CHAPTER 8 – Breakeven and Payback Analysis

Lecture 26
Breakeven Analysis for a Single Project

In many cases, the question is not "Is this project economically viable?" or "Should I choose Project A or Project B?" but "What needs to happen for this project to be economically viable" or "Under what circumstances would I choose Project A over Project B?" This is called ***breakeven analysis***.

In this lecture, we'll look at the former question, which deals with breakeven analysis of single projects. In the next lecture, we'll address the latter question, which deals with breakeven analysis between two alternatives.

In breakeven analysis, one or more of the problem parameters is represented by a variable or some function of a variable rather than by a constant. The goal is to determine the value of that variable (called the ***breakeven value***) that makes a relation involving that variable equal to zero. In other words, we're finding the roots of an equation.

This is a deliberately broad statement because breakeven analysis takes many different forms.

In Chapter 8.1 in the textbook, the authors use breakeven analysis to find the production volumes needed to make a profit in a manufacturing setting. The authors define profit as simply the difference between ***sales revenue*** (R) and ***total costs*** (TC):

$$\text{Profits} = \text{Revenue} - \text{Total Costs} = R - TC$$

The breakeven point is the production level that produces a profit of exactly zero:

$$R - TC = 0$$

In most manufacturing problems, the ***total costs*** are composed of both ***fixed costs*** (FC) and ***variable costs*** (VC):

$$TC = FC + VC$$

Fixed costs are costs that don't depend on the production levels. Things like mortgages, property taxes, rents, salaries of front-office staff, utilities, etc. These are often referred to as ***overhead costs***.

Variable costs are costs that vary with production levels. Things like raw materials, machining costs, the hourly wages and benefits of the machinists and assembly line workers.

We often approximate the variable costs as a linear function of the production volume. So if you double your production volume, you double your variable costs. We can write this as

$$VC = v\,Q$$

where v is the variable cost per unit of output and Q is the quantity of output.

In other cases, the relationship may be nonlinear. Economies of scale may make it cheaper to manufacture large quantities of an item rather than small quantities. Maybe you can get price breaks on the raw materials if you order them by the truckload.

Going the other direction, maybe an increase in production volume beyond a certain point will require you to add a second shift, thereby increasing your per-unit labor costs, at least until your sales increase to the point where the second shift is fully utilized.

Revenue, too, can be a linear or nonlinear function of production quantities. If the unit price is independent of the sales volume, then the revenue is a linear function of the volume:

$$R = r\,Q$$

Of course, the economies of scale apply to revenues, too. Manufacturers often offer price breaks to retailers who purchase in large quantities or, as with Walmart, the retailer demands a price break from the manufacturer to sell its products in large quantities.

If both the revenues and the variable costs are linear functions of the production volume, we can easily solve for the breakeven production volume:

$$R - TC = 0$$

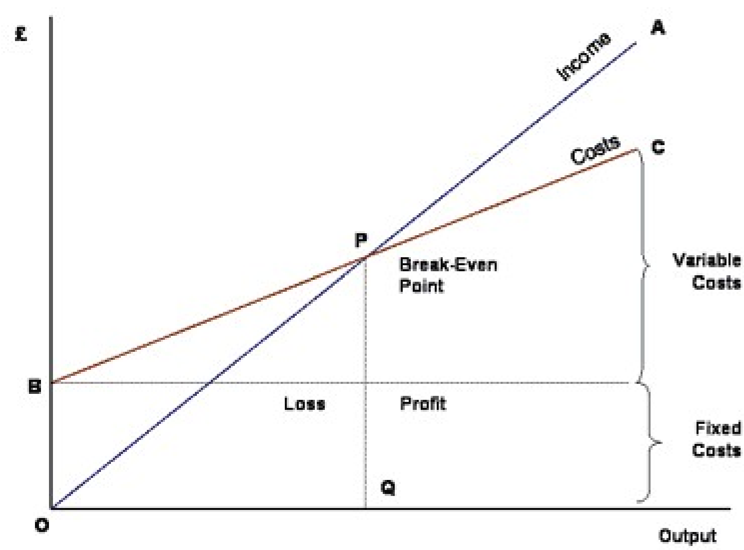
$$R - VC - FC = 0$$

$$r\,Q_{be} - v\,Q_{be} - FC = 0$$

$$(r-v)\,Q_{be} = FC$$

$$Q_{be} = \frac{FC}{r-v}$$

This can be shown graphically as follows:



Let's look at an example (although not strictly in a manufacturing setting):

Sue's Daycare Example 1

Sue has decided to start a day-care business. She has found a house to rent for $1100 per month. Utilities will cost her another $400 per month. She expects to spend $5 per child per day for breakfast, lunch and snacks plus another $0.50 per child per day for expendable supplies such as crayons, glue, paper, coloring books, and so on.

Sue thinks she can charge $18 per day and still be competitive. How many children must she enroll in order to break even?

Sue's fixed costs are her monthly rent and utilities:

$$FC = \$1100 + \$400 = \$1500$$

If we assume 20 weekdays per month then her monthly variable costs are

$$VC = (\$5.50/\text{day})\ (20\ \text{days})\ Q = \$110\ Q$$

and her monthly revenue is

$$R = (\$18.00/\text{day})\ (20\ \text{days})\ Q = \$360\ Q$$

So Sue's breakeven volume is

$$Q_{be} = \frac{\$1500}{\$360 - \$110} = 6$$

So Sue needs to watch 6 children per day to break even on her childcare business.

Now let's take this one step further:

Sue's Daycare Example 2

Sue has decided to start a day-care business. She has found a house to rent for $1100 per month. Utilities will cost her another $400 per month. She expects to spend $5 per child per day for breakfast, lunch and snacks plus another $0.50 per child per day for expendable supplies such as crayons, glue, paper, coloring books, and so on.

