Lecture 23
Calculating Bond Yields and the True Cost of a Loan

In this lecture, we'll apply what we've learned about calculating rate of return to some things you'll encounter in your personal life. Specifically, we'll look at how to calculate the ***yield*** for a bond and the ***true cost*** of a loan whose repayment terms are specified but not the interest rate.

BONDS

Recall that a bond is a certificate of debt that is issued by a government or corporation to raise money. It has a ***face value*** that represents what the bondholder will get paid when the bond ***matures*** and a ***coupon rate*** that is used to calculate the semi-annual (typically) interest payments that the bondholder receives as long as he owns the bond.

Because the coupon rate is a nominal annual interest rate (an APR), the interest rate per coupon period is calculated as

$$i = \frac{r}{m}$$

where "r" is the annual coupon rate and "m" is the number of coupon dates per year.

The coupon amount is calculated by multiplying the face value of the bond (B_0) by the interest rate per coupon period:

$$\text{coupon amount} = B_0\left(\frac{r}{m}\right)$$

Remember that we said you can buy and sell bonds on the open market. If we buy a bond, we'd like to know what rate of return we'll earn if we hold the bond until it matures. This is called the ***yield to maturity*** or simply the ***yield***. It is usually expressed as a nominal annual interest rate (an APR) because bond coupon rates are usually expressed as nominal annual rates.

If you buy a bond the day it is issued and pay face value, then the yield is simply the coupon rate. The harder problem is to determine the yield if you buy the bond after it has been issued or pay something other than face value for the bond.

Let's look back at the railroad bond we introduced in Chapter 4.

Bond Example 2

This Louisville and Nashville Railroad bond was issued on October 1, 1968 and matures on September 30, 1993. The face value is $1000, the coupon rate is 7¾% per year and coupons are paid twice per year. Assume I purchased this bond for $945 on October 1, 1983. What will my yield to maturity be for this investment?

To answer that question, we need to calculate the interest rate that makes the present worth of all the cash flows equal to zero. The cash flows include (a) the amount I paid for the bond, (b) all of the coupons that I will cash in between the time I buy the bond and the day it matures, and (c) the face value that I will receive on the day the bond matures.

To begin, each coupon will be worth

$$\$1000\left(\frac{7.75\%\text{ per year}}{2\text{ coupons per year}}\right) = \$38.75\text{ per coupon}$$

Since there are 20 coupon periods remaining until maturity (remember, you have to work in coupon periods, not years), the present value of the cash flows can be written as

$$\text{PW} = -\$945 + \$38.75\left(\text{P}\mid\text{A}, i^*, 20\right) + \$1000\left(\text{P}\mid\text{F}, i^*, 20\right) = 0$$

To solve for i* we'll have to use trial-and-error. ***What would be a good first guess?***

Since the coupon rate is 3.875% per coupon period, that rate would be a good place to start. Unfortunately, there is no table for an interest rate of 3.875%, so let's make our lives easier by starting with a guess of 4% per coupon period, instead:

$$\text{PW} = -\$945 + \$38.75\underbrace{\left(\text{P}\mid\text{A}, 4\%, 20\right)}_{13.5903} + \$1000\underbrace{\left(\text{P}\mid\text{F}, 4\%, 20\right)}_{0.4564} = \$38.02$$

We're a bit high, so our rate of return must be a bit more than 4% per coupon period. The next higher table is for 5% interest, so let's try that next:

$$\text{PW} = -\$945 + \$38.75\underbrace{\left(\text{P}\mid\text{A}, 5\%, 20\right)}_{12.4622} + \$1000\underbrace{\left(\text{P}\mid\text{F}, 5\%, 20\right)}_{0.3769} = -\$85.19$$

This is negative, so we must earn less than 5% per coupon period, meaning the solution is somewhere between 4% and 5% per coupon period. We can approximate the answer using linear interpolation:

$$i^* = 4\% + \left(5\% - 4\%\right)\left[\frac{0 - 38.02}{-85.19 - 38.02}\right] = 4.31\%\text{ per coupon period}$$

So our ***yield to maturity*** (which, remember, is a nominal <u>annual</u> rate) is

4.31% per coupon period × 2 coupon periods per year = 8.62%

This is higher than the bond's coupon rate because we purchased the bond at a discount (below face value) so we earn a better return than had we purchased it at face value.

Bond prices and bond yields always move in opposite directions. As the price drops (due to changes in the bond rating or changes in the interest rate environment) the yield rises and vice versa.

TRUE COST OF A LOAN

Sometimes, you run across a loan where the interest rate is never specified, just the payment plan. The ***true cost of the loan*** is hidden.

This is commonly encountered in rent-to-own situations or situations where you're buying something on a payment plan ("Just six easy payments of $19.99!").

