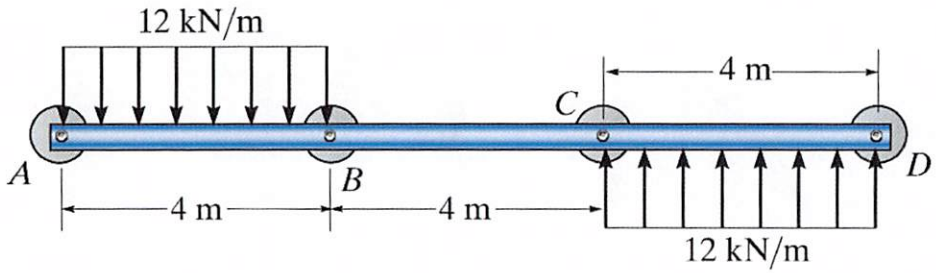
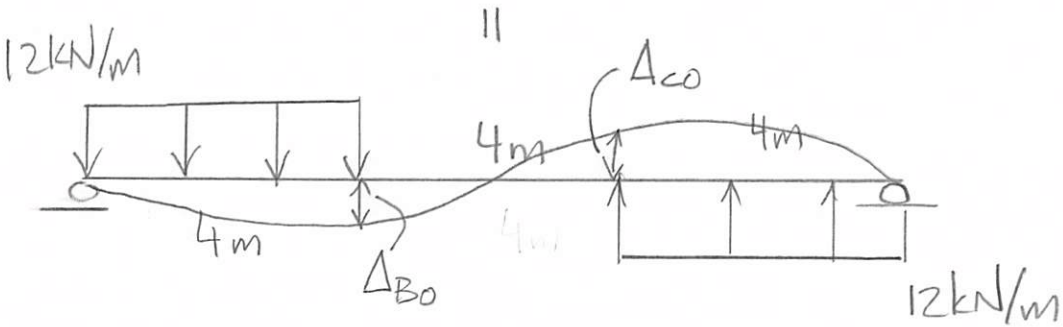
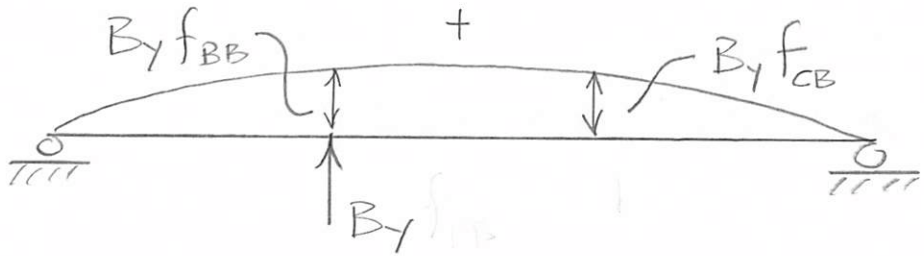
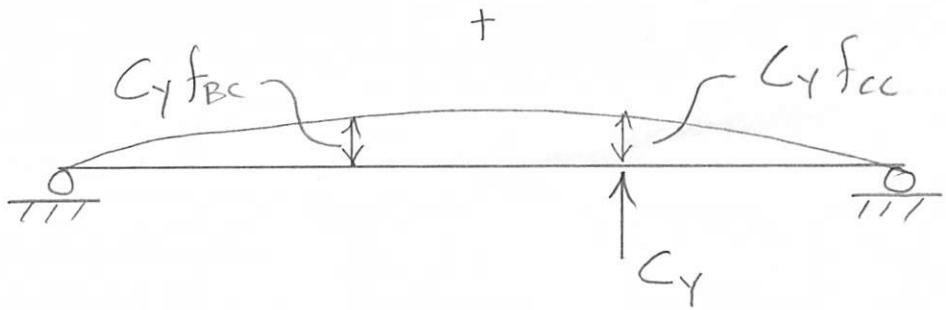
Problem 11-6 – Determine the support reactions. The supports at A, B, C, and D are pins. Assume the horizontal reactions are zero. EI is constant.