Sue thinks she can charge $18 per day and still be competitive. What is her annual pre-tax profit if she can look after 10 children per day?

Let's go back to our original equation and substitute the appropriate relations:

$$\text{Profits} = \text{Revenues} - \text{Total Costs}$$

$$\text{Profits} = r\,Q - FC - v\,Q$$

$$\text{Monthly Profits} = \$360\,Q - \$1500 - \$110\,Q$$

$$\text{Monthly Profits} = \$360\,(10) - \$1500 - \$110\,(10) = \$1000$$

$$\text{Annual Profits} = \$1000\,(12) = \$12{,}000$$

To complete this example, let's take the next step:

Sue's Daycare Example 3

Sue has decided to start a day-care business. She has found a house to rent for $1100 per month. Utilities will cost her another $400 per month. She expects to spend $5 per child per day for breakfast, lunch and snacks plus another $0.50 per child per day for expendable supplies such as crayons, glue, paper, coloring books, and so on.

If Sue can look after 12 children per day, how much must she charge to earn a pre-tax profit of $24,000 per year?

If Sue wants to earn $24,000 per year ($2000 per month) then

$$\text{Monthly Profits} = r\,Q - FC - v\,Q = \$2000$$

$$\text{Monthly Profits} = r\,(12) - \$1500 - \$110\,(12) = \$2000$$

$$12\,r = \$2000 + \$1500 + \$1320 = \$4820 \text{ / month}$$

$$r = \$400 \text{ / month / child}$$

Assuming 20 days per month, she must charge $400/20 = $20 per child per day.

Now let's look at a different application of breakeven analysis. This time we'll take a broader approach and include the time value of money.

NOTE: This is actually covered in the last paragraph of Section 8.1 of the textbook.

Our goal is to determine the value of some problem parameter (revenue, expenses, salvage value, initial cost) that makes the project earn exactly the MARR.

For example, how many widgets per year must I make to earn my MARR? How low must my labor costs be to earn my MARR? How high must my salvage value be to earn my MARR? What is the most I can pay for this machine and still earn my MARR?

In this case the relation being set equal to zero is the present worth or annual worth. Recall that if the present worth is exactly equal to zero, then the company earns exactly its MARR.

Let's go back to the Lockheed-Martin problem to see how this might work. When we first introduced the problem, we said:

> *Lockheed-Martin signed a contract with a European communications company that will pay them €3.9M per year for the next 8 years … what is their rate of return on this project?*

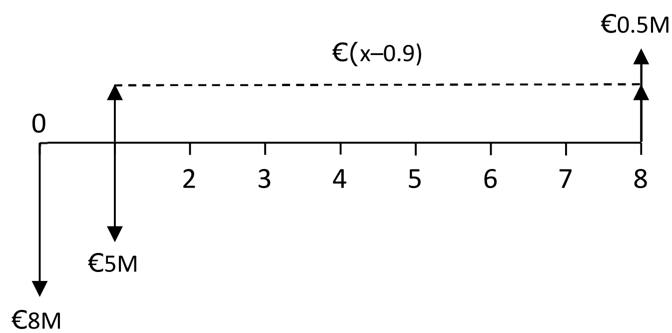
What if we turned this question around and asked, instead

> *How much would Lockheed-Martin have to charge the communications company each year to earn a return of 15% per year?*

To perform this break-even analysis, you need to determine the value of the parameter in question (in this case the annual charge) that makes the PW, FW, or AW exactly zero.

Recall that Lockheed-Martin was planning to invest €13M in a satellite tracking system (€8M at Time 0 and the remaining €5M during the first year of operation) and expected to spend €0.9M per year to operate the facility. The salvage value was estimated at €0.5M.

Let *x* denote the amount they will have to charge the European communications company in order to break even. Then the cash flow diagram looks like this:



The PW of the cash flows can be written as

$$PW = -8^M - 5^M \left(\overset{0.8696}{P \mid F, 15\%, 1}\right) + \left(x - 0.9^M\right)\left(\overset{4.4873}{P \mid A, 15\%, 8}\right) + 0.5^M \left(\overset{0.3269}{P \mid F, 15\%, 8}\right) = 0$$

Rearranging:

$$(x - €0.9^M)\ (4.4873) = €8^M + €5^M\ (0.8689) - €0.5^M\ (0.3269)$$
$$(x - €0.9^M)\ (4.4873) = €12.185^M$$
$$(x - €0.9^M) = €2.715^M$$
$$x = €3.615^M$$

So Lockheed-Martin needs to charge €3.615^M per year if they are to break even (i.e., to earn exactly their MARR).

Most of the time, the goal is not to break even at the end of the project. That doesn't leave much room for error. Most companies would like to make back their initial investment in 2-3 years. So a better question might be ***how much does Lockheed-Martin need to charge in order to break even by the end of Year 3?***

If we let *x* denote the annual revenue, then the PW can be written as

$$PW = -8^M - 5^M \overset{0.8696}{(P \mid F, 15\%, 1)} + \left(x - 0.9^M\right) \overset{2.2832}{(P \mid A, 15\%, 3)} = 0$$

Note that we've omitted the salvage value because it's beyond Year 3. Rearranging:

$$(x - €0.9^M)\ (2.2832) = €8^M + €5^M\ (0.8696)$$

$$2.2832\ (x - €0.9^M) = €12.348^M$$

$$(x - €0.9^M) = €5.408^M$$

$$x = €6.308^M$$

This is, of course, considerably more than in the previous example because we want to break even in three years instead of eight.