> Rent-to-Own Example 1
>
> Chris wants a new plasma-screen TV so he can watch his beloved Tigers play in the NCAA tournament. But Chris can't afford the $1350 price tag. A local rent-to-own company is advertising a March Madness Special of $30 per week. The manager tells Chris the TV will be his in just 60 weeks. The $30 per week rental cost fits into Chris' budget, so he signs the contract. What implicit annual interest rate is Chris paying to purchase this television?

Chris is, in effect, borrowing $1350 from the local rent-to-own company and paying back that loan at a rate of $30 per week over 60 weeks. The ***true cost*** of the loan is just the interest rate that makes the loan principal economically equivalent to the loan payments. In other words, it's the internal rate of return for the loan.

> Note that the "cost" in "true cost" is an interest rate, just as the "cost" in WACC is an interest rate.

Setting the present value of the loan payments equal to the cost of the television:

$$\$30\left(P \mid A, i^*, 60\right) = \$1350$$

We could solve this by trial-and-error as we did earlier in the chapter, but the simplicity of this equation allows us to take a short cut. Rearranging to get (P|A) on the left side:

$$\left(P \mid A, i^*, 60\right) = \frac{\$1350}{\$30} = 45$$

Looking through the interest factor tables in the appendix, we find

$$\left(P \mid A, 1.0\%, 60\right) = 44.9550$$

So the ***true cost*** of this loan is, for all intents and purposes, 1% per week.

What is the equivalent APR (remember that the interest rate on consumer loans must be reported as an APR, so if they advertised the true cost it would have to be an APR)?

$$\text{APR} = \left(1\% \text{ / week}\right)\left(52 \text{ weeks / year}\right) = 52\% \text{ per year}$$

This is actually pretty low for a rent-to-own purchase. Many of them run between 75% and 150% per year!

Let's look at another example. This one is from Memphis, TN.

Rent-to-Own Example 2

The ad says that the "90 Days Same As Cash" price is \$1799.99 or you can pay \$34.99 per week and, after 104 payments, you will own it. If you take the rent-to-own route, your payments will total 104 $\times$ \$34.99 = \$3638.96, which is twice the cash price!

As before, we'll set the cost of the TV at Time 0 (the "same as cash" price) equal to the Time 0 equivalent of all the loan payments and solve for the unknown i*:

$$\$34.99\left(\mathrm{P} \mid \mathrm{A}, \mathrm{i}^*, 104\right) = \$1799.99$$

$$\left(\mathrm{P} \mid \mathrm{A}, \mathrm{i}^*, 104\right) = \frac{1799.99}{34.99} = 51.44$$

Normally, we would look through the tables at the back of the book until we find one table with a P|A value slightly above 51.44 and an adjacent table with a value slightly below 51.44. Unfortunately, there is no row for n = 104. But there are rows for n = 100 and n = 108 and the P|A value for n = 104 would have to be somewhere in between.

Looking through the tables at the back of the textbook:

(P|A,1.0%,104) is somewhere between 63.0 and 65.9, which is too high.

(P|A,1.5%,104) is somewhere between 51.6 and 53.3, which is just slightly too high.

(P|A,2.0%,104) is somewhere between 43.1 and 44.1, which is too low.

So the answer is somewhere between 1.5% and 2.0% (but closer to 1.5% than 2.0%).

Using the formulas for P|A:

$$(P \mid A, 1.5\%, 104) = \frac{(1.015)^{104} - 1}{0.015(1.015)^{104}} = 52.49$$

$$(P \mid A, i^*, 104) = 51.44$$

$$(P \mid A, 2.0\%, 104) = \frac{(1.02)^{104} - 1}{0.02(1.02)^{104}} = 43.62$$

At this point we can just linearly interpolate.

BE CAREFUL! We're **not** looking for the interest rate that makes PW = 0; we're looking for the interest rate that makes (P|A,i*,104) = 51.44.

i	P|A
1.5%	52.49
i*	51.44
2.0%	43.62

Using (2%,43.62) as our "anchor" we can write

$$\frac{i^* - 2\%}{1.5\% - 2\%} = \frac{51.44 - 43.62}{52.49 - 43.62} = 0.8816$$

$$i^* - 2\% = 0.8816(-0.5\%)$$

$$i^* = 2\% + 0.8816(-0.5\%) = 1.559\% \text{ per week}$$

So what is the APR for this loan?

$$\text{APR} = 52 \text{ weeks per year} \times 1.559\% \text{ per week} = 81.1\% \text{ per year}$$

Note that we also could have solved this by setting PW=0 as we did at the start of the chapter, but we'd have to guess a value of i* out of the blue to start our search and hope that we were close. We wouldn't have the advantage of being able to quickly look through the tables in the back of the textbook to narrow our search.