3 EQUATIONS OF EQUALIBRIUM



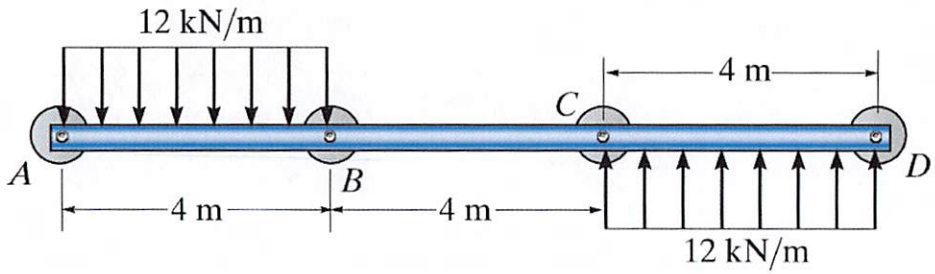


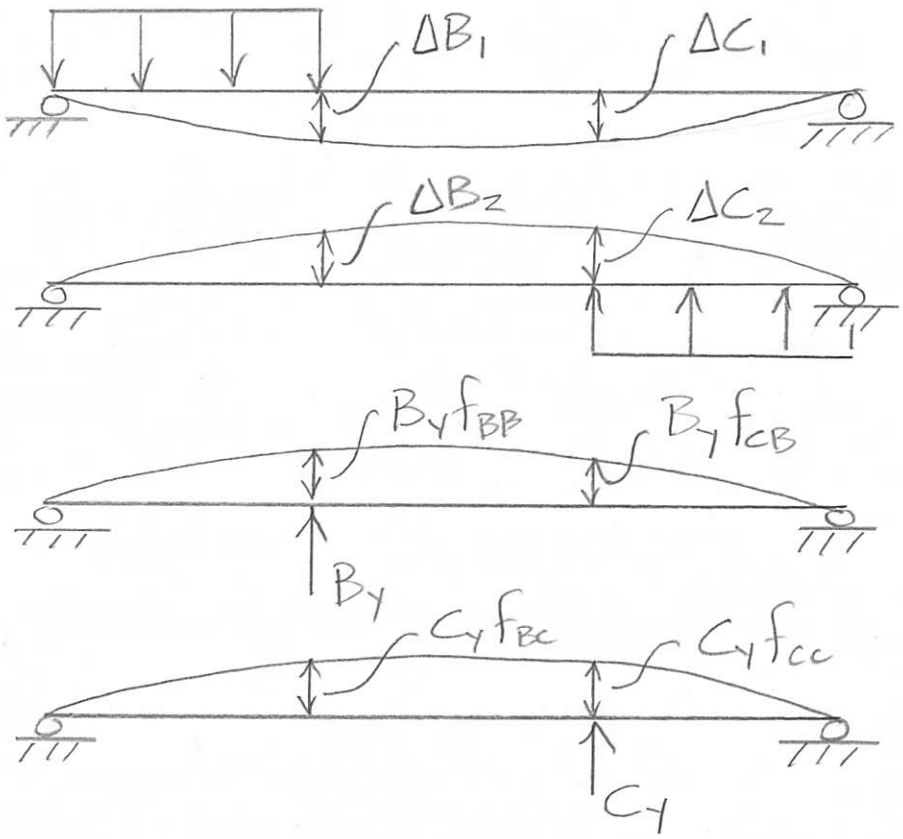


$\Delta_B = 0 = \Delta_{B_o} + B_y f_{BB} + C_y f_{BC}$

$\Delta_C = 0 = \Delta_{C_o} + B_y f_{CB} + C_y f_{CC}$

Problem 11-6 – Determine the support reactions. The supports at A, B, C, and D are pins. Assume the horizontal reactions are zero. EI is constant.





$$\Delta_B = 0 = \Delta B_1 + \Delta B_2 + B_y f_{BB} + C_y f_{BC}$$

$$\Delta_C = 0 = \Delta C_1 + \Delta C_2 + B_y f_{CB} + C_y f_{CC}$$

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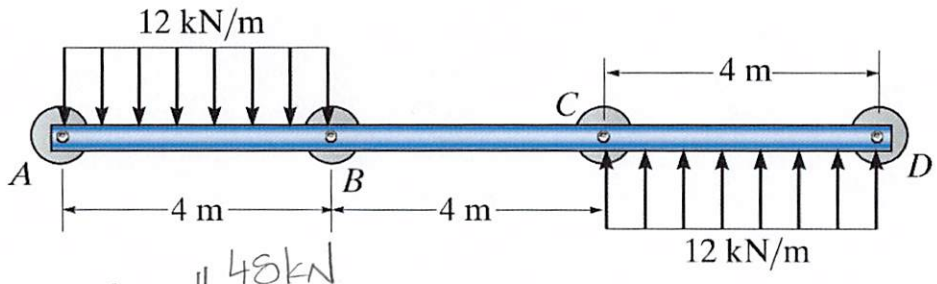
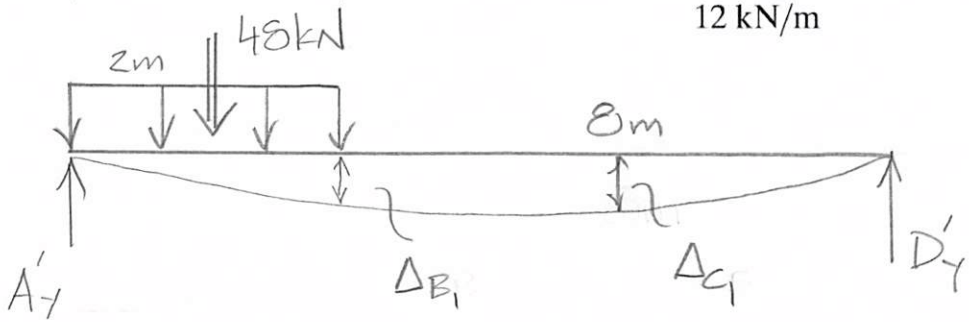


* USE VIRTUAL WORK TO COMPUTE Δ_{BO} & Δ_{CO}



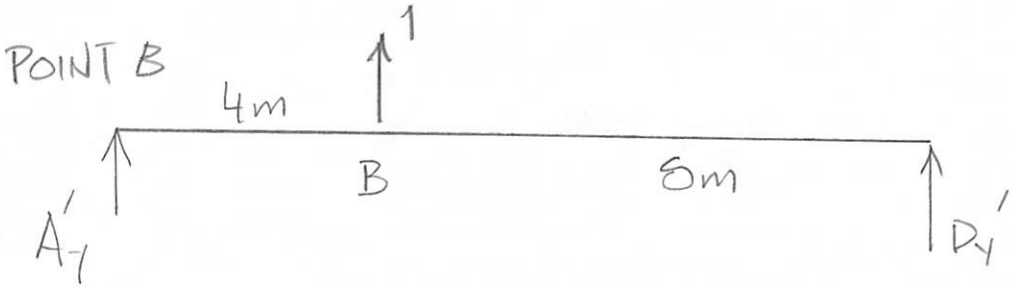
REAL BEAM

$$\circlearrowleft + \sum M_D = 0 = 48\text{kN}(10\text{m}) - A'_y(12\text{m})$$

$$A'_y = 40\text{kN}$$

$$+\uparrow \sum F_y = 0 = A'_y + D'_y - 48\text{kN}$$

$$D'_y = 8\text{kN}$$



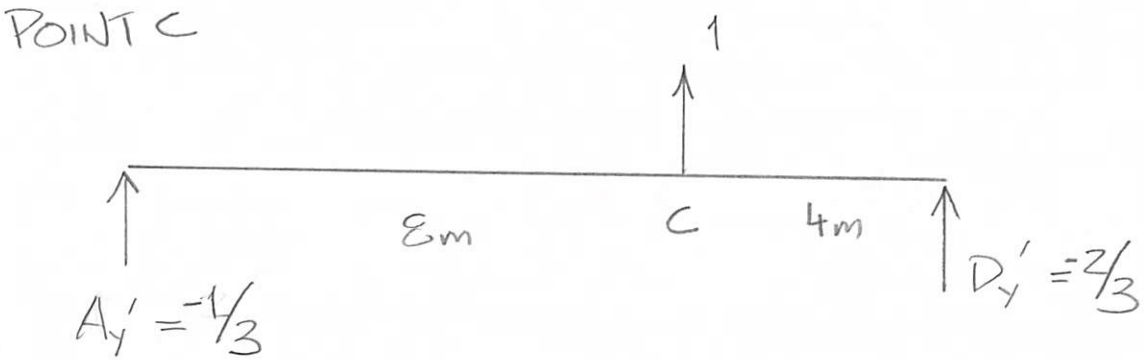
VIRTUAL BEAM

$$\circlearrowleft + \sum M_D = 0 = -1(8\text{m}) - A'_y(12\text{m})$$

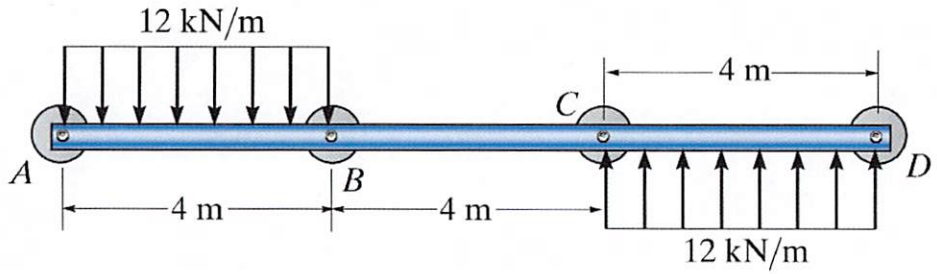
$$A'_y = -2/3$$

$$+\uparrow \sum F_y = 0 = A'_y + D'_y + 1$$

$$D'_y = -1/3$$



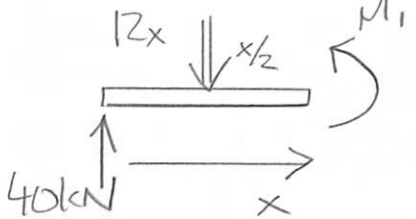
Problem 11-6 – Determine the support reactions. The supports at A, B, C, and D are pins. Assume the horizontal reactions are zero. EI is constant.



FIND Δ_{B_1}

REAL MOMENTS

$0 \le x \le 4$

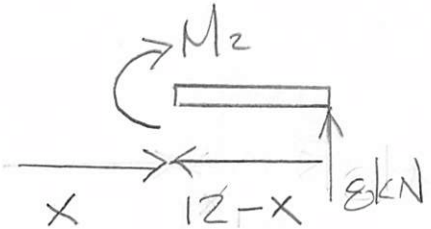


$$\circlearrowright \sum M_{CUT} = 0$$

$$= M_1 + 12x\left(\frac{x}{2}\right) - 40x$$

$$M_1 = \left[-6x^2 + 40x\right] kN\,m$$

$4 \le x \le 12$



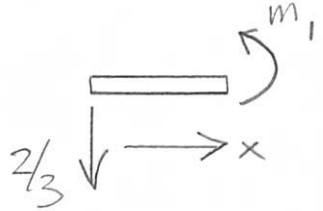
$$\circlearrowright \sum M_{CUT} = 0$$

$$= -M_2 + 8(12-x)$$

$$M_2 = \left[8(12-x)\right] kN\,m$$

VIRTUAL MOMENTS

$0 \le x \le 4$

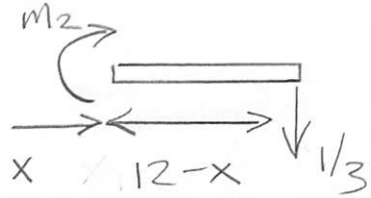


$$\circlearrowright \sum M_{CUT} = 0$$

$$= m_1 + \tfrac{2}{3}x$$

$$m_1 = -\tfrac{2}{3}x$$

$4 \le x \le 12$



$$\circlearrowright \sum M_{CUT} = 0$$

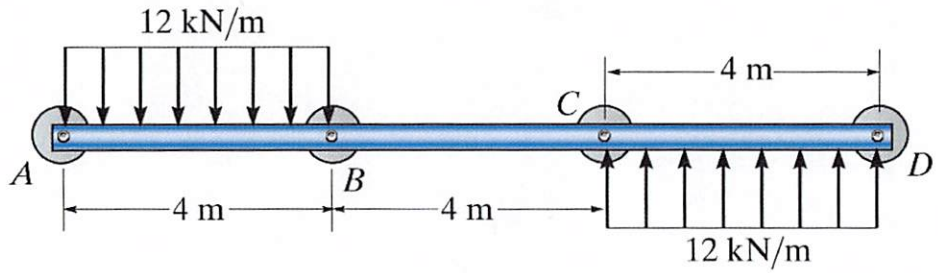
$$= -m_2 - \tfrac{1}{3}(12-x)$$

$$m_2 = -\tfrac{1}{3}(12-x)$$

$$\Delta_{B_1} = \frac{1}{EI}\left[\int_0^4 M_1 m_1\,dx + \int_4^{12} M_2 m_2\,dx\right] = \frac{1}{EI}\left[\int_0^4 \frac{2x}{3}\left(6x^2 - 40x\right)dx + \int_4^{12} -\frac{8}{3}(12-x)^2\,dx\right]$$

$$= \underline{-768\,kN\,m^3/EI}$$

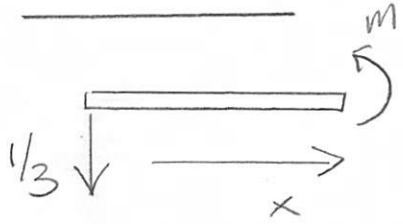
Problem 11-6 – Determine the support reactions. The supports at A, B, C, and D are pins. Assume the horizontal reactions are zero. EI is constant.



FIND Δ_{C_1}

VIRTUAL MOMENTS

$0 \le x \le 8$

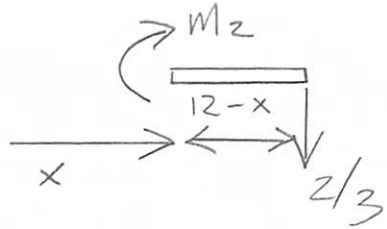


$$\circlearrowleft + \sum M_{cut} = 0$$

$$= m_1 + \tfrac{1}{3}x$$

$$m_1 = -x/3$$

$8 \le x \le 12$



$$\circlearrowleft + \sum M_{cut} = 0$$

$$= -m_2 - \frac{2}{3}(12-x)$$

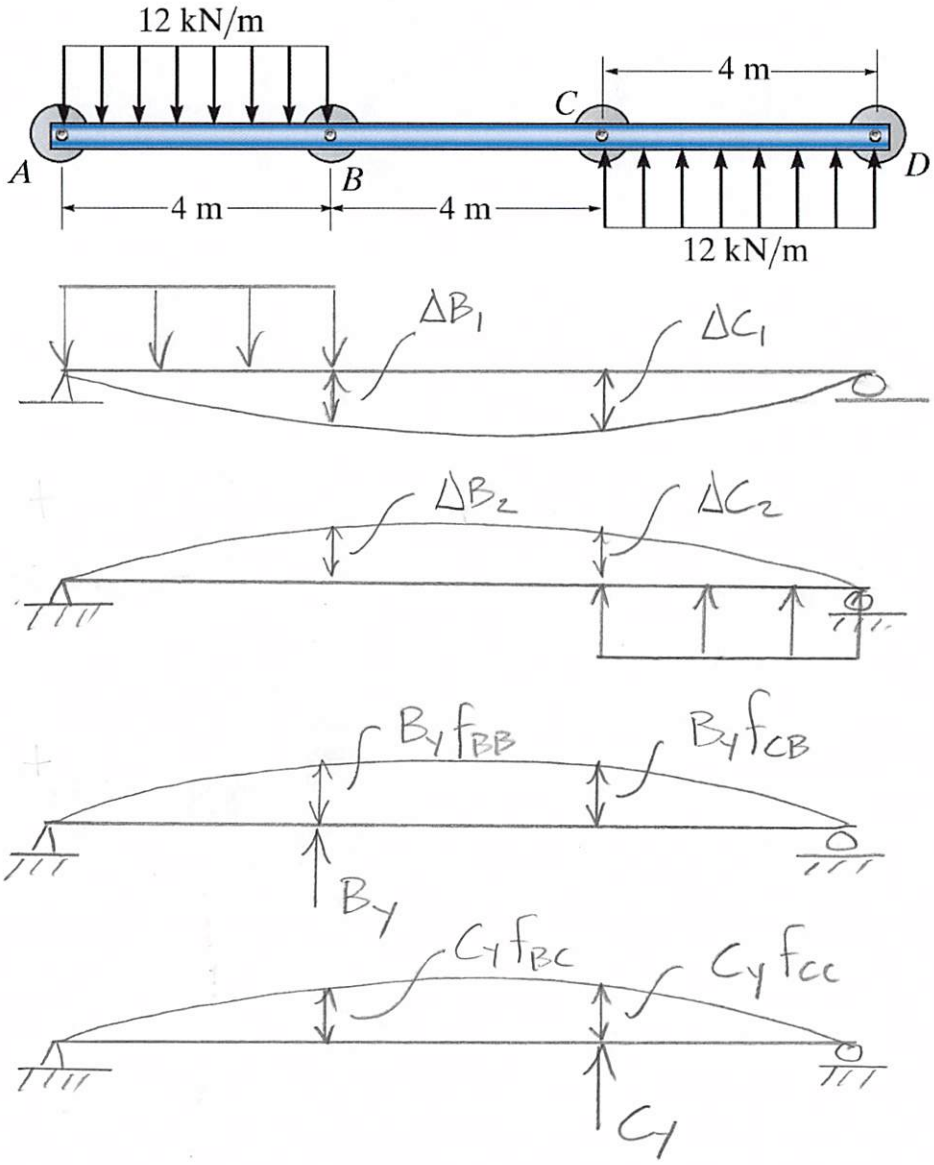
$$m_2 = -\frac{2}{3}(12-x)$$

$$\Delta_{C_1} = \frac{1}{EI}\left[\int_0^4 M_1 m_1\,dx + \int_4^8 M_2 m_1\,dx + \int_8^{12} M_2 m_2\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^4 \frac{x}{3}(6x^2-40x)\,dx + \int_4^8 -\frac{8x}{3}(12-x)\,dx + \int_8^{12} \frac{16}{3}(12-x)^2\,dx\right]$$

$$= -\frac{640\ \text{kNm}^3}{EI}$$

Problem 11-6 – Determine the support reactions. The supports at A, B, C, and D are pins. Assume the horizontal reactions are zero. EI is constant.



$$\Delta B_1 = -\Delta C_2 \qquad \Delta C_1 = -\Delta B_2$$

* FROM TABLE, WE CAN FIND f

$$f_{BB} = \frac{Pbx}{6LEI}\left(L^2 - b^2 - x^2\right)$$

$L = 12m,\ x = 8m,\ b = 4m,\ a = 8m$

$$f_{BB} = \frac{256\,m^3}{9EI}$$

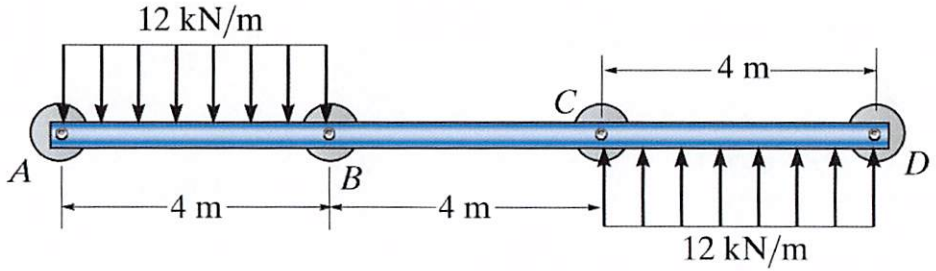
$$f_{CB} = \frac{Pbx}{6LEI}\left[L^2 - b^2 - x^2\right]$$

$L = 12m,\ x = 4m,\ b = 4m,\ a = 8m$

$$f_{CB} = \frac{224\,m^3}{9EI}$$

$$f_{CC} = f_{BB} \qquad f_{CB} = f_{BC}$$

Problem 11-6 – Determine the support reactions. The supports at A, B, C, and D are pins. Assume the horizontal reactions are zero. EI is constant.



$$\Delta_B = 0 = \Delta B_1 + \Delta B_2 + B_y f_{BB} + C_y f_{BC}$$

$$= \frac{1}{EI}\left[-768 + 640 + \frac{256}{9} B_y + \frac{224}{9} C_y\right] kNm^3 \quad (1)$$

$$\Delta_C = 0 = \Delta C_1 + \Delta C_2 + B_y f_{CB} + C_y f_{CC}$$

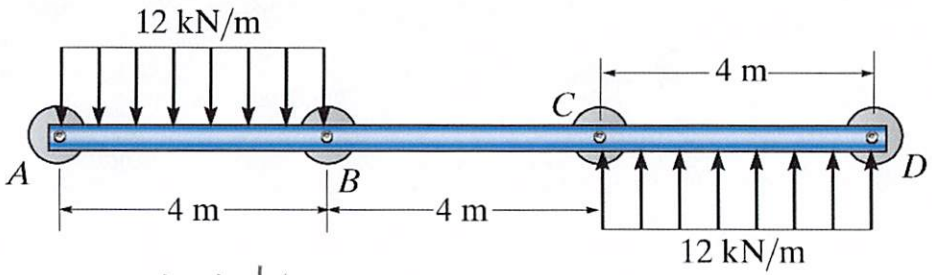
$$= \frac{1}{EI}\left[768 - 640 + \frac{224}{9} B_y + \frac{256}{9} C_y\right] kNm^3 \quad (2)$$

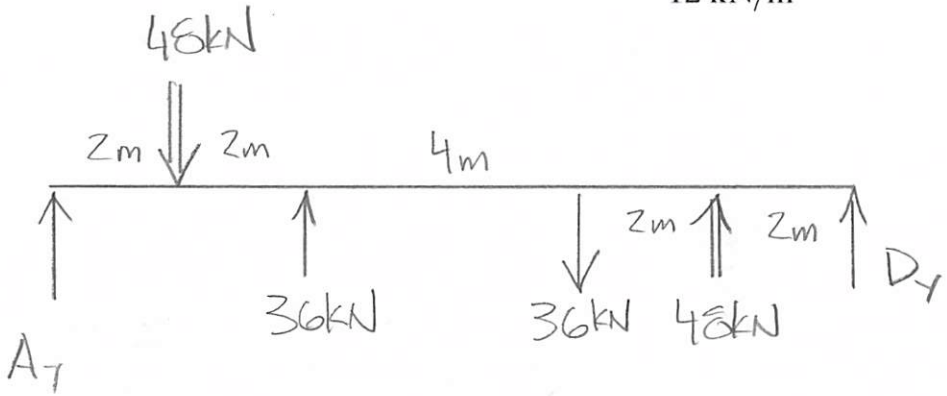
$(1) + (2) \Rightarrow \underline{C_y = -B_y}$

$(1) \Rightarrow 1{,}152 = 256 B_y + 224(-B_y) \Rightarrow 1{,}152 = 32 B_y$ $\quad \underline{B_y = 36 kN}$

$\underline{C_y = -36 kN}$

Problem 11-6 – Determine the support reactions. The supports at A, B, C, and D are pins. Assume the horizontal reactions are zero. EI is constant.





$$\overset{+}{\circlearrowleft}\Sigma M_D = 0 = 48kN(10m) - 36kN(8m) + 36kN(4m) - 48kN(2m) - A_y(12m)$$

$$\underline{\underline{A_y = 20kN}}$$

$$+\uparrow \Sigma F_y = 0 = A_y + D_y - 48kN + 48kN + 36kN - 36kN$$

$$D_y = -A_y$$

$$\underline{\underline{D_y = -20kN}}